Two-level multicriteria optimization of maintenance activities on power distribution networks

Fábio Luiz Usberti, Christiano Lyra Filho,
Celso Cavellucci, José Federico Vizcaino González
Faculdade de Engenharia Elétrica e de Computação - UNICAMP
Cidade Universitária Zeferino Vaz, Av. Albert Einstein, 400. CEP: 13083-852, Campinas - SP
e-mail: fusberti@yahoo.com

Abstract

Power distribution utilities must transport electric energy to attend all clients in a given network within specified targets for quality and reliability. The occurrence of a failure from one of the network components is the main factor which compromises the system reliability. Maintenance actions such as repairs and component replacements are employed to avoid power interruptions or resume the network healthy operation. This paper represents the relationship between maintenance activities and reliability as an optimization problem with two criteria: cost of maintenance activities and maximum value for the *system average interruption frequency index*, along the planning period. Solution of the problem for each local network gives the local *efficient solutions* and the associated *trade-off* curves. These solutions are optimally combined to solve higher levels multiple criteria optimization problems, unveiling *efficient solutions* and associated *trade-off* curves for each group of networks and for the whole company. The procedure makes the *hierarchical multiple criteria optimization of maintenance activities* which provides information to assess good decisions on maintenance activities at all decision levels of the network management. Application to both illustrative and real case studies certifies its benefits.

KEYWORDS: maintenance resources allocation, distribution networks reliability, multiple criteria optimization, hierarchical multiple criteria optimization, two-level maintenance resources optimization, power distribution networks.

1 Introduction

Given a power distribution network, the *hierarchical multiple criteria optimization of maintenance activities*, *HMO* for short, is to define the best compromise of maintenance policies for the network at both local and global levels, along a given period. The "best compromise" encodes a comprehensive representation of the search for the minimum cost in maintenance activities and the highest possible reliability for the networks.

The paper proposes to address the problem with an hierarchy optimization processes which mirrors the flow of informations and decisions in the management of utilities. Multiple criteria optimization at the local (lower) levels search the best compromise between the cost of maintenance activities and maximum value for the *system average interruption frequency index SAIFI*, along the planning period. Solutions of the problem for each local network give a set of local efficient solutions and the associated trade-off curves.

Maintenance policies defined at the upper management levels, for the utility as a whole, must take into account the available budget, minimum reliability requirements established by regulatory agencies (usually for a group of networks) and customer relationships. In order to assess these upper level decisions, local efficient solutions and associated trade-off curves are optimally combined to solve higher levels multiple criteria optimization problems, unveiling efficient solutions and associated trade-off curves for each group of networks and for the whole company.

Contrary to other multiple criteria decision problems on power systems that require agreements between different agents that strive to meet their objectives (Lyra and Ferreira 1995, Lyra et al. 1996), the decision process about maintenance policies studied in the paper is essentially internal to the utilities. It is supposed to exist a cooperative effort to allow both a reliable flow of informations in the bottom-up optimization process and in the top-down implementation of the decisions that should represent the best benefits for the utility.

At first glance, the *HMO* might be liken with a bilevel optimization problem (Vicente and Calamai 1994), where the set of feasible points of the upper level problem is given by the solution set of a lower level problem. The variables of the upper level are the parameters of the lower level problem and again the solutions of the optimization problem on the lower level influence the upper level objective function value.

A new area of research is the multiobjective bilevel optimization problem, where one or both levels hold a vector-valued objective function. For instance, Eichfelder (2010) proposes the first algorithm to solve bicriteria upper and lower level problems with a one-dimensional variable on the upper level. For that purpose, the author uses an scalarization technique combined with an adaptive parameter control.

A major difference between the *HMO* and the traditional multiobjective bilevel optimization problem is that *HMO* holds not one, but several multiobjective lower level problems which must be cooperatively combined into an efficient set for the upper level problem. In the best of our knowledge, no algorithm has been proposed for this sort of problem.

The research methodology used to solve the HMO consisted in modeling the problem in order to represent the process of deterioration which the components are subject to. This normative modeling allows maintenance decisions based on quantitative informations (Bertrand and Fransoo 2002). Analysis of the components failure rates are made according to the preventive maintenances applied. A multi-objective optimization technique was implemented in the search for non-dominated solutions, which comprehend the best strategies for maintenance actions.

The primary goal of the paper is to discuss the problem that motivated the design of *HMO* and to develop the theory and algorithms that make up the methodology. Section 2 formulates the multiple criteria decision problems at both local and higher levels; it also presents several necessary definitions. Section 3 presents the main theoretical results and solution strategies. Case Studies and Conclusions follow.

2 Problem Definition

Most power distribution networks operate with a radial configuration, which means that, using a graph terminology, the network can be represented as a tree rooted at a substation which provides a unique path to every section s. Each section attends a given number of clients and contains a set of electrical equipments (components) subject to failure. The occurrence of any equipment failure causes power supply interruption to the corresponding section and recursively to all of his offsprings sections. Fig. 1 and Fig. 2 illustrate how a power distribution network, for purposes of maintenance planning, may be organized as coupled sections, which include part of the primary and secondary networks (Costa et al. 2010). Table 1 gives some notation used throughout the mathematical formulation.

2.1 Mathematical Formulation of the Local Level Problems

The Multi-Objective Maintenance Resources Allocation Problem for a local network, MRAP, is defined for a planning horizon of $t = 1, \ldots, T$ years. The nature by which the equipments deteriorate or ameliorate along this time horizon, as they receive or not maintenance, is defined by a failure

Table 1: Notation.

x_e^t	binary decision variable of whether equipment e receives maintenance (1) or not (0).
δ_e^t	failure rate of equipment e in year t.
N_s	number of clients affected (including offsprings) by failure on section s.
E_s	set of equipments belonging to section s.
p_e	preventive maintenance cost for equipment e.
c_e	corrective maintenance cost for equipment e.
m_{e0}	failure rate multiplier for equipment e on lack of maintenance.
m_{e1}	failure rate multiplier for equipment e when maintenance is executed.
m_e^t	failure rate multiplier applied on equipment e failure rate in year t.

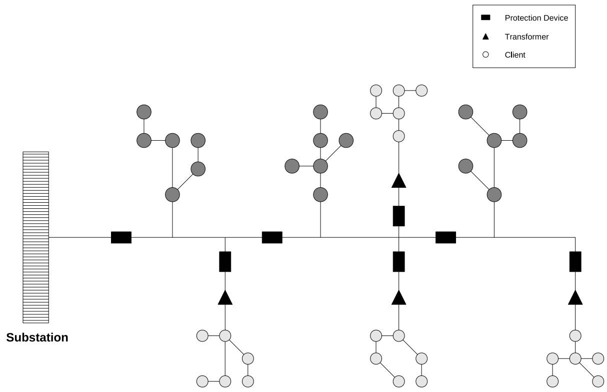


Fig. 1: A simple power distribution network.

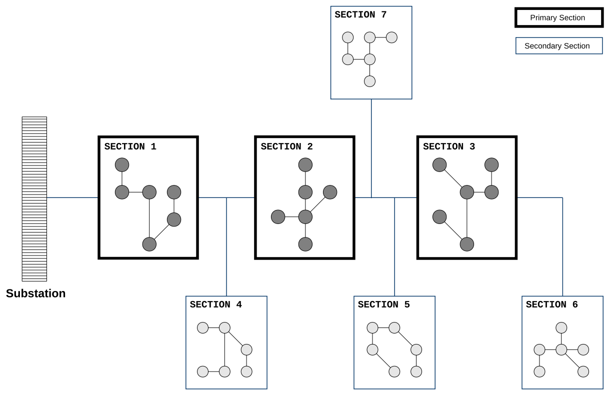


Fig. 2: The same power distribution network represented by sections.

rate model given by Eq. (1).

$$\delta_e^{t+1} = \delta_e^t m_e^t \tag{1}$$

The equipment failure rates are updated along the time horizon by the failure rate multiplier, according to the actions applied. When maintenance is carried out, the equipment failure rate is reduced (or sustained) by the multiplier m_{e1} ($0 < m_{e1} \leqslant 1$); otherwise the equipment failure rate is increased by m_{e0} ($m_{e0} > 1$).

One of the MRAP objectives is the maintenance cost, which is expressed in terms of preventive maintenance cost and corrective maintenance cost. The preventive cost during time t (PC^t) is the sum of all costs from the performed maintenance.

$$PC^t = \sum_s \sum_{e \in E_s} p_e x_e^t \tag{2}$$

The corrective cost during time t (CC^t) is the sum of every equipment corrective maintenance cost multiplied by its failure rate.

$$CC^t = \sum_s \sum_{e \in E_s} c_e \delta_e^t \tag{3}$$

The second MRAP objective is related to the network reliability, given by the *system average interruption frequency index* (Brown 2002), *SAIFI* for short, given by Eq. (4), which represents the expected number of failures for each client, per year.

$$SAIFI^t = \frac{\sum_s (N_s \sum_{e \in E_s} \delta_e^t)}{\sum_s N_s} \tag{4}$$

Considering the previous equations, the MRAP for a single local level network, such as the one illustrated in Fig. 1, can be summarized in the following bi-objective integer non-linear mathematical model:

(MRAP)

$$MIN \quad CT = \sum_t \frac{1}{(1+i)^t}(CC^t + PC^t) \tag{5}$$

$$MIN \quad SAIFI = \max_t (SAIFI^t) \tag{6}$$

st

$$m_e^t = x_e^t m_{e1} + (1 - x_e^t) m_{e0} \qquad \forall e, \forall t \tag{7}$$

$$\delta_e^t = \delta_e^{t-1} m_e^{t-1} \qquad \forall e, \forall t \tag{8}$$

$$x_e^t \in \{0, 1\} \qquad \forall e, \forall t \tag{9}$$

The first objective function (5) represents the maintenance total cost, adjusted by the present value, given an interest rate i. The second objective function (6) represents the maximum SAIFI obtained for the planning horizon. Constraints (7) and (8) represents the failure rate model. The binary constraints on the decision variables are expressed by (9). The initial failure rates δ_e^0 are presumed known for every equipment. The number of decision variables x_e^t grows according with the number of equipments and periods of the planning horizon ($|x_e^t| = |e||t|$). The same growth is observed in the number of constraints (7) and non-linear constraints (8).

The following definitions define the nature of the solutions that one should accomplish by solving the MRAP.

Definition 1 (Feasible Solution) *Each feasible solution* $\mathbf{x} = [x_e^t]\,(\forall e, \forall t)$ *for the MRAP has an image in the criteria space* $z(\mathbf{x}) = (CT(\mathbf{x}), SAIFI(\mathbf{x}))$*. Henceforth, each solution will be represented by the image* $z = (CT, SAIFI)$*.*

Definition 2 (Solution Set) *A solution set* $S = \{z_1, z_2, \ldots, z_p\}$ *contains* p *feasible solutions referring to a network with NC clients. Every network* i *will have its own solution set* $S^i = \{z_1^i, z_2^i, \ldots, z_{p^i}^i\}$ *and a given number of clients* NC^i*. Whenever possible, the solutions subindexes will be ommited.*

Definition 3 (Dominance) *A solution* $\bar{z} = (\overline{CT}, \overline{SAIFI})$ *dominates* $z = (CT, SAIFI)$ *if* $\overline{CT} \leqslant CT$ *and* $\overline{SAIFI} \leqslant SAIFI$*, with at least one of the inequalities being strict (Roy and Vincke 1981). This is represented by* $\bar{z} \prec z$*.*

Definition 4 (Nondominated Solution) *A solution* $z = (CT, SAIFI)$ *is nondominated (or efficient) in* S *if, and only if, there does not exist a solution* $\bar{z} \in S$ *such that* $\bar{z} \prec z$ *(Roy and Vincke 1981, Ehrgott 2005).*

When dealing with a multi-objective problem, such as the MRAP, it is expected to achieve a solution set S containing all (and only) nondominated solutions (efficient solutions). Notwithstanding the size of S can be theoretically infinite, in practical applications only a subset with a representative number of nondominated solutions is adequate.

2.2 Mathematical Formulation of the Higher Level Problem

Power distribution utilities, with several (electrically independent) networks, are interested in solving the MRAP for the power distribution system as a whole. Theoretically, in this case the MRAP model can still be used, by considering the collection of networks as a single entity. The fact that this instance will no longer be a tree but a forest is irrelevant to the model. This approach, however simple and intuitive, is impractical due to the scale of this instance. Also, it would not provide a realistic representation for the flow of information and decisions in a distribution utility.

In order to deal with these large scale instances that are formed by a group of networks, we formulated this higher level problem (which also benefits from a divide and conquer paradigm), where first we obtain the solution set for each network and compose this collection of sets into a single set of nondominated solutions for the entire power distribution system. This set represents the best possible global maintenance policies for the power distribution company considering the trade-offs between maintenance investment and system reliability. For this reason we call it the *Global Composite Solution Set* (GCSS). The following definitions present three new operators which will serve as building blocks for the formal definiton of the GCSS, and for the algorithm to efficiently determine it, presented in Section 3.2.

Definition 5 (Filter π) *We define the operation $\pi(S)$ that filters all the dominated solutions from the set S. That said, $\tilde{S} = \pi(S)$ implies that $\tilde{S} \subseteq S$, $\forall \bar{z} \in S$ if $\bar{z}$ is nondominated then $\bar{z} \in \tilde{S}$, and $\forall z \in S$ if z is nondominated then $z \notin \tilde{S}$.*

Definition 6 (Aggregation $\oplus$) *Consider two solutions $z^i = (CT^i, SAIFI^i) \in S^i$ and $z^j = (CT^j, SAIFI^j) \in S^j (i \neq j)$ (original solutions). The operator aggregation $\oplus$ under these solutions, generates a third solution z^k (aggregate solution) formed by the sum of the costs and the weighted average of the SAIFIs from the original solutions. In this sense, this aggregate solution represents the total amount of maintenance investment, and the average failure rate for both networks i and j.*

$$z^i \oplus z^j = (CT^i + CT^j, \frac{NC^i SAIFI^i + NC^j SAIFI^j}{NC^i + NC^j}) = z^k \tag{10}$$

Definition 7 (Composition $\otimes$) *Let two networks i and j with NC^i and NC^j clients have solution sets S^i and S^j (original sets), respectively. The operator composition $\otimes$ under these two sets generates a third solution set S^k (composite set) which contains all possible aggregated solutions produced from the original sets. In addition, the number of clients associated with this composite solution set will be naturally $NC^k = NC^i + NC^j$.*

$$\begin{aligned} S^i \otimes S^j = \{ & z_1^i \oplus z_1^j, z_1^i \oplus z_2^j, \ldots, z_1^i \oplus z_{p^j}^j, z_2^i \oplus z_1^j, z_2^i \oplus z_2^j, \ldots, z_2^i \oplus z_{p^j}^j, \ldots, \\ & z_{p^i}^i \oplus z_1^j, z_{p^i}^i \oplus z_2^j, \ldots, z_{p^i}^i \oplus z_{p^j}^j \} = S^k \end{aligned} \tag{11}$$

Definition 8 (Global Composite Solution Set) *The GCSS is the set of all global Pareto optimal (nondominated) solutions concerning a collection of n sets $\{S^1, S^2, \ldots, S^n\}$. This means that each solution from the GCSS is the result of the aggregation of n solutions, one from each set.*

$$GCSS = \pi(S^1 \otimes S^2 \otimes S^3 \otimes \cdots \otimes S^n) \tag{12}$$

3 Solution Strategy

3.1 Solution of the Local Level Problems

To solve the MRAP model, a classical multi-objective scalarization technique was used, the ε-*Constraint Method* (Ehrgott 2005), where given the multi-objective optimization problem, we minimize only one of the objectives, the maintenance cost, while the SAIFI is transformed into a constraint (14). Therefore, in this technique the original multi-objective model (MRAP) is converted into a mono-objective model (ε-MRAP):

(ε-MRAP)

$$MIN \quad CT = \sum_t \frac{1}{(1+i)^t}(C_c^t + C_p^t) \tag{13}$$

st

$$\max_t (SAIFI^t) \leqslant \varepsilon_k \tag{14}$$

$$m_e^t = x_e^t m_{e1} + (1 - x_e^t) m_{e0} \qquad \forall e, \forall t \tag{15}$$

$$\delta_e^t = \delta_e^{t-1} m_e^{t-1} \qquad \forall e, \forall t \tag{16}$$

$$x_e^t \in \{0, 1\} \qquad \forall e, \forall t \tag{17}$$

It can be verified in (14) that the SAIFI is no longer part of the objective, but, instead, constrained by the parameter ε_k. The ε-MRAP, despite having only one objective, remains difficult to be solved optimally, since it is a non-linear integer optimization problem. Nevertheless good solutions were attained heuristically by means of a genetic algorithm based on the work of Reis (2007). Some characteristics of this metaheuristic are listed in the following:

i. The representation of the individuals were maid through a vector of real numbers in which each position (locus) represents an equipment subject to maintenance, and the corresponding value represents the investment destinated to that component.

ii. The initialization of population (30 individuals) was made through a greedy constructive heuristic, where the individuals were generated considering only a restricted list of components that could receive maintenance investment based on their potential to improve the system reliability.

iii. The recombination operator, due to the decision of using a real vector representation, was an arithmetic crossover, where the offsprings atributes are generated as a linear combination of their parents atributes.

iv. The mutation operation (with 0.5% probability) is basically a random perturbation of the generated solutions through recombination.

v. The population update is made using a steady-state strategy, where only one individual is generated in each iteration, which replaces his worst parent.

vi. A local search is used to further improve the solution quality by reallocating investments between pairs of components.

To obtain the set of p solutions, the ε-MRAP is solved p times under distinct values of ε_k, where $\varepsilon_1 = SAIFI_{\max}$ and $\varepsilon_p = SAIFI_{\min}$. The values of $SAIFI_{\max}$ and $SAIFI_{\min}$ can be trivially calculated by considering $x_e^t = 0$ (not a single maintenance) and $x_e^t = 1$ (all possible maintenance) $(\forall t, \forall e)$, respectively. To distribute p values of ε_k uniformly between the $SAIFI$ extreme values, they are computed by Equation (18).

$$\varepsilon_k = SAIFI_{min} + \frac{(SAIFI_{max} - SAIFI_{min})(k-1)}{p-1} \qquad k = 1,\ldots,p \tag{18}$$

Thus, given a power distribution network, its solution set is obtained after solving p times the ε-MRAP model and filtering any nondominated solutions.

3.2 Solution of the Higher Level Problem

By definition, GCSS is the solution set resulting from the composition of a collection of n solution sets S^i $(i = 1,\ldots,n)$ after filtering only the nondominated solutions. However, determining GCSS by means of Equation (12) is a naive strategy. Since if we have p solutions in each set S^i, it would be necessary p^n aggregations to generate GCSS, which is highly impractical.

This work proposes a more efficient way to compute the GCSS. Before describing it, some properties of the operators $\oplus$ and $\otimes$ are provided.

Property 1 *The aggregation operator $\oplus$ is comutative.*

Proof:

$$\begin{aligned} z^i \oplus z^j &= \left(CT^i + CT^j, \frac{NC^i SAIFI^i + NC^j SAIFI^j}{NC^i + NC^j}\right) \\ &= \left(CT^j + CT^i, \frac{NC^j SAIFI^j + NC^i SAIFI^i}{NC^j + NC^i}\right) = z^j \oplus z^i \end{aligned} \tag{19}$$

■

Property 2 *The aggregation operator $\oplus$ is associative.*

Proof:

$$\begin{aligned} (z^i \oplus z^j) \oplus z^k &= \left((CT^i + CT^j) + CT^k, \frac{NC^{ij}\left(\frac{NC^i SAIFI^i + NC^j SAIFI^j}{NC^i + NC^j}\right) + NC^k SAIFI^k}{NC^{ij} + NC^k}\right) \\ &= \left(CT^i + (CT^j + CT^k), \frac{NC^i SAIFI^i + NC^{jk}\left(\frac{NC^j SAIFI^j + NC^k SAIFI^k}{NC^j + NC^k}\right)}{NC^i + NC^{jk}}\right) = z^i \oplus (z^j \oplus z^k) \end{aligned}$$

■

Property 3 *The operator $\otimes$ is comutative and associative.*

Proof: Given that the aggregation $\oplus$ is both comutative and associative, extending these properties to $\otimes$ is trivial. ■

The conditions by which a solution belonging to a composite set is nondominated are the objects of interest for Lemmas (1) and (2).

Lemma 1 (Necessary condition for an aggregated nondominated solution) *Let $z^k \in S^k$ be an aggregate solution from $z^i \in S^i$ and $z^j \in S^j$ $(z^k = z^i \oplus z^j,\ S^k = S^i \otimes S^j)$. A necessary condition for z^k to be nondominated in S^k is that z^i and z^j are also nondominated in S^i and S^j, respectively.*

Proof: By contradiction, suppose that one of the original solutions that generates the nondominated solution z^k, for instance z^i, is dominated. In that case there exist a solution $\overline{z}^i$ which dominates z^i.

$$\overline{z}^i \prec z^i \Rightarrow \begin{cases} \overline{CT}^i \leqslant CT^i \\ \overline{SAIFI}^i \leqslant SAIFI^i \\ \overline{z}^i \neq z^i \Rightarrow (\overline{CT}^i < CT^i) \vee (\overline{SAIFI}^i < SAIFI^i) \end{cases}$$

Let $\overline{z}^k$ be the aggregate solution from $\overline{z}^i$ and z^j ($\overline{z}^k = \overline{z}^i \oplus z^j$). Comparing solutions z^k and $\overline{z}^k$:

$$\begin{cases} \overline{CT}^k = \overline{CT}^i + CT^j \leqslant CT^i + CT^j = CT^k \\ \overline{SAIFI}^k = \frac{NC^i\overline{SAIFI}^i + NC^jSAIFI^j}{NC^i+NC^j} \leqslant \frac{NC^iSAIFI^i + NC^jSAIFI^j}{NC^i+NC^j} = SAIFI^k \end{cases}$$
$$\Rightarrow \overline{z}^k \prec z^k$$

However, if $\overline{z}^k$ dominates z^k then it is a contradiction with the hypothesis that z^k is nondominated. The contradiction is a consequence of the assumption that z^i was dominated.

■

Lemma 2 (Non-sufficient condition for an aggregated nondominated solution) *Let $z^k \in S^k$ be an aggregate solution from $z^i \in S^i$ and $z^j \in S^j$ ($z^k = z^i \oplus z^j$, $S^k = S^i \otimes S^j$). It is not a sufficient condition that z^i and z^j are nondominated in S^i and S^j (respectively) for z^k to be nondominated in S^k.*

Proof: A simple procedure to produce a dominated solution from the aggregation of two nondominated solutions will be given. Consider two nondominated solutions z^i_1 and z^i_2 in S^i, and two nondominated solutions z^j_1 and z^j_2 in S^j.

$$\left.\begin{aligned} z^i_1 = (CT^i_1, SAIFI^i_1) \\ z^i_2 = (CT^i_2, SAIFI^i_2) \end{aligned}\right\} \quad (CT^i_1 < CT^i_2, SAIFI^i_1 \geqslant SAIFI^i_2)$$
$$\left.\begin{aligned} z^j_1 = (CT^j_1, SAIFI^j_1) \\ z^j_2 = (CT^j_2, SAIFI^j_2) \end{aligned}\right\} \quad (CT^j_1 < CT^j_2, SAIFI^j_1 \geqslant SAIFI^j_2)$$

Aggregating solution z^i_1 with z^j_2 to generate z^k_1, and solution z^i_2 with z^j_1 to generate z^k_2.

$$z^k_1 = z^i_1 \oplus z^j_2 = \left(CT^i_1 + CT^j_2, \frac{NC^iSAIFI^i_1 + NC^jSAIFI^j_2}{NC^i + NC^j}\right)$$
$$z^k_2 = z^i_2 \oplus z^j_1 = \left(CT^i_2 + CT^j_1, \frac{NC^iSAIFI^i_2 + NC^jSAIFI^j_1}{NC^i + NC^j}\right)$$

Now suppose that z^k_1 dominates z^k_2:

$$z^k_1 \prec z^k_2 \Rightarrow \begin{cases} CT^i_1 + CT^j_2 \leqslant CT^i_2 + CT^j_1 \\ \dfrac{NC^iSAIFI^i_1 + NC^jSAIFI^j_2}{NC^i + NC^j} \leqslant \dfrac{NC^iSAIFI^i_2 + NC^jSAIFI^j_1}{NC^i + NC^j} \\ z^k_1 \neq z^k_2 \end{cases}$$
$$\Rightarrow \begin{cases} CT^j_2 - CT^j_1 \leqslant CT^i_2 - CT^i_1 \\ NC^i(SAIFI^i_1 - SAIFI^i_2) \leqslant NC^j(SAIFI^j_1 - SAIFI^j_2) \\ z^k_1 \neq z^k_2 \end{cases} \tag{20}$$

The conditions expressed by (20) are feasible. Therefore, Lemma 2 gives a procedure to generate a dominated solution through aggregation of two nondominated solutions. Also, it gives the conditions that lead to z_1^k dominating z_2^k.

■

Property 3 with Lemmas 1 and 2 allows to draw the following conclusions.

i. The result from one or several compositions is independent of the order by which the solution sets are composed.

ii. To obtain a nondominated solution of a composite set it is necessary to aggregate nondominated solutions from the original sets. Therefore, dominated solutions from the original sets can be discarded.

iii. Aggregating two nondominated solutions is not a sufficient condition to generate a nondominated solution for the composite set. Hence, many dominated solutions may be generated through a composition, even if the original sets contain only nondominated solutions.

Based on these properties, Theorem 1 describes an alternative method to determine the GCSS, which is more efficient when compared to using the definition (12).

Theorem 1 (Alternative strategy to determine GCSS) *Given a collection of n solution sets S^i ($i = 1, \ldots, n$), the GCSS can be determined by iteratively composing and filtering these sets two-by-two, in any order, until only one set remains, that one being the GCSS.*

$$GCSS = \pi(\cdots\pi(\pi(S^1 \otimes S^2) \otimes S^3)\cdots \otimes S^n) \tag{21}$$

Proof:

The GCSS is defined as the set of global Pareto optimal (nondominated) solutions result from the composition of the original solution sets. To demonstrate that Equation (21) can successfully determine GCSS, it must be shown that the π filters applied on each composition do not lose any solution in GCSS. This is guaranteed by Lemma 1, since nondominated solutions from a composite set are always generated by nondominated solutions from the original set. Thus removing dominated solutions after the composition of two sets does not interfere in the result of the suceeding compositions.

■

3.3 On the Complexity of Determining GCSS

In practice, Equation (21) is much more efficient in determining the GCSS than by Equation (12), since several dominated solutions are filtered during the process, which in turn can dramatically reduce the computational effort. However, an algorithm implementing Equation (21) will also have a worst-case complexity of $O(p^n)$. This is true for the reason that the GCSS may theoretically contain p^n nondominated solutions (i.e., all solutions are nondominated); hence, the simple enumeration of the solutions would be exponential. For these pathological cases, this work proposes an $O(np^2)$ approximate algorithm to find the GCSS. In this algorithm, the solution sets are still composed and filtered two-by-two; however a limited number of p nondominated solutions are carried out on each composite set. The criteria to select these p nondominated solutions can be, for instance, uniform distribution, lowest costs or lowest SAIFIs. Once again, this algorithm is iterated until only one set remains, which will be an approximation of the GCSS. This algorithm requires at most $(n-1)p^2$ aggregations; therefore, its complexity grows polynomially with the size of the instance.

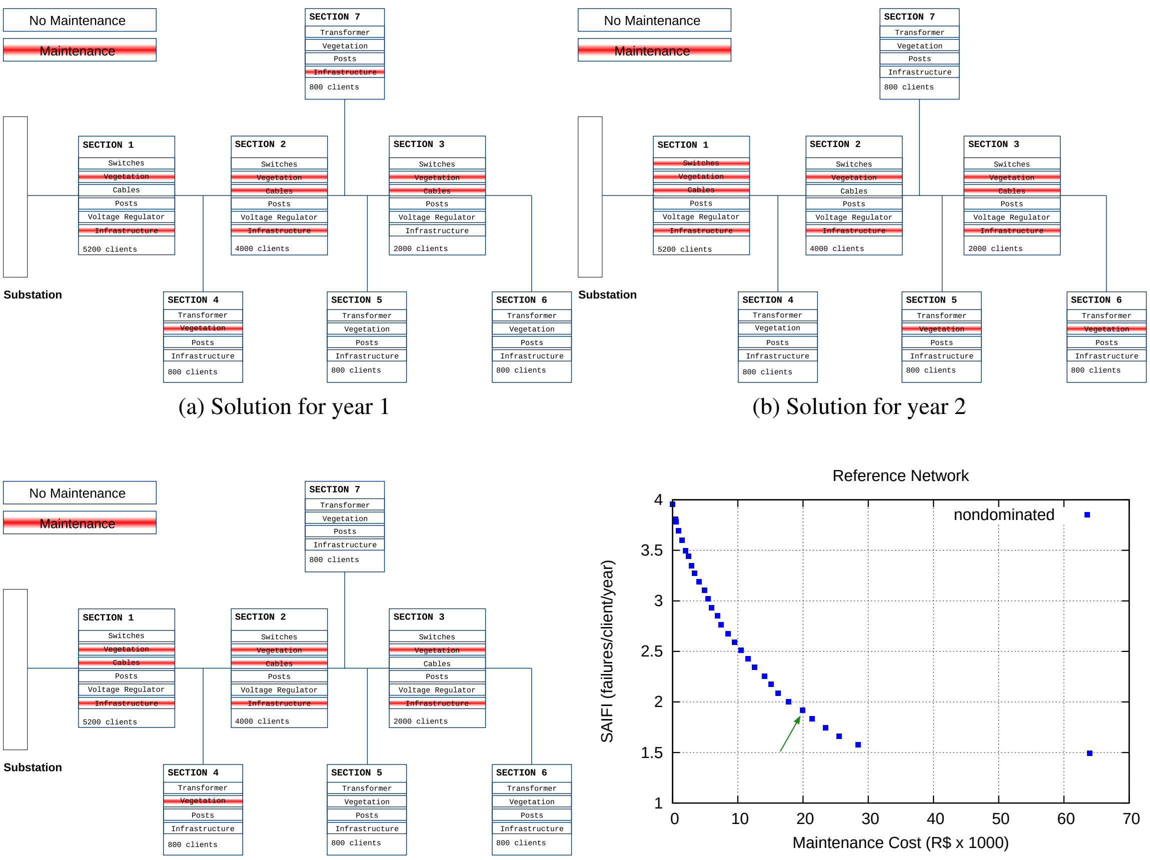
4 Case Studies

On this Section, the ε-MRAP model has been solved to one small idealized network, based on the instance used by Sittithumwat et al. (2004), using the genetic algorithm discussed in Section 3.1. Three real networks have also been solved, and their solution sets composed using Equations (12) and (21) to determine the GCSS. All algorithms were implemented in Java, and the computational tests executed in a Intel Quad Core 3.0Ghz, 4 Gb RAM.

4.1 Case Studies of the Local Level Problems

The MRAP was applied to a small network, called reference network, already depictured in Fig. 1. This network is formed by eight sections (3 primary and 5 secondary), 34 components, and 5200 clients. The ε-MRAP was solved by a genetic algorithm earlier described on Section 3.1, assuming 30 uniformly distributed values of $\varepsilon = SAIFI \in [1.49, 3.96]$. The genetic algorithm found one nondominated solution at every iteration, taking 5.74 seconds of CPU time.

Fig. 3d shows the set of nondominated solutions for the reference network. Each one of these correspond to a maintenance policy for the reference network, thus we can find trade-off policies which go from zero cost (simply allowing the natural degradation of the components) to almost $R\$64,000.00$ (intensive maintenance on every component). With respect to the system reliability, there are solutions which have an expected number of failures down from 1.5 up to 4.0 per client per year.



(a) Solution for year 1

(b) Solution for year 2

(c) Solution for year 3

(d) Trade-off curve

Fig. 3: Local level solution for the reference network.

To further illustrate what each solution of the set represents, Fig. 3d has a marked solution

$z = (19.9, 1.92)$, for which the complete 3-years maintenance policy is described in Fig. 3a, 3b and 3c. In these figures, the components are displaced according to the section they belong. The number of clients represents not only the clients residing in the section, but also includes all the clients affected by an eventual power interruption on it. The highlighted components were chosen by the optimization process to go under maintenance. A quick observation is that mainly the components on the primary sections have been selected for maintenance, which can be explained by the greater number of clients depending on the good operation of these components.

4.2 Case Studies of the Higher Level Problems

The object of study for determining the Global Composite Solution Set (GCSS) was a group of three real large scale distribution networks, refered as Network 1, 2, and 3. The characteristics of these network are shown in Table 2.

Table 2: Characteristics of Netwoks 1, 2, and 3.

	Network 1	Network 2	Network 3
Vegetation	178	447	820
Posts	178	447	820
Cables	178	447	820
Infrastructure	178	447	820
Transformers	52	269	200
Switches	1	4	8
Total	765	2061	3488
Sections	178	447	820
Clients	4513	18268	25466

The genetic algorithm solved 30 iterations of the ε-MRAP for each network separately, thus obtaining three solution sets, depicted in Fig. 4. Some of the solutions from these sets are dominated, what is undesirable. However, it is expected that this can happen once in a while, since we are not in the possession of an exact algorithm. The CPU time necessary to generate these solution sets were 104, 653 and 1840 seconds, for Network 1, 2, and 3, respectively.

Equation (21) was implemented to obtain the GCSS for the three solution sets. It starts by composing two solution sets, and the filtered result was then composed with the third solution set. The GCSS emerged after filtering the result from the second composition; it can be visualized as the nondominated points of the composite solution set in Fig. 4. The CPU time of 2.35 seconds to generate the GCSS was practically irrelevant, compared to the genetic algorithm CPU time. In order to exemplify how Equation (21) is computationally more efficient than Equation (12) in practical situations, we have also used the Eq. (12) to determine the same GCSS, taking 4.543 seconds of CPU time, say 93.3% higher then the previous strategy, for composing only three networks.

Fig. 4 illustrates that every solution in the composite set results from the aggregation of the original solutions, one from each original set. In addition, several dominated solutions were generated during the compositions (also shown in Fig. 4), which is consistent with Lemma 2.

5 Conclusions

This paper proposed the *hierarchical multiple criteria optimization of maintenance activities* (*HMO*), which allows to define the best compromise of maintenance policies for power distribution networks at both local and global levels, along a given period. The best compromise is obtained by evaluating *efficient solutions* and associated *trade-off* curves that consider the search for the

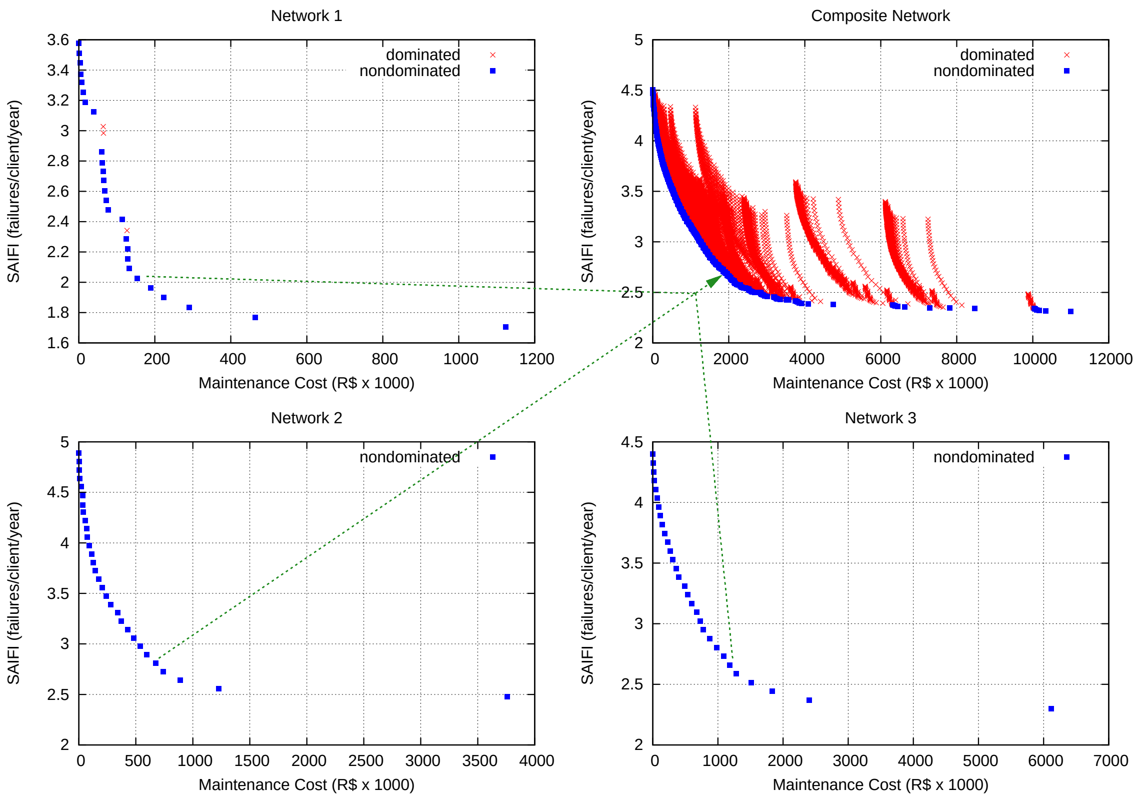


Fig. 4: Composition of the solution sets from Network 1, 2 and 3.

minimum cost in maintenance activities and the highest possible reliability for the networks. The development of the approach was motivated by a decision problem faced by distribution utilities in Brazil, where reliability indicators, such as maximum values for the system average interruption frequency index (*SAIFI*), are establish and supervised by the Brazilian Electricity Regulatory Agency, for each group of networks operated by a distribution company in a geographical area.

Solutions of the problem for each network give local efficient solutions and the associated trade-off curves. Local solutions are optimally combined to solve the higher level multiple criteria optimization problem, unveiling efficient solutions and associated trade-off curves for the whole company. This way, the flow of informations in a bottom-up multiple criteria optimization procedure provide the necessary information to allow well informed decisions about maintenance policies for the utility as a whole. The top-down flow of these decisions meets the previously defined trade-off curves and associated efficient solutions, which indicate the best use of maintenance resources at all local networks. To sum up, the methodology provides information to assess good decisions on maintenance activities at all decision levels of a power distribution utility.

6 Acknowledgements

The authors kindly acknowledge support from the Brazilian National Council for Scientific and Technological Development (CNPq), "Companhia Paulista de Força e Luz" (CPFL-Paulista) and "Companhia Piratininga de Força e Luz" (CPFL-Piratininga). They would also like to thank helpful comments from their colleagues Antônio Claudinei Simões (CPFL), Fernando José Von Zuben (UNICAMP), Hamilton Melo Ferreira (UNICAMP) and Guilherme Palermo Coelho (UNICAMP).

References

Bertrand, J. and Fransoo, J.: 2002, Operations management research methodologies using quantitative modeling, *International Journal of Operations & Production Management* **22**(2), 241–264.

Brown, R. E.: 2002, *Electric Power Distribution Reliability*, CRC Press, New York.

Costa, A. M., França, P. M. and Lyra, C.: 2010, Two-level network design with intermediate facilities: an application to electrical distribution systems, *OMEGA* . DOI: 10.1016/j.omega.2010.01.005.

Ehrgott, M.: 2005, *Multicriteria Optimization*, 2nd edn, Springer, Heidelberg.

Eichfelder, G.: 2010, Multiobjective bilevel optimization, *Mathematical Programming* **123**(2), 419–449.

Lyra, C., Castro, R. and Ferreira, L. R. M.: 1996, Assessing decisions on multiple uses of water and hydroelectric facilities, *International Transactions in Operational Research* **3**(3-4), 281–292. DOI: 10.1111/j.1475-3995.1996.tb00053.x.

Lyra, C. and Ferreira, L. R. M.: 1995, A multiobjective approach to the short-term scheduling of a hydroelectric power system, *IEEE Transactions on Power Systems* **10**(4), 1750–1755. DOI: 10.1109/59.476038.

Reis, P. A.: 2007, *Reliability based optimization of maintenance plans for power distribution systems*, Master's thesis, Campinas State University. In Portuguese.

Roy, B. and Vincke, P.: 1981, Multicriteria analysis: Survey and new directions, *European Journal of Operational Research* **8**(3), 207–218. DOI: 10.1016/0377-2217(81)90168-5.

Sittithumwat, A., Soudi, F. and Tomsovic, K.: 2004, Optimal allocation of distribution maintenance resources with limited information, *Electric Power Systems Research* **68**(3), 208–220. DOI: 10.1016/j.epsr.2003.07.001.

Vicente, L. N. and Calamai, P. H.: 1994, Bilevel and multilevel programming: A bibliography review, *Journal of Global Optimization* **5**(3), 291–306.