PARAMETRIC DEA MODELS WITH WEIGHTS RESTRICTIONS

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Resumo: Neste artigo, discutimos uma metodologia de distribuição de um novo insumo ou produto a um conjunto de unidades tomadoras de decisão de tal forma que ao término da distribuição todas as unidades sejam consideradas eficientes no conceito DEA-CCR. A formulação segue o que a literatura vem chamando de DEA paramétrico. A contribuição deste artigo está na introdução da possibilidade de modelagem com a inclusão de restrições aos pesos relativos das variáveis (insumos e produtos) previamente existentes no problema.

Palavras-chave: DEA, DEA paramétrico, distribuição de insumos e produtos

Abstract: In this article, we discuss a method for distributing a new input or output to a set of decision making units so that by the end of the distribution all units are considered efficient in the DEA-CCR concept. The formulation follows the pattern referred in the literature as parametric DEA. This paper’s contribution is introducing the possibility of modeling with the inclusion of constraints on the relative weights of the variables (inputs and outputs) previously present in the problem.

Keywords: ENADE, statistical evaluation, assessment engineering education

1. Introduction

The allocation of scarce resources is an important problem faced by any kind of organization. Commonly, organizations are entities composed by a number of independent units, in the sense that these units have some freedom in determining their input and output levels. For example, within a university these units may correspond to the different departments, another case is the various distinct bank agencies that compose a banking organization.

An organization will have to set targets for the future performance of these individual units and, consequently, to execute a sound distribution of resources within these units in order to guarantee the feasibility to achieve the proposed targets.

Another possibility is that the organization has a new resource available. In this case, we can consider that a reasonable objective for the organization is to distribute this new resource across the units in a way to increase the overall efficiency of the organization.

Data envelopment analysis (DEA) is a well known technique used to evaluate the efficiency of decision making units (DMU`s) that perform similar activities, as well as to determine the benchmark units in the efficiency frontier and prescribing, quantitatively, the improvements needed for underperforming units to achieve efficiency. These units are assessed under Farrel's concept of efficiency (Farrel and Fieldhouse, 1962) by solving a set of linear programming problems.

The first DEA formulation, which became well known as the CCR model (Charnes et al, 1978), operates under constant returns to scale (CRS); on the other hand, the DEA formulation known as the BCC model (Banker et al, 1984) assumes variable returns to scale (VRS).

The classic DEA models make use of the hypothesis of freedom of action, in the sense that the levels of inputs and outputs can be increased or decreased infinitely. However, many situations appear in which this freedom does not exist. An easy to understand illustration regarding the evaluation of countries efficiency in the Olympic Games is presented at (Estellita Lins et al 2003), in which countries are DMU's generating medals as outputs and making use of a set of inputs formed by the country's wealth (GDP) and population. In order for an inefficient country to attain efficiency it is required for this country, *ceteris paribus*, to win more medals, but since the number of medals available is constant the consequence is that other countries will have to lose these same medals. Problems of this nature are called DEA models with constant sum of outputs.

In the above mentioned paper (Estellita Lins et al, 2003) the authors utilized the Zero Sum Gains (ZSG) DEA model, which can be used to allocate inputs or outputs. In the so called proportional strategy of the ZSG, a DMU wishing to reach the efficient frontier will have to increase (decrease) its output (input) levels and consequently, obligate the other DMU's to decrease (increase) its output (input) levels proportionally to their respective efficiencies (see also Gomes et al 2003 and Gomes and Estellita Lins, 2008).

The first DEA model with constant sum of inputs was put forward by Cook and Kress (Cook and Kress, 1999) in which they seek to make an equitable distribution of fixed cost among a set of DMU's assuming CRS and maintaining the efficiency of the DMU's unchanged after the resource allocation. The extension of this work to operate with different model orientations and the possibility to comprise the VRS hypothesis is presented in Cook and Zhu (Cook and Zhu, 2005).

Beasley (Beasley, 2003) proposed a method regarding the allocation of inputs or outputs that consists in the solution of a series of non-linear programming problems. In the formulation of Beasley, the individual units look for a distribution of the resources that maximize the average efficiency of the DMU's. Furthermore, in Beasley's method all inputs and outputs are considered and CRS or VRS assumption can be applied.

Lozano and Villa (Lozano and Villa, 2004) presented the concept of "centralized DEA" in which the objective is to make the DMU's achieve the efficient frontier and, simultaneously, reduce the total quantity of resource consumed by the units. In this model different orientations can be used under the VRS condition, highlighting that weight restrictions can be applied, but in this case there is no that all DMUs can be projected onto the efficient frontier (see also Lozano and Villa, 2005). A modification of this method made to permit only the adjustment of the inefficient DMU's is presented in Asmild et al (Asmild et al, 2009).

Avellar et al (Avellar et al, 2007) proposed the spherical frontier DEA model (SFM), in which a fixed input needs to be assigned to a set of DMU's in order to locate these units in an efficient frontier pre-defined to have a spherical shape and, for this reason, this model is said to be parametric differently from the other models of the literature. SFM assumes CRS and, by nature of the parametric imposition, guarantee that all the DMU's achieve strong efficiency, i.e., all units technically efficient and zero slacks. With this assumption, the problem of allocating a new input, which is known to be a difficult one, becomes relatively easy to be solved. Guedes (Guedes et al, 2009) proposed the adjusted spherical frontier DEA model (ASFM), an improvement over the SFM that provides the additional property known as the coherent redistribution property, an important property present only in the parametric model ASFM.

However an important shortcoming observed in the ASFM is related to its lack of capacity in incorporating *a priori* knowledge of the situation being modeled, the so called weight restrictions. Among a number of reasons that led to the development of weight restrictions presented in Allen et al (Allen et al, 1997) and Lins et al (Estellita Lins et al, 2006), we emphasize that a severe limitation of conventional DEA is the excessive freedom in the choice of the weights by the individual DMU's, resulting in units with inflated efficiency by selecting a set of weights that is inconsistent, because ignores variables considered significant in the model or questionable, because is incompatible with the expert knowledge available. Additional information on this topic can be obtained in (Pedraja-Chaparra et al, 1997) and (Angulo Meza and Estellita Lins, 2002).

The purpose of the model here proposed is to allow the employment of the ASFM in the presence of weight restrictions. In this model, a linear programming formulation that generalizes the ASFM algebraic formulation is developed in which the available weight restrictions theory can be directly applied, allowing the interaction Decision Maker-model.

Considering that the important problem of (re)distribution of resources is an open one, in the sense that none of the solutions of the literature considered possess a set of characteristics that dominate all of the others, we believe that the unique features contained in our model makes a relevant contribution to elucidate this subject.

This paper is organized as follows. In section 2, the ASFM-lp formulation is developed. In section 3 we present numerical examples to illustrate the application o the method. Finally, conclusions and proposals for future research are presented in section 5.

2. Theoretical Development

The Adjusted Spherical Frontier Model (ASFM)

In order to present the structure that allows the imposition of weight's restrictions to the ASFM, it is convenient to make a brief revision of the ASFM itself.

Let

s the number output measures

m the number of input measures

n the number of DMU's being evaluated

$y_{rj}\,(\geq 0)$ be the value of output r (r=1,2,...,s) for DMU j (j=1,2,...,n)

$x_{ij}\,(\geq 0)$ be the value of input i (i=1,2,...,m) for DMU j (j=1,2,...,n)

$F\,(\geq 0)$ be the total fixed input to distributed to all DMU's

$f_j\,(\geq 0)$ be the value of input to be assigned to DMU j (j=1,2,...,n)

Guedes (Guedes et al, 2009), showed that the fraction of the new input to be allocated to each DMU j in the problem with n DMU's, m inputs and s outputs is determined by:

$$f_j = \left(\frac{1}{R} \left(\sqrt{\sum_{r=1}^{s} \left(\frac{y_{rj}}{\sum_{j=1}^{n} y_{rj}} \right)^2} \right) - \sum_{i=1}^{m} \frac{x_{ij}}{\sum_{j=1}^{n} x_{ij}} \right) \frac{1}{m} \qquad (1)$$

In which R is the radius of the spherical efficiency frontier. We can see that the allocation of the new input is completely determined by the solution of a simple system containing $n \times n$ linear equations.

Observing the formulation of the ASFM model – equation (1) - one can perceive that it makes no mention to the input or output weights, meaning that the decision maker has no possibility to intervene in the formulation and thus cannot impose weights restrictions (*a priori* knowledge) on the model. That is a significant weakness of the ASFM algebraic formulation, since the introduction of weights restrictions is an important area of development in DEA. Differently to the standard ASFM, which is founded on the algebraic formulation, we propose an equivalent linear programming formulation, named ASFM-lp which allows the ASFM to operate under imposed weights restrictions.

In order to motivate our approach to the ASFM-lp model consider the simple example presented in Table 1. In that we have three DMUs and two outputs.

Table 1: Short example.

DMU	Output 1	Output 2
1	1	10
2	2	9
3	3	8
soma	**6**	**27**

Let's suppose now that we need to distribute a new input with constant sum F among the individual units. According to our idea of a fair distribution of inputs, all DMUs must have efficiency one after the distribution, i.e. it enables a DMU to have maximum efficiency when compared with its peers.

To proceed we introduce the following nomenclature:

u_{rj} be the weight related to output r (r=1,2,...,s) by DMU j (j=1,2,...,n)

v_{ij} be the weight related to input i (i=1,2,...,m) by DMU j (j=1,2,...,n)

e_{pj} be the (relative) efficiency of DMU j (j=1,2,...,n),

evaluated when using the weights of DMU p (p=1,2,...,n)

ε be a very small "non-Archimedean" number ($0<\varepsilon \ll 1$).

Note that the absolute efficiency of a DMU j is denoted by e_{jj}.

For the example presented in Table 1 we have that:

$$e_{11} = \frac{\frac{1}{F}\left(1u_{11}+10_{21}\right)}{f_1 v_{11}} = 1 \qquad (2)$$

$$e_{22} = \frac{\frac{1}{F}\left(2u_{12} + 9_{22}\right)}{f_2 v_{12}} = 1 \qquad (3)$$

$$e_{33} = \frac{\frac{1}{F}\left(3u_{13} + 8_{23}\right)}{f_3 v_{13}} = 1 \qquad (4)$$

$$f_1 + f_2 + f_3 = F \qquad (5)$$

$$0 \le e_{pj} \le 1, \qquad p=1,2,...,n; \;\; j=1,2,...,n; \;\; p \ne j \qquad (6)$$

$$u_{rj} \ge \varepsilon, \qquad r=1,2,...,s; \;\; j=1,2,...,n \qquad (7)$$

$$v_{ij} \ge \varepsilon, \qquad i=1,2,...,m; \;\; j=1,2,...,n \qquad (8)$$

The multiplication by the inverse of F in equations (2) to (4) is needed to compensate the values of the new inputs f_j, for they are defined to be a percentage of F.

Notice now that is possible to add a set of weights restrictions to the model. Traditional approaches to deal with value judgments such as the Assurance Region, Cone Ratio and Virtual Inputs/Outputs restrictions can be directly applied to our formulation. We will present a formulation with the Cone Ratio method presented in the work of Charnes et al (Charnes et al, 1990) because both the Assurance Region and Virtual Inputs/Outputs methods can be expressed in the Cone Ratio formulation.

In order to complete the formulation, we must guarantee that all DMUs are positioned in an efficiency frontier described by a spherical locus of points, condition that also guarantees the strong DEA efficiency for all DMUs. This can be accomplished by expressing equation (1) in the form of a linear programming problem.

In the light of our simple example and the comments previously made we can present the general approach for (re)distributing a new input to a set of DMU's via parametric DEA as follows:

$$\text{Minimize } W_{\max} - W_{\min} \qquad (9)$$

$$W_{\max} \ge \frac{m \cdot f_j + \sum_{i=1}^{m} \frac{x_{ij}}{\sum_{j=1}^{n} x_{ij}}}{\sqrt{\sum_{r=1}^{s}\left(\frac{y_{rj}}{\sum_{j=1}^{n} y_{rj}}\right)^2}} \ge W_{\min}, \qquad \forall j \qquad (10)$$

$$\sum_{j=1}^{n} f_j = 1 \qquad (11)$$

$$W_{\max} \geq \frac{1}{R} \geq W_{\min} \qquad (12)$$

$$e_{pj} = \frac{\frac{1}{F}\sum_{r=1}^{s} u_{rp} y_{rj}}{\frac{1}{F}\sum_{i=1}^{m-1} v_{ip} x_{ij} + f_j}, \qquad p=1,2,...,n; \quad j=1,2,...,n \qquad (13)$$

$$e_{jj} = \frac{\frac{1}{F}\sum_{r=1}^{s} u_{rj} y_{rj}}{\frac{1}{F}\sum_{i=1}^{m-1} v_{ij} x_{ij} + f_j} = 1, \qquad j=1,2,...,n \qquad (14)$$

$$0 \leq e_{pj} \leq 1, \qquad p=1,2,...,n; \quad j=1,2,...,n; \quad p \neq j \qquad (15)$$

$$u_{rj} \geq \varepsilon, \qquad r=1,2,...,s; \quad j=1,2,...,n \qquad (16)$$

$$v_{ij} \geq \varepsilon, \qquad i=1,2,...,m; \quad j=1,2,...,n \qquad (17)$$

$$\mathbf{u} = \sum_{q=1}^{s\times n} \mathbf{a}_q \mathrm{w}_q = \mathbf{Aw}, \qquad w_q \geq 0 \quad \forall q \qquad (18)$$

$$\mathbf{v} = \sum_{q=1}^{m\times n} \mathbf{b}_q \mathrm{z}_q = \mathbf{Bz}, \qquad z_q \geq 0 \quad \forall q \qquad (19)$$

In which

$$\mathbf{A} = \left(\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, ..., \mathbf{a}_k\right) \in R^{m\times n} \qquad (20)$$

$$\mathbf{B} = \left(\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3, ..., \mathbf{b}_l\right) \in R^{s\times n} \qquad (21)$$

The objective function in (9) seeks a solution with a unique radius for the efficiency frontier. Observe that if the objective function achieves the value of zero, i.e., a unique value for the radius represented in its inverse form by $W_{\max}$ and $W_{\min}$, then a spherical frontier was encountered. Notice too that this formulation provides some flexibility for the shape of the efficiency frontier in the sense that, case the problem has no possible spherical frontier solution the model seeks a near-spherical efficiency frontier. By near-spherical solution we mean one that minimizes the absolute difference between the inverse radiuses as described in equation (9).

It's important to highlight the fact that the loss of the spherical frontier implies the loss of the coherent redistribution property.

Equation (10) is simply a rearrangement of the algebraic formulation of the ASFM method described in equation (1) by means of the inverse of the radiuses.

Restriction (11) imposes that the distribution of the new input is given as a percentage of the total value of the new input F, which is necessary condition for the coherent redistribution property.

The Cone Ratio restrictions are expressed in equations (18) and (19).

The complete linear programming problem has $n\left[2\left(s+m\right)+1\right]+2$ variables and $n\left[n+2\left(s+m\right)+2\right]+1$ restrictions.

3. Numeric example

For illustrative purposes, we will apply our methodology to a model with restrictions imposed only on the outputs.

We consider the dataset presented in Table 1 where 2 outputs are given for 10 DMU's and a new input with constant sum of 100 units needs to be allocate among these DMU's.

With these data, the distribution of the new input is calculated via ASFM-lp without the introduction of weights restrictions and the result is summarized in Table 3.

A spherical frontier solution was achieved for this data set, fact that can be confirmed by the objective function value receiving value of zero. Looking for the pure weights value presented in Table 3 one can notice that two of ten DMUs (DMU 1 and 10) have showed weights' value equal to zero and that may be unacceptable to the decision maker.

Table 2: Dataset for the example

DMU	Output 1	Output 2
1	1	10
2	2	9
3	3	8
4	4	7
5	5	6
6	6	5
7	7	4
8	8	3
9	9	2
10	10	1
Sum	**55**	**55**

Table 3: Results without weights restrictions

DMU	Benchmarks	Output 1	Output 2	I_{new}
		Pure weights (u_{1j})	Pure weights (u_{2j})	
1	0	**0**	0,100	11,50
2	0	0,024	0,105	10,55
3	0	0,042	0,109	9,78
4	0	0,062	0,107	9,23
5	0	0,082	0,098	8,94
6	0	0,098	0,082	8,94
7	0	0,107	0,062	9,23
8	0	0,109	0,042	9,78
9	0	0,105	0,024	10,55
10	0	0,100	**0**	11,50
Sum				**100**
Objective Function: ($W_{max} - W_{min}$) = 0				

Suppose now that, after consultation, the specialists arrived at the conclusion that a reasonable set of bounds must be according to the one presented in inequality (22) below,

$$0,16 \leq \frac{u_{1j}}{u_{2j}} \leq 6,25 \qquad \forall j \tag{22}$$

Using these bounds, we executed again the ASFM-lp and obtained the results showed in Table 4. Again, a spherical solution was achieved and the weights restrictions satisfied. We can notice that the bounds on DMUs 0 and 10 are tight (indicated with bold figures).

Table 4: Results with weights restrictions.

DMU	Benchmarks	$\frac{u_{1j}}{u_{2j}}$	Output 1	Output 2	I_{new}
			Pure weights (u_{1j})	Pure weights (u_{2j})	
1	0	**0,160**	0,015	0,098	11,50
2	0	0,233	0,024	0,105	10,55
3	0	0,386	0,042	0,109	9,78
4	0	0,581	0,062	0,107	9,23
5	0	0,838	0,082	0,098	8,94
6	0	1,193	0,098	0,082	8,94
7	0	1,719	0,107	0,062	9,23
8	0	2,586	0,109	0,042	9,78
9	0	4,285	0,105	0,024	10,55
10	0	**6,250**	0,098	0,015	11,50
Sum					**100**
Objective Function: ($W_{max} - W_{min}$) = 0					

Now, let's consider the case in which the specialists concluded that the bounds on the weights should be equal the one presented in equation (23) that follows:

$$0,20 \leq \frac{u_{1j}}{u_{2j}} \leq 6,00 \qquad \forall j \tag{23}$$

Table 5, executing the ASFM-lp with restriction (23) results in a positive value for the objective function and so a spherical frontier cannot be achieved. All DMUs achieved DEA 100% efficiency, but DMU1 and DMU10 are not strongly efficient since both have benchmark DMUs. As said before, an additional implication of losing the parametric solutions is that the coherent redistribution property is no longer guaranteed.

The efficiencies were computed using the opens software EMS 1.3 (Scheel, 2000).

As saw in the previous example, a problem can arrive when using the ASFM-lp method, the infeasibility of the parametric frontier. Since the spherical frontier has only degree of freedom, the sphere radius, it has little flexibility on the imposition of weighs restrictions.

In order to overcome the "rigidity" of this parametric model (ASFM-lp), other efficiency frontier methods are being developed like the Ellipsoidal Frontier Method (EFM) (Milioni, 2010), a generalization of the ASFM model.

Table 5: Results with weights restrictions (No spherical solution).

DMU	Benchmarks	$\frac{u_{1j}}{u_{2j}}$	Output 1 Pure weights (u_{1j})	Output 2 Pure weights (u_{2j})	I_{new}
1	**2**	**0,200**	0,019	0,098	11,46
2	0	0,251	0,026	0,105	10,56
3	0	0,386	0,042	0,109	9,78
4	0	0,581	0,062	0,107	9,23
5	0	0,838	0,082	0,098	8,94
6	0	1,193	0,098	0,082	8,94
7	0	1,719	0,107	0,062	9,23
8	0	2,586	0,109	0,042	9,78
9	0	4,233	0,105	0,024	10,56
10	**9**	**6,000**	0,098	0,016	11,50
Sum					**100**
Objective Function: ($W_{max} – W_{min}$) = 0,002864					

4. Conclusions

In this paper we have presented a new formulation for the ASFM (Guedes et al, 2009), called ASFM-lp, a parametric DEA model to (re)distribute a new input based that is based on properties of the efficiency frontier. The ASFM-lp is a linear programming formulation that generalizes the algebraic formulation of the standard ASFM allowing the imposition of weights restrictions to this parametric model.

The introduction of weights restrictions in DEA models has an important reason to be imposed; it avoids DMUs to overestimate their efficiencies by means of attaching unacceptable weights to their inputs and/or outputs.

Important features of the ASFM-lp model comprise the low computational costs of solving large problem instances, due to its linear programming formulation and the strong DEA efficiency and coherent redistribution property not shared by other models of the literature.

Suggestion for future works includes: the development of a linear programming formulation for the Hyperbolical Frontier Model (HFM) developed by Avellar (Avellar et al, 2005), a parametric model to allocate outputs, in order to allow this method to work in the presence of weights restrictions and the extension of the method here proposed to deal with more flexible parametric models such as the Ellipsoidal Frontier Model (EFM) (Milioni, 2010).

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