OPTIMIZATION UNDER UNCERTAINTY FOR OPERATIONAL PLANNING OF PETROLEUM REFINERIES

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ABSTRACT

In this paper, we describe a nonlinear programming (NLP) model for operational planning of oil refineries, considering uncertainties related to oil supply and refinery capacity. Three mathematical models based on stochastic programming (two-stage stochastic model) and robust programming (min-max regret model and max-min model) are developed to address these uncertainties. The main purpose of this paper is to address the impact of uncertainty on the operational planning of oil refineries by using different risk-profiles. The stochastic approach corresponds to a risk-neutral attitude, whereas the robust approach corresponds to a risk-averse attitude. A study based on real data from a Brazilian refinery demonstrates the performance of the various approaches. After analyzing the oil purchase decisions, we identify a clear relationship between the adopted risk attitude and the quantity and quality of the purchased oil. We also show the strong influence of the product specification constraints on the model decisions.

KEYWORDS. Optimization under uncertainty. Refinery planning. Scenario analysis. EA – Applications to Energy.

1. Introduction

Oil refineries are increasingly interested in improving the planning of their operations. One of the major driving factors is the dynamic nature of the refining business. Companies want to assess the potential impact of various refinery shifts on the overall performance, such as the final product specifications, the crude oil composition as well as other operational variations including the available capacity of the refinery. It has also been shown that the integration of new technologies for process operations is an essential profitability factor (Joly *et al*., 2002). The use of mathematical programming in the planning activities was shown to lead to potential gains of US $10 per ton of refined product in large refineries, which corresponds to savings of more than one million dollars per year (Moro, 2003). However, such a gain is extremely difficult to achieve because of the complexity of oil refining activities.

In the literature, many operational planning models have been tested in real refineries around the world. Gao *et al.* (2008) developed a mixed integer linear programming model (MILP) to address the production planning problem of a large-scale fuel oil-lubricant plant in China. The authors considered the choice of operational modes at each processing unit as the main optimization decision of the model.

The MILP proposed by Micheletto *et al.* (2007) optimizes the operation of a refinery plant in Brazil by considering mass and energy balances, operational mode of each unit, and demand satisfaction over multiple periods of time. Moro *et al*. (1998) also employed their model for studying a refinery in Brazil. They developed a nonlinear planning model, which was applied to the particular case of diesel production to maximize the profit of the refinery.

Other applications in Brazil can be found in Neiro and Pinto (2004, 2005). In their early work (Neiro and Pinto, 2004), these authors developed a general framework for the modeling of petroleum supply chains. The resulting multi-period mixed-integer nonlinear programming (MINLP) model was tested in a supply chain consisting of four Brazilian refineries. A nonlinear integer programming application associated with uncertainty was investigated in the work by Neiro and Pinto (2005). They formulated a stochastic multi-period model for which the uncertainty is related to the prices of petroleum and product as well as to the product demand.

Pongsakdi *et al.* (2006) treated the uncertainty and financial risk in the planning of operations for a refinery in Thailand using a two-stage linear stochastic model. The problem consists in determining how much of each crude oil had to be purchased and the anticipated production level of different products based on demand forecasts. The uncertainty was introduced by means of the demand and product price parameters. The first-stage decisions were represented by the amount of crude oil purchased for each period. Lakkhanawat and Bagajewicz (2008) extended the work of Pongsakdi *et al.* (2006) by incorporating the product pricing in their study.

Among all of these applications, only Lakkhanawat and Bagajewicz (2008), Neiro and Pinto (2005), and Pongsakdi *et al.* (2006) addressed the uncertainty in the operational planning of refineries. Despite of these significant contributions, these applications are mainly based on stochastic programming techniques and still present very simplified models that exclude important aspects of a real refinery operation, such as constraints for the specification of final products. Therefore, the refinery operational planning problem under uncertainty is still an open issue, which is relevant for both mathematical modeling and actual applications.

Several approaches based on different mathematical methods can be equally used to optimize the operational planning of a refinery. This work aims at evaluating the main techniques used to treat uncertainty (stochastic and robust programming) and to show that optimization models under uncertainty can be used to support the production planning at a real refinery. The main purpose of this paper is to address the impact of uncertainty on the operational planning of oil refineries by using different risk approaches. We introduce a nonlinear programming (NLP) model for the refinery operational planning by considering two uncertainty sources: the oil supply and the capacity of the process units, while taking into account the equipment maintenance. To address these uncertainties, three formulations are proposed: (1) a two-stage stochastic model (Dantzig, 1955) with a finite number of realizations, (2) a min-max regret model (Kouvelis and Yu, 1997), and (3) a max-min model (Kouvelis and Yu, 1997).

In our model, the refineries are represented in great detail by considering the nonlinearities inherent to the process, which allows us to gain a great precision in the information regarding the operational planning that is provided as solution. Without any loss of generality, we use a case study with real data from a Brazilian oil refinery to evaluate the three optimization models that we propose. Finally, we discuss the main differences in the three approaches and their impact on the refinery operational planning.

The remainder of this paper is organized as follows. In section 2, we present the problem to be addressed. In section 3, we use three different approaches to treat uncertainties associated with the refinery operational planning problem. Section 4 presents our results and a discussion based on a numerical example. Finally, we draw some conclusions in section 5.

2. Problem Statement

The oil refining activity is certainly one of the most complex activities in the chemical industry, because refineries carry different processes with several possible configurations and structures (Khor and Elkamel, 2008). The main objective of a refinery is to transform crude oil into refined products of higher aggregate value, in addition to maximize the profit. The refinery topology is defined by a set of process units, storage tanks for final and intermediary products, and pipes interconnecting all the components. Refineries carry process units and tanks to blend products and produce several streams of intermediate products that can be blended to create distinct commercial offerings. Figure 1 depicts a simplified refining flowchart of a refinery located in Brazil:

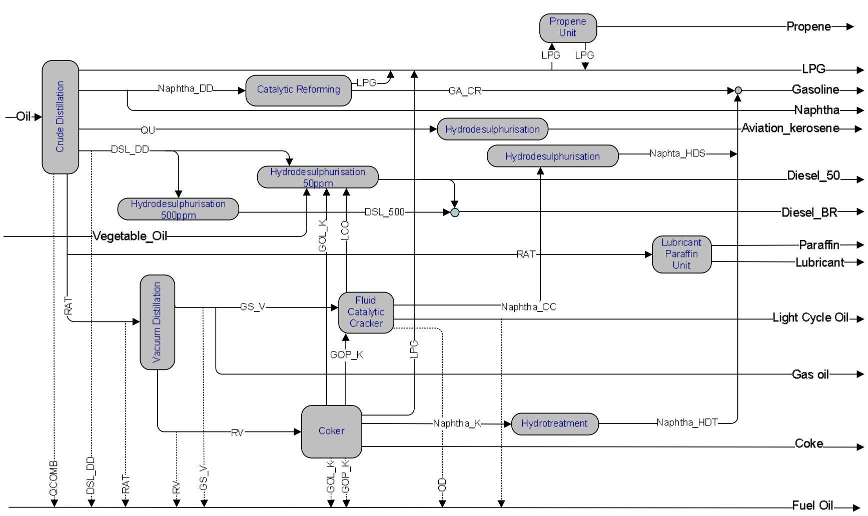

Figure 1. Refinery flowchart

The refinery processes the crude oil into marketable products through three main types of process: separation, conversion and treatment. Separation processes (Crude Distillation and Vacuum Distillation) are designed to separate the oil into its basic fractions or to process a previously generated fraction to produce a specific group of components. Conversion processes (Cocker, Fluid Catalytic Cracker, and Catalytic Reforming) transform a fraction into another one or change the molecular structure of a fraction. Treatment processes (Hydrodesulphurisation and Hydrotreatment) provide better cutting of semi-finished products by reducing contaminants (sulfur, nitrogen, and metals) or removing them from their structure. The refinery produces light

(Propene, LPG, Gasoline, and Naphta), medium (Aviation kerosene, and Diesel) or heavy (Paraffin, Lubricants, Light Cycle Oil, Gas Oil, Coke, and Fuel Oil) fractions.

A good planning model for oil refineries must allow for the proper selection of oil blending and consider an appropriate manipulation of intermediary streams to obtain the final products in the desired quantities and qualities (Moro, 2000). Choosing the ideal plan for refinery operations and the best configuration for each process unit is a difficult task due to the high number of variables and constraints present in these processes. Mathematical programming plays a crucial role in solving this problem, assisting in the decision-making process.

3. Mathematical Model

This study presents a nonlinear programming (NLP) model for the refinery operational planning by considering two uncertainty sources: the oil supply and the capacity of the process unit depending on the equipment maintenance. We assume that the associated prices, costs, and demands are externally imposed. To address these uncertainties, three different formulations are proposed: (1) a two-stage stochastic model with fixed recourse, (2) a min-max regret robust model, and (3) a max-min model. The classical two-stage expected profit maximization model (Section 3.1) provides a traditional risk-neutral approach. This two-stage model was based on the stochastic formulation proposed by Neiro and Pinto (2005). In Section 3.2 of this paper, we introduce the risk-averse point of view using robust programming.

3.1. Risk-neutral Attitude: Stochastic Model

The two-stage stochastic linear model with fixed recourse represents uncertainties in terms of discrete random experiments s ($s \in S$). We assume that the probability that the *s-th* scenario will occur is represented by p_s ($p_s \geq 0, \sum_{s=1}^{S} p_s = 1$). The general formulation of the stochastic approach was defined by Dantzig (1955) as follows:

$$Max\ c^T x + \sum_{s \in S} p_s q_s^T y_s \tag{1}$$

$$s.t.$$

$$Ax \leq b \tag{2}$$

$$Wy_s \leq h_s - Tx \qquad \forall s \in S \tag{3}$$

$$x \geq 0 \quad \text{and} \quad y_s \geq 0 \qquad \forall s \in S$$

First-stage decisions are assumed to be taken before the realization of random variables (here-and-now decisions), represented by a vector x, while second-stage decisions, denoted by y_s, are taken under complete information about the realization of s, becoming scenario-dependent variables.

The objective function in Equation (1) contains a deterministic term $c^T x$, which models the oil purchase decisions, and the expected value of the second-stage objective $\sum_{s \in S} p_s q_s^T y_s$, which models the stochastic operative profit due to the first-stage decision. In this model, a set of deterministic inequalities (2) is used to model the oil purchase in the spot market. Stochastic constraints (3) are used to represent refinery operation and to model all operative relations between the inputs (or different petroleum types) and the outputs (or final products).

Based on these assumptions, the complete stochastic model in this paper can then be represented by (for definitions, see nomenclature section at the end of this paper):

Maximize

$$OM = \left(\begin{array}{l} -\sum_{c\in C_u}\sum_{u\in UC}\sum_{s\in SS_{u,cc}}\sum_{t\in T} CFPA^t_{u,s} ca^t_{u,c,s} \\ +\sum_{sc\in SC} P^{sc}\left(\begin{array}{l} \sum_{c\in C_u}\sum_{u\in UE}\sum_{s\in SE_{u,ce}}\sum_{t\in T} PFP^t_{u,s} qis^{t,sc}_{u,c,s} - \sum_{u\in UP}\sum_{c\in C_u}\sum_{l\in L_c}\sum_{t\in T} CUT^t_{u,c,l} qi^{t,sc}_{u,c} \\ -\sum_{c\in C_u}\sum_{u\in UC}\sum_{s\in SS_{u,cc}}\sum_{t\in T}\left(CFPF^t_{u,s} QOCF^{t,sc}_{u,c,s}\right) \\ -\sum_{u\in UA}\sum_{c\in C_u}\sum_{s\in SS_{u,c}}\sum_{t\in T} CINV^t_{u,s} vo^{t,sc}_{u,c} \end{array} \right) \end{array} \right) \tag{4}$$

Process unit capacities

$$QIL^t_{u,c} \le qi^{t,sc}_{u,c} \le QIU^t_{u,c} \qquad \forall u\in UP\bigcup UT\bigcup UD, \forall c\in C_u, \forall t\in T, \forall sc\in SC \tag{5}$$

$$QISL^t_{u,c,s} \le qis^{t,sc}_{u,c,s} \le QISU^t_{u,c,s} \qquad \forall u\in UPS\bigcup UT\bigcup UD, \forall c\in C_u, \forall t\in T, \forall sc\in SC \tag{6}$$

$$QL^{t,sc}_u \le \sum_{c\in C_u} qi^{t,sc}_{u,c} \le QU^{t,sc}_u \qquad \forall u\in UP\bigcup UT\bigcup UD, c\in C_u, \forall t\in T, \forall sc\in SC \tag{7}$$

$$QSL^t_{u,s} \le \sum_{c\in C_u} qis^{t,sc}_{u,c,s} \le QSL^t_{u,s} \qquad \forall u\in UPS\bigcup UT\bigcup UD, \forall s\in SI_{u,c}, \forall t\in T, \forall sc\in SC \tag{8}$$

Process constraints and material balances

$$qi^{t,sc}_{u,c} = \sum_{(u',c',s,u,c)\in F} q^{t,sc}_{u',c',s,u,c} \qquad \forall u\in UP\bigcup UT\bigcup UD, \forall c\in C_u, \forall t\in T, \forall sc\in SC \tag{9}$$

$$qis^{t,sc}_{u,c,s} = \sum_{(u',c',s,u,c)\in F} q^{t,sc}_{u',c',s,u,c} \qquad \forall u\in UPS\bigcup UT\bigcup UD\bigcup UE, \forall c\in C_u, \forall s\in SI_{u,c}, \forall t\in T, \forall sc\in SC \tag{10}$$

$$qo^{t,sc}_{u,c,s} = \sum_{(u,c,s,u',c')\in F} q^{t,sc}_{u,c,s,u',c'} \qquad \forall u\in UP\bigcup UT\bigcup UD\bigcup UC, \forall c\in C_u, \forall s\in SO_{u,c}, \forall t\in T, \forall sc\in SC \tag{11}$$

$$vo^{t,sc}_{u,c} = VOLI^t_{u,c} + vo^{t-1,sc}_{u,c} + qi^{t,sc}_{u,c} - \sum_{s\in SS_{u,c}} qo^{t,sc}_{u,c,s} \qquad \forall u\in UA, \forall c\in C_u, \forall t\in T, \forall sc\in SC \tag{12}$$

$$\sum_{s\in SS_{u,c}} qo^{t,sc}_{u,c,s} = qi^{t,sc}_{u,c} \qquad \forall u\in UD\bigcup UM, \forall c\in C_u, \forall t\in T, \forall sc\in SC \tag{13}$$

$$qo^{t,sc}_{u,c,s} = \sum_{s'\in SE_{u,c}} qis^{t,sc}_{u,c,s'} YUPS_{u,c,s,s'} \qquad \forall u\in UPS, \forall c\in C_u, \forall s\in SO_{u,c}, \forall t\in T, \forall sc\in SC \tag{14}$$

$$qo^{t,sc}_{u,c,s} = qi^{t,sc}_{u,c} YUPC_{u,c,s} \qquad \forall u\in UPC, \forall c\in C_u, \forall s\in SO_{u,c}, \forall t\in T, \forall sc\in SC \tag{15}$$

$$\sum_{(u',c',s,u,c)\in F} q^{t,sc}_{u',c',s,u,c} = RUT_{u,c,s} qi^{t,sc}_{u,c} \qquad \forall u\in UTR, \forall c\in C_u, \forall s\in SI_{u,c}, \forall t\in T, \forall sc\in SC \tag{16}$$

Supply plant constraints

$$QOCF^{t,sc}_{u,c,s} \le qo^{t,sc}_{u,c,s} \le QOCF^{t,sc}_{u,c,s} + ca^t_{u,c,s} \qquad \forall u\in UC, \forall s\in SO_{u,c}, \forall t\in T, \forall sc\in SC \tag{17}$$

$$ca^t_{u,c,s} \le QOCA^t_{u,c,s} \qquad \forall u\in UC, \forall s\in SO_{u,c}, \forall t\in T, \forall sc\in SC \tag{18}$$

Stock constraints

$$VOLL^t_{u,c} \le vo^{t,sc}_{u,c} \le VOLU^t_{u,c} \qquad \forall u\in UA, \forall c\in C_u, \forall s\in SO_{u,c}, \forall t\in T, \forall sc\in SC \tag{19}$$

Demand constraints

$$DEML^t_{u,c,s} \le qis^{t,sc}_{u,c,s} \le DEMU^t_{u,c,s} \qquad \forall u\in UE, \forall c\in C_u, \forall s\in SI_{u,c}, \forall t\in T, \forall sc\in SC \tag{20}$$

$$po^{t,sc}_{u,c,s,p} qo^{t,sc}_{u,c,s} = \sum_{s'\in SE_{u,c}} qis^{t,sc}_{u,c,s'} YUPS_{u,c,s,s'} POASSAY^t_{u,c,s,s'p} \qquad \forall u\in UPS, \forall c\in C_u, \forall s\in SO_{u,c}, \forall p\in PO_{u,c,s}, \forall t\in T, \forall sc\in SC \tag{21}$$

$$po^{t,sc}_{u,c,s,p} = PBS_{u,c,s,p} \qquad \forall u\in UPC, \forall c\in C_u, \forall s\in SO_{u,c}, \forall p\in PO_{u,c,s}, \forall t\in T, \forall sc\in SC \tag{22}$$

$$\left(vo_{u,c}^{t-1,sc} + VOLI_{u,c}^{t} + qi_{u,c}^{t,sc}\right) po_{u,c,s,p}^{t,sc} = vo_{u,c}^{t-1,sc} po_{u,c,s,p}^{t-1,sc} + VOLI_{u,c}^{t} POI_{u,c,p}^{t} + pi_{u,c,p}^{t,sc} qi_{u,c}^{t,sc}$$
$$\forall u \in UA, \forall c \in C_u, \forall s \in SO_{u,c}, \forall p \in PO_{u,c,s}, \forall t \in T, \forall sc \in SC \quad (23)$$

$$po_{u,c,s,p}^{t,sc} = pi_{u,c,p}^{t,sc} \quad \forall u \in UD \bigcup UM, \forall c \in C_u, \forall s \in SO_{u,c}, \forall p \in P, \forall t \in T, \forall sc \in SC \quad (24)$$

$$pi_{u,c,p}^{t,sc} qi_{u,c}^{t,sc} = \sum_{(u',c',s,u,c) \in F} q_{u',c',s,u,c}^{t,sc} po_{u',c',s,p}^{t,sc} \quad \forall u \in UP \bigcup UT \bigcup UD, \forall c \in C_u, \forall p \in P, \forall t \in T, \forall sc \in SC \quad (25)$$

$$POL_{u,c,p}^{t} \le po_{u,c,p}^{t,sc} \le POU_{u,c,p}^{t} \quad \forall u \in UT, \forall c \in C_u, \forall s \in SO_{u,c}, \forall p \in P, \forall t \in T, \forall sc \in SC \quad (26)$$

$$ca_{u,c,s}^{t}, q_{u',c',s,u,c}^{t,sc}, qis_{u,c,s}^{t,sc}, qi_{u,c}^{t,sc}, qo_{u,c,s}^{t,sc}, vo_{u,c}^{t,sc} \in \Re^{+}; pi_{u,c,p}^{t,sc}, po_{u,c,s,p}^{t,sc} \in \Re \quad (27)$$

The oil purchase in the spot market ($ca_{u,c,s}^{t}$) constitutes the first stage decisions. The oil supply for each refinery is defined by long-term contracts. However, the oil delivery is subject to delays, which must be corrected by the purchase of additional oil in the spot market. Thus, the oil availability from long-term contracts is represented by a stochastic parameter ($QOCF_{u,c,s}^{t,sc}$) in the model. The second-stage decisions are related to the refinery operations, such as flows between units ($q_{u',c',s,u,c}^{t,sc}$), inventory level ($vo_{u,c}^{t,sc}$), and refined products qualities ($pi_{u,c,p}^{t,sc}$ and $po_{u,c,s,p}^{t,sc}$).

The objective function (Equation 4) maximizes the expected operating margin. This margin includes the revenue from the products sales minus the processing costs, the raw materials costs and the inventory costs.

Equation (5) restricts the feed flow rate of each unit *u* for each operational mode *c*. Equation (6) limits the inlet flow rate of stream *s* for the unit *u* and for each operational mode *c*. Equation (7) controls the feed flow rate of the unit *u*. Equation (8) limits the inlet flow rate of stream *s* for the unit *u*.

Equation (9) describes the mass balance at the inlet stream of the unit *u* ($qi_{u,c}^{t,sc}$). Equation (10) represents the mass balance at the inlet stream *s* of the unit *u* ($qis_{u,c,s}^{t,sc}$). Equation (11) describes the mass balance at the outlet stream s of the unit u ($qo_{u,c,s}^{t,sc}$). The stock balance in the storage unit UA is represented by Equation (12). Equation (13) corresponds to the mass balance for the blending units UM and pipelines UD.

Equation (14) describes the process in the separation unit UPS, where the outlet flow rate of stream *s* ($qo_{u,c,s}^{t,sc}$) is a function of the feed flow rate of stream *s'* (mainly oils) and of the separation unit yield. Equation (15) describes the process in the conversion unit UPC, where the outlet flow rate of stream *s* ($qo_{u,c,s}^{t,sc}$) is a function of the feed flow rate and the conversion unit yield. Equation (16) determines the blending recipe of the feed flow rate of stream *s* for the tank unit by revenue UTR as a function of the feed flow rate of the tank unit UTR ($qi_{u,c}^{t,sc}$) and the parameter $RUT_{u,c,s}$.

Equation (17) limits the outlet flow rate for raw material tanks UC. Fixed and additional raw materials are available. The refinery consumes all the fixed raw material and purchases the additional raw material necessary for its operation through the first stage variable $ca_{u,c,s}^{t}$. Equation (18) limits the additional raw materials available for purchase. Equation (19) represents the inventory level for product tanks at every time period t under scenario sc. Equation (20) limits the inlet flow rate for the final products in the delivery units UE.

Equation (21) and Equation (22) define the properties of the outlet stream at the separation unit *ups* and at the conversion unit UPC, respectively. Equation (23) refers to the properties for storage units, which consider the stock at the time interval before *t-1* and the inlet flow rate at the time interval *t*. Equation (24) refers to the properties for blending units and pipelines , where the properties of the outlet stream is equal the properties of the inlet stream. Equation (25) defines the properties of the inlet stream for all types of units. Equation (26) specifies the property range.

3.2. Risk-averse Attitudes: Min-Max Regret Model and Max-Min Model

The classical two-stage expected profit maximization model (4)-(27) provides a traditional risk-neutral approach to choose the best operational plan among a set of candidate periods. In this section, we introduce the risk-averse point of view using robust programming. The robustness definition is largely discussed in the decision theory literature. Although many other relevant definitions are available (see Ben-Tal and Nemirovski, 2000; Bertsimas and Sim, 2004) in this work, we adopt the robustness concept defined by Kouvelis and Yu (1997).

The first robust model is based on the fear of regret when taking a decision in the first stage before knowing the uncertainties. The objective is to find the most robust first stage decision for the potential regret that a decision maker can take after the realization of the uncertainty parameters. The min-max regret model aims at finding an operational plan, under uncertainty, such that the worst (greater) absolute deviation between the achieved objective function and the per-scenario optimal objective value, obtained under perfect information, is minimized. In this context, the regret is modeled as the loss of optimality in each scenario incurred by the achieved solution, due to the degree of uncertainty faced in the parameters. The robustness of the solution is characterized by the worst case regret minimization.

The general form of the min-max regret robust model can be represented as follows:

$$Min\ \alpha \tag{28}$$

$$s.t.$$

$$z\left(x_s^*, y_s^*\right) - \left[c^T x + q_s^T y_s\right] \le \alpha \qquad \forall s \in S \tag{29}$$

$$Ax \le b \tag{30}$$

$$Wy_s \le h_s - Tx \qquad \forall s \in S \tag{31}$$

$$x \ge 0 \text{ and } y_s \ge 0 \qquad \forall s \in S \tag{32}$$

$$\alpha \ge 0 \tag{33}$$

In this model, the constraints (30)-(31) represent the oil purchase and operational decisions in the same way as in (2)-(3). The constraints (29) define the deviation (or the regret) as the difference between the optimum deterministic solution $z\left(x_s^*, y_s^*\right)$ and the profit, $c^T x + q_s^T y_s$ (provided by the "robust solution") for each scenario s. The parameter $z\left(x_s^*, y_s^*\right)$ is an upper bound (pre-calculated) of the final profit for each scenario s. Therefore, the objective function assumes the highest deviation between the solution obtained under perfect information and the robust solution by minimizing α.

A second risk-averse profile is the so-called max-min model. The model aims at maximizing the profit of the worst-case profit scenario. The decision maker is not "worried" about the potential regret due to the loss of perfect information optimality but in defining a first stage decision that minimize the worst financial loss it can generate. In this context, the max-min model appears to be an interesting comparative risk-profile because it provides the maximum conservative (or pessimistic) plan.

The mathematical formulation of the max-min model is given in (28)-(33) by substituting the objective definition to maximize the worst profit scenario:

$$Max\ \beta \tag{34}$$

$$s.t.$$

$$\left[c^T x + q_s^T y_s\right] \ge \beta \qquad \forall s \in S \tag{35}$$

$$Ax \le b \qquad \forall s \in S \tag{36}$$

$$Wy_s \le h_s - Tx \qquad \forall s \in S \tag{37}$$

$$x \ge 0 \quad \text{and} \quad y_s \ge 0 \tag{38}$$

$$\beta \in \Re \tag{39}$$

The min-max regret robust model and max-min model contain the same two-stage decision structure present in the stochastic model, where the first stage decisions include the purchase of additional oil in the spot market and the second-stage decisions are related to the refinery operations. The main difference between these two models and the stochastic model previously discussed resides in the objective function. Thus, the min-max regret robust model and the max-min model can be expressed as in (5)-(27), by substituting the objective definition to minimize the worst (greater) regret and maximize the worst profit scenario, respectively.

4. Numerical Example

The oil industry is one of the most important and dynamic industry in Brazil. The participation of the oil sector in the Brazil's gross domestic product (GDP) increased from 2.5% in 1996 to 9.8% in 2007. Its estimated investments in 2007 for 2010 represent 48% of the total amount to be invested by all industries in Brazil.

Brazil's current refining system includes seventeen refineries and three main petrochemical plants, which also produce refined products. Our case study focuses on a Brazilian refinery, which carries three processes units and 32 tanks. This refinery is supplied by two types of oils (named here as A and B) and process up to 69 intermediate products with 8 properties that need to be controlled to specify the 17 final products.

To create the scenarios used in this study, we considered two stochastic parameters: the oil supply, and the available capacity in the process units taking into account the equipment maintenance . The first parameter has two possible realizations (20,000 m3 or 10,000 m3 of oil supply defined by long-term contracts, representing a delay in oil delivery in the first period) and the second parameter has three possible realizations (5, 0 or 7 day stops for maintenance, affecting the available total unit capacity in the second period).

Assuming that the random variables are independent, we can combine the two stochastic parameters and create six scenarios, as shown in Figure 2.

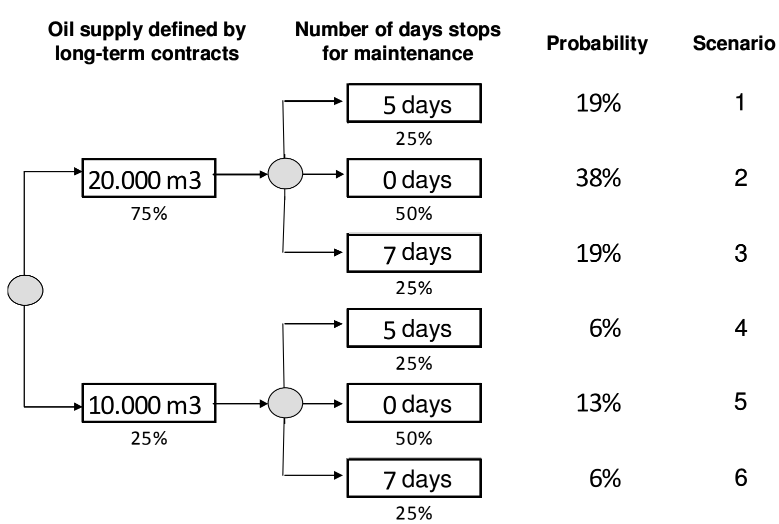

Figure 2. Scenarios of case study

The base case (scenario 2) used data from the current planning system on going in Brazilian refineries (PIMS - Process Industry Modeling System). PIMS addresses only one deterministic case used to generate the base case. The other scenarios were constructed based on the expertise of employees (engineers, production managers, and coordinators) of the refinery under study.

4.1. Computational results and discussion

A study using real data from a Brazilian refinery was used to evaluate the performance of the proposed models in optimizing an operational planning problem. The models were implemented using the AIMMS software package (Advanced Integrated Multidimensional Modelling Software - Bisschop and Roelofs, 2007) and solved using SNOPT 7.2 and Knitro 6.0. SNOPT 7.2 provides a sequential quadratic programming algorithm used to generate initial points. This solver was particularly efficient for our large problem because led to solutions that were locally optimal in very reduced times (see Table 1 for solving time). Afterwards an interior-point method (also known as barrier method) provided by Knitro 6.0 was used to improve the quality of the solution. A PC using an Intel® Core™ 2 Duo processor at 2.1GHz with 3.0Gb RAM was used for all computational results described in Table 1.

Table 1. Computational Results

Model	Base Case (scenario 2)	Stochastic	Min-Max Regret	Max-Min
# scenarios	1	6	6	6
# constraints	640	3,775	3,780	3,780
# continuous variables	532	3,127	3,127	3,127
# nonzeros	2,345	13,975	13,788	13,788
Snopt 7.2 - solving time (s)	0.36	2.82	2.78	3.25
Knitro 6.0 - solving time (s)	16.82	1,207.93	1,967.22	1,207.10
Total solving times (s)	17.18	1,210.75	1,970.00	1,210.35

Table 2 summarizes the model solutions for the oil purchase in the spot market (first stage decision):

Table 2. Model Solutions

Oil	Period	Base Case (scenario 2)	Stochastic	Min-Max Regret	Max-Min
A	1	11,593	20,000	18,468	17,606
A	2	4,207	5,800	2,019	1,815
B	1			3,863	4,673
B	2			1,450	1,706

The refinery can be supplied by oil type A and type B. Oil A is the most appropriate oil to produce refined products with a minimum demand. The oil B is less expensive than oil A and can also be used to produce refined products with small demand, although is more difficult to respect the specifications constraints due to the oil B lower quality. The long-term contracts are associated only with oil type A and the spot market offers oil A and B.

The base case (scenario 2) considers 20,000 m3 of oil supply defined by long-term contracts, without any delay in the delivery, and the refinery capacity is not affected by stop for maintenance in this scenario. The deterministic solution for the base case, where there is already a high supply of oil A (20,000 m3), was only to buy additional oil A in the spot market to complete the refinery capacity in order to maximize the profit.

The stochastic solution also recommended to purchase only oil type A in the spot market. The quantity of oil A purchased in the stochastic solution was used to meet the demand and capacity constraints, while avoided large inventories of oil in the scenarios where a high oil supply exists. The robust models (Min-Max regret and Max-Min) recommended a combined purchase of oil A and B in the spot market to optimize their objective and maintain the feasibility for the other scenarios. The relationship between the robust objective and the first stage decision (quantity of oil purchase in the spot market) is described in more details below.

Table 3 shows the profit obtained for each studied scenario based on the three models used in this work:

Table 3. Profit solution for each model ($)

Scenario	Wait and See	Stochastic	Min-Max Regret	Max-Min
1	12,076,420.38	9,920,532.61	9,901,196.58	9,884,574.63
2	11,773,824.45	9,921,038.99	9,873,733.38	9,885,575.51
3	12,085,951.41	9,236,999.26	**9,262,900.64**	**9,264,854.89**
4	12,076,420.31	11,769,903.50	11,772,895.26	11,769,720.11
5	11,773,824.44	11,773,824.45	11,754,699.48	11,750,867.41
6	12,085,907.91	11,769,900.37	11,772,872.43	11,769,617.04
E[OF]		10,255,392.65	10,236,866.16	10,237,675.85

The stochastic model maximizes the profit expected value (E[profit]) and, therefore, showed the best performance. The stochastic model was evaluated using Expected Value of Perfect Information (EVPI) (Birge and Louveaux, 1997). The EVPI result ($1,672,109.80) shows the difference between the solution obtained by the agent able to make the perfect prediction (wait-and-see solution – i.e., the solution in which the oil purchase decisions are postponed until that the uncertainty is unfolded) and that obtained by the agent that solved the problem under uncertainty (recourse problem). The lower the EVPI, the better the stochastic model accommodates uncertainties. In our case study, the EVPI reached only 14% of the wait-and-see solution.

The min-max regret model minimizes the gap between the target and the robust solution. The greater regret was observed for scenario 3, with no delay in the oil supply and the longest operation stop for maintenance (7 days). The 7 days stop affects the refinery capacity reducing the minimum capacity require for its operation. For that reason, the optimum first stage decision was to purchase the quantity of oil A to minimize the regret in scenario 3 and to guarantee the final products specifications in all scenarios. An additional quantity of oil B was also purchased to respect the capacity constraints in the scenarios with the smaller stops for maintenance (0 and 5 days).

The max-min model maximizes the result of the worst case scenario (scenario 3), the same scenario optimize in the min-max regret approach. The optimum first stage decision was to purchase in the spot market a combination of oils A and B, which optimized the profit for scenario 3 and ensured the feasibility of other scenarios. Observe that compare to the min-max regret model, the max-min model purchased a smaller quantity of oil type A and a larger quantity of oil type B. This decision reflects the fact that to optimize the worst case scenario a lower quantity of oil A is necessary and to respect the minimum capacity constraints the cheaper oil is used. This difference also appears in the objective function value, where the max-min objective function is slightly higher than the min-max regret.

Conclusions

The purpose of this paper was to develop a mathematical model to improve the operational planning of oil refineries considering uncertainties related to oil supply and process unit capacity based on equipment maintenance.

We were able to develop a nonlinear programming model for the refinery operational planning with a great level of detail, including the use of three different approaches based on stochastic and robust programming, and the use of sources of uncertainty which have hardly been studied in the literature.

The model was applied to an actual refinery in Brazil. Computational results for the three approaches were compared. We concluded that depending on the agent risk profile, each of the implemented models is more or less adequate in optimizing the decision-making process. The stochastic model addresses the expected value of profit. The min-max regret model corresponds to scenarios with the highest deviation and the max-min model to scenarios with the worst profit. The optimization results for the three models are suitable to the real planning activities of an actual refinery.

Nomenclature

Table 4. Sets and variables

Sets			
Set of process units ***(u, u')***	U	Tank units (storage and blending)	$UT \subset U$
Set of operational modes ***(c, c')***	C	Storage units	$UA \subset UT$
Set of streams ***(s, s')***	S	Blending units	$UM \subset UT$
Set of utilities ***(l)***	L	Tank units (storage and blending) by revenue	$UTR \subset U$
Set of properties ***(p, p')***	P	Processing units (separation and conversion)	$UP \subset U$
Time periods $\{n \mid n = 1,\dots, T\}$	T	Conversion units	$UPC \subset UP$
Set of scenarios ***(sc)***	SC	Separation units	$UPS \subset UP$
Operational modes c performed in unit u	$C_u \subset C$	**First stage decisions variables**	
Utilities l consumed in the mode c	$L_c \subset L$	Quantity of oil s purchased in the spot martket	$ca^{t}_{u,c,s}$
Properties p of intlet stream s	$PI_{u,c} \subset P$	**Second stage decisions variables**	
Properties p of outlet stream s	$PO_{u,c,s} \subset P$	Flow rate of stream s between (u,c) and (u',c')	$q^{t,sc^o}_{u',c',s,u,c}$
Inlet streams s of unit u	$SI_{u,c} \subset S$	Inventory level of u	$vo^{t,sc^o}_{u,c}$
Outlet streams s of unit u	$SO_{u,c} \subset S$	Property p of the feed stream of unit u	$pi^{t,sc}_{u,c,p}$
Tanks of raw material	$UC \subset U$	Property p of the outlet stream s at unit u	$po^{t,sc}_{u,c,s,p}$
Delivery units for final products	$UE \subset U$	Feed flow rate of unit u	$qi^{t,sc^o}_{u,c}$
Pipelines	$UD \subset U$	Feed flow rate of stream s *at* unit u	$qis^{t,sc^o}_{u,c,s}$
Flows (u,c,s,u',c'), where the pair (u,c) is origin and the pair (u',c') is the destine	$F \subset S$	Outlet flow rate of stream s at unit u	$qo^{t,sc}_{u,c,s}$

Table 5. Parameters

Parameters			
Probability of scenario sc^o	P^{sc^o}	Initial stock of unit u	$VOLI^{t}_{u,c}$
Product price	$PFP^{t}_{u,s}$	Minimum storing capacity of unit u	$VOLL^{t}_{u,c}$
Cost of additional raw material	$CFPA^{t}_{u,s}$	Maximum storing capacity of unit u	$VOLU^{t}_{u,c}$
Cost of fixed raw material	$CFPF^{t}_{u,s}$	Minimum demand	$DEML^{t}_{u,c,s}$
Utility cost	$CUT^{t}_{u,c,l}$	Maximum demand	$DEMU^{t}_{u,c,s}$
Inventory cost	$CINV^{t}_{u,s}$	Minimum feed flow rate of u at mode c	$QIL^{t}_{u,c}$
Separation unit yield	$YUPS_{u,c,s,s'}$	Maximum feed flow rate of u at mode c	$QIU^{t}_{u,c}$
Conversion unit yield	$YUPC_{u,c,s}$	Minimum feed flow rate of stream s	$QISL^{t}_{u,c,s}$
Blending revenue of stream s	$RUT_{u,c,s}$	Maximum feed flow rate of stream s	$QISU^{t}_{u,c,s}$
Lower bound of outlet property p of unit u	$POL^{t}_{u,c,p}$	Minimum feed flow rate of unit u	QL^{t,sc^o}_{u}
Upper bound of outlet property p of unit u	$POU^{t}_{u,c,p}$	Maximum feed flow rate of unit u	QU^{t,sc^o}_{u}
Property value of the initial stock	$POI^{t}_{u,c,p}$	Maximum of additional raw material	$QOCA^{t}_{u,c,s}$
Property of outlet stream s at separation unit u as a function of feed flow rate s'	$POASSAY^{t}_{u,c,s,s'p}$	Quantity of oil from long-term contracts at tank unit u	$QOCF^{t,sc^o}_{u,c,s}$

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