LIFTED INEQUALITIES FOR A SOFTDRINK LOTSCHEDULING MODEL

Michelli Maldonado Carretero, Socorro Rangel
UNESP/IBILCE/DCCE
Rua Cristóvão Colombo, 2265, Jd Nazareth - CEP 15054-000 - S. J. do Rio Preto - SP, Brasil
e-mail: michellimaldonado@hotmail.com, socorro@ibilce.unesp.br

Alistair Clark
University of the West of England - UWE - Department of Engineering Design and Mathematics
e-mail: alistair.clark@uwe.ac.uk

RESUMO

Neste trabalho propomos a inclusão de inequações válidas obtidas a partir da operação de *lifting* em um modelo integrado de dimensionamento e sequenciamento da produção aplicado ao setor de bebidas. As inequações válidas são obtidas a partir do conjunto de restrições associado ao sequenciamento de lotes. O estudo computacional desenvolvido com o objetivo de avaliar a qualidade do modelo reformulado e a interação das inequações válidas propostas com os planos de corte incluídos em um sistema comercial é descrito. Os resultados obtidos usando dados da literatura mostram que sob certas condições as inequações validas propostas são úteis no processo de solução do problema.

PALAVRAS CHAVE: Dimensionamento e Sequenciamento de Lotes, Problema do Caixeiro Viajante, Restrições de Eliminação de Subrotas.

Área Principal: Otimização Combinatória

ABSTRACT

In this paper, we propose the inclusion of a set of valid inequalities derived using a lifting procedure into a lot scheduling model applied to the soft-drink production. The set of valid inequalities is derived from constraints associated with the scheduling decisions. A computational study conducted to evaluate the quality of the proposed reformulation and how the lifted inequalities relate to the cutting planes included in a commercial solver is described. The results, using data from the literature, show that under certain conditions the proposed inequality is useful to improve the solution process of the lot scheduling model.

KEYWORDS: Lot sizing and Scheduling Problems, Traveling Salesman Problem, Subtour elimination constraints.

Main area: Combinatorial Optimization

1 Introduction

The development of a formulation that approximates the convex hull of a set of feasible points can facilitate the resolution of Mixed Integer Programmes by methods such as *Branch and Bound* or *Branch and Cut* and may lead to smaller solution times. Speeding up the solution process of a given optimization model is an important aspect to be addressed in order to develop a flexible decision support system. In this paper we are interested in speeding up the solution process of the integrated lot size and scheduling problem (ILS) associated with the production planning of soft drinks.

Several mathematical formulations are proposed in the literature for the ILS problem (*e.g.* Fleischmann and Meyr (2007), Pochet and Wolsey (2006), Bernado *et al.* (2007), Toso *et al.* (2008)). Two main approaches have been used to model the decisions associated with lot scheduling: the division of a period of the planning horizon into subperiods (GLSP approach), and the use of the assignment and subtour elimination constraints associated with the asymmetric travelling salesman problem (ATSP approach). In the latter, there are formulations that use subtour elimination constraints proposed by Dantzig, Fulkerson e Johnson (DFJ) and others that use those proposed by Miller, Tucker e Zemlim (MTZ) (*e.g* Lawler (1985)). The solution process of instances of both formulations varies with the application type, and in general, instances of realistic size have been proven difficult to solve even when state of the art commercial solvers are used.

In the context of soft drink production, Defalque *et al.* (2010) present a formulation for the associated ILS problem based on the ATSP approach (model P1S1MTS - one-stage one-machine traveling salesman lot-scheduling problem). The model P1S1MTS performs well, particularly when compared to the model of Ferreira *et al.* (2010) which is based on the GLSP approach. However, the solution gaps for instances of realistic size showed that there is scope for further improvements. In this paper we apply a proposal from the literature to obtain valid inequalities for the P1S1MTS model by applying a lifting procedure to the associated subtour elimination constraints. The aim is to obtain a reformulation of the P1S1MTS model that has a better approximation of the convex hull of the feasible set (Nemhauser and Wolsey (1998)).

This paper is organized as follows. In section 2 we briefly present the P1S1MTS model. In Section 3 the proposed valid inequalities are derived. The results of a computational study to evaluate the model efficiency are presented in Section 4, and conclusions are given in Section 5.

2 Brief description of the P1S1MTS model

In this section we review the mathematical model P1S1MTS proposed by Defalque *et al.* (2010) to represent the production process of small scale soft drink plants. The production process of soft drinks in different sizes and flavours is carried out in two stages: liquid flavor preparation (Stage I) and bottling (Stage II). The model P1S1MTS considers that there are J soft drinks (items) to be produced from L liquid flavors (syrup) on one production line (machine). To model the decisions associated with Stage I, it is supposed that there are several tanks to store the syrup and that it is ready when needed. Therefore, it is not necessary to consider the scheduling of syrups in the tanks, nor the changeover times since it is possible to prepare a new lot of syrup in a given tank, while the machine is bottling the syrup from another tank. However, the syrup lot size needs to satisfy upper and lower bound constraints in order to not overload the tank and to guarantee syrup homogeneity. In Stage II, the machine is initially adjusted to produce a given item. To produce another item it is necessary to stop the machine and make all the necessary adjustments (another bottle size and/or syrup flavor). Therefore, in this stage, changeover times from one product to another may affect the machine capacity and thus have to be taken into account. The P1S1MTS model addresses the problem of defining the lot size and lot schedule taking into account the demand for items and the capacity of the machine and syrup tanks, minimizing the overall production costs. It assumes that there is an unlimited quantity of other supplies (*e.g.* bottles, labels, water).

In the P1S1MTS model the decisions associated with lot sizing are based on the Capacitated Lot Sizing Problem (CLSP) (*e.g.* Karimi *et al.* (2003)). The scheduling decisions use the ATSP approach with the MTZ constraints to eliminate subtours. To present the model, let the following parameters define the problem size:

J : number of soft-drinks (items);
L : number of syrup flavors;
T : number of periods;

and let (i, j, k, l, t) be the index set defined as: $i, j, k \in \{1, \ldots, J\}; l \in \{1, \ldots, L\}; t \in \{1, \ldots, T\}$. The data and variables described below with superscript I relate to Stage I (syrup preparation) and with superscript II relate to Stage II (bottling).

Data

a_j^{II} : machine production time for one lot of item j;
b_{ij}^{II} : machine changeover time from item i to j;
d_{jt} : demand for item j in period t;
g_j : non-negative backorder cost for item j;
h_j : non-negative inventory cost for item j;
I_{j0}^{+} : initial inventory for item j;
I_{j0}^{-} : initial backorder for item j;
K_t^{II} : total time capacity of the machine in period t;
s_{ij}^{II} : machine changeover cost from item i to j;
S_t : maximum number of tank setups in períod t;
K^I : total capacity of the tank, in liters of syrup;
q_l : minimum quantity of syrup l to guarantee homogeneity;
r_{lj} : quantity of syrup l necessary for the production of one lot of item j;
γ_l : set of items that need syrup l;

Variables

I_{jt}^{+} : inventory for item j at the end of period t;
I_{jt}^{-} : backorders for item j at the end of period t;
x_{jt}^{II} : production quantity of item j in period t;
z_{ijt}^{II} : changeover on machine (stage II) from item i to item j in period t.
u_{jt} : auxiliary variable - might be used to indique the production order of item j in period t;
w_{lt} : number of tanks to be prepared with syrup l in period t;
n_{lt} : fraction of tank capacity used to produce syrup l in period t;
y_{lt}^{I} : is equal to 1 if the tank is setup for syrup l in period t;

The complete description of the P1S1MTS model is given by expressions (1)-(15).

$$\text{Min } Z = \sum_{j=1}^{J}\sum_{t=1}^{T}(h_j I^+_{jt} + g_j I^-_{jt}) + \sum_{t=1}^{T}\sum_{i=1}^{J}\sum_{j=1, j\neq i}^{J} s^{II}_{ij} z^{II}_{ijt} \tag{1}$$

such that

Stage I: Syrup preparation

$$\sum_{j\in\gamma_l} r_{lj} x^{II}_{jt} = K^I(w_{lt} - n_{lt}), \qquad \forall l, \forall t. \tag{2}$$

$$n_{lt} \leq 1 - \left(\frac{q_l}{K^I}\right), \qquad \forall l, \forall t. \tag{3}$$

$$y^I_{lt} \leq w_{lt} \leq S_t y^I_{lt}, \qquad \forall l, \forall t. \tag{4}$$

$$\sum_{l\in L} w_{lt} \leq S_t; \qquad \forall t. \tag{5}$$

Stage II (bottling) - lot sizing:

$$I^+_{j(t-1)} + I^-_{jt} + x^{II}_{jt} - I^+_{jt} - I^-_{j(t-1)} = d_{jt}, \qquad \forall j, \forall t \tag{6}$$

$$\sum_{j=1}^{J} a^{II}_j x^{II}_{jt} + \sum_{i=1}^{J}\sum_{j=1, j\neq i}^{J} b^{II}_{ij} z^{II}_{ijt} \leq K^{II}_t, \qquad \forall t \tag{7}$$

$$a^{II}_j x^{II}_{jt} \leq K^{II}_t \sum_{i=1, i\neq j}^{J} z^{II}_{ijt}, \qquad \forall j, \forall t \tag{8}$$

$$\sum_{i=1}^{J}\sum_{j=1, j\neq i}^{J} z_{ijt} \leq S_t, \qquad \forall t \tag{9}$$

Stage II (bottling) - scheduling:

$$\sum_{j=1, j\neq i_0}^{J} z^{II}_{i_0 jt} \geq \sum_{i=1, i\neq k}^{J} z^{II}_{ikt}, \qquad \forall k, \forall t \tag{10}$$

$$\sum_{j=1, j\neq i}^{J} z^{II}_{ijt} \leq 1, \qquad \forall i, \forall t \tag{11}$$

$$\sum_{i=1, i\neq k}^{J} z^{II}_{ikt} = \sum_{j=1, j\neq k}^{J} z^{II}_{kjt}, \qquad \forall k, \forall t \tag{12}$$

$$u_{jt} \geq u_{it} + 1 - (J-1)(1 - z^{II}_{ijt}); \quad \forall i, \forall j; i \neq j; \forall t \tag{13}$$

$$1 \leq u_{jt} \leq J - 1 \qquad \forall j, \forall t \tag{14}$$

$$x^{II}_{jt} \geq 0,\ z^{II}_{ijt},\ y^I_{lt} = 0/1,\ w_{lt} \in \mathbb{Z}_+,\ n_{lt} \geq 0, \qquad \forall i, j; \forall t; \forall l. \tag{15}$$

The optimization criterion (1) is to minimize the overall costs taking into account inventory, backorder and machine changeover costs. In Stage I, constraints (2)-(5) control the syrup production. Constraints (2) guarantee that if the tank is ready for production of syrup l, then there will be production of item j and the quantity produced uses all the syrup prepared in that period. The variables n_{lt} allow partial use of the tank and is controlled to respect the minimum amount needed to ensure syrup homogeneity, as specified by constraints (3). Constraints (4) ensure that there is production of the syrup l only if the tank is prepared. According to constraints (5), the total number of tanks produced in period t is limited by the total number of tank setups.

In Stage II, constraints (6) represent the flow conservation of each item in each time period. Constraints (7) represent the machine capacity in each time period. Constraints (8) guarantee that there is production of item j only if the machine is prepared. Note that the setup variable is considered implicitly in terms of the changeover variables and that production may not occur although the machine might be prepared. Constraints (9) control the maximum number of setups in each period.

Constraints (10)-(14) model the order in which the items will be produced in a given period t. They are based on the ATSP model. Constraints (10) consider that in each period the machine is initially setup for a ghost item i_0. The changeover costs associated with the ghost item are zero and do not interfere in total solution cost. Constraints (11) guarantee that each item j is produced at maximum once in each period t. Constraints (12) conserve flow and ensure that if there is a changeover from an item i to any item k then there is a changeover from that item k to an item j.

Constraints (10) and (12) alone might generate subtours, that is disconnected cycles, and thus do not guarantee a proper sequence of items. The MTZ type subtour elimination constraints (13) avoid this situation. With the inclusion of constraints (14) the variable u_{jt} gives the order position in which item j is produced. Finally constraints (15) define the variables' domain. More details on the P1S1MTS model can be obtained from Defalque *et al.* (2010) and Defalque (2010). Other formulations of the soft drink production process can be found in Toledo *et al* (2007), Ferreira *et al.* (2009 and 2010).

3 Lifted valid inequalities

In model P1S1MTS the constraints associated with the lot scheduling decisions are formulated based on the constraints proposed by Miller, Tucker and Zemlim (MTZ) to eliminate subtours in a formulation of the traveling salesman problem (Lawer, 1985), hereafter called MTZ-TSP. These constraints are of polynomial order, thus allowing their inclusion *a priori*. However, the MTZ constraints produce a weak linear relaxation of the associated formulation. Since the MTZ constraints allow a compact polynomial representation of the subtour elimination constraints, they are very attractive when the TSP emerges as submodels in the context of large-scale models such as those for the vehicle routing and scheduling problems. Motivated by this fact, Desrochers and Laporte (1991) proposed a new class of valid inequalities to improve the MTZ-TSP formulation. A lifting procedure (Wolsey (1998)) is applied to the subtour elimination constraints to obtain stronger valid inequalities. In what follows, we will use the same technique to obtain stronger valid inequalities for the P1S1MTS model.

The MTZ subtour elimination constraints (13) can be rewritten as follows:

$$u_{it} - u_{jt} + (J-1)z_{ijt}^{II} \leq J-2 \quad t, i, j = 1, \ldots, J; \quad i \neq j. \tag{16}$$

We wish to construct a valid inequality for model P1S1MTS including the variable z_{jit}^{II} with coefficient α_{jit} in (16):

$$u_{it} - u_{jt} + (J-1)z_{ijt}^{II} + \alpha_{jit} z_{jit}^{II} \leq J-2 \quad t, i, j = 1, \ldots, J; \quad i \neq j. \tag{17}$$

We must find the largest value that coefficient α_{jit} can to take so that the inequality (17) continues to be valid for P1S1MTS, through the use of the *lifting* technique. This is done by solving a new optimisation problem:

$$\text{Max } \ Z = \alpha_{jit} \tag{18}$$

$$\alpha_{jit} z^{II}_{jit} \leq J - 2 - u_{it} + u_{jt} - (J-1) z^{II}_{ijt}. \tag{19}$$

We will consider two cases when solving the optimization problem (18)-(19): $z^{II}_{jit} = 0$ and $z^{II}_{jit} = 1$. If $z^{II}_{jit} = 0$, then we will have the following maximisation problem:

$$\text{Max } \ Z = \alpha_{jit} \tag{20}$$

such that:

$$0 \leq J - 2 - u_{it} + u_{jt} - (J-1) z^{II}_{ijt}. \tag{21}$$

Thus, whatever the value of α_{jit}, the inequality (17) is valid for P1S1MTS, as any value of α_{jit} satisfies the constraint (21).

If $z^{II}_{jit} = 1$ then $z^{II}_{ijt} = 0$, and we must solve the following problem:

$$\text{Max } \ Z = \alpha_{jit} \tag{22}$$

such that:

$$\alpha_{jit} \leq J - 2 - u_{it} + u_{jt}. \tag{23}$$

However, we know that if $z^{II}_{jit} = 1$, then item i is produced after item j, so the inequality (24) is valid for the P1S1MTS.

$$u_{it} = u_{jt} + 1. \tag{24}$$

Thus, substituting (24) in (23), we obtain the following optimization problem:

$$\text{Max } \ Z = \alpha_{jit} \tag{25}$$

such that:

$$\alpha_{jit} \leq J - 3 \tag{26}$$

As we are maximizing, the largest value that coefficient α_{jit} can take is $J - 3$. Thus

$$u_{it} - u_{jt} + (J-1) z^{II}_{ijt} + (J-3) z^{II}_{jit} \leq J - 2 \quad t, i, j = 1, \ldots, J; \quad i \neq j. \tag{27}$$

is valid for P1S1MTS.

The proposed model, denoted P1S1MTS-DL, is constructed using the same constraints as model P1S1MTS, but substituting constraints (13) by constraints (27) and removing constraints (14).

4 Computational Experiments

This section presents the computational results from evaluating the performance of model P1S1MTS-DL proposed in section 3. The tests carried out for model P1S1MTS were newly repeated due to the use of a different computer processor and RAM to that used by Defalque *et al.* (2010) resulting in a small difference between this paper and Defalque *et al.* (2010). The models were implemented in the AMPL modelling language and solved using CPLEX 10.0 with its default parameters. The tests were carried out on an Intel Core i7 2.93 GHz processor with 1.87 Gb of RAM under Windows 7.

The computational tests were divided into 2 parts. In the first, we evaluated the performance of model P1S1MTS-DL using instances obtained from randomly generated data. In the second part, we used instances generated from real data. Both sets of instances were obtained from Defalque *et al* (2010).

4.1 Results from random generated data

Three classes of random instances were generated, 10 instances in each class. The first class (Class 1) considers that more than one item uses the same syrup for their production. The second class (Class 2) is a modification of the instances in Class 1 such that the changeover costs are equal to 10 % of the changeover times. For the instances in the third class (Class 3), different items use different syrups. All the instances were generated considering that there are $J = 4$ items. For the instances in Classes 1 and 2, there are $L = 2$ syrups and for the instances in Class 3, $L = 4$. The planning horizon is $T = 2$ periods, the tank capacity is $K^I = 1000$ liters and the minimum syrup quantity for liquid homogeneity is $q_l = 176.802$ liters. The other parameters were randomly generated by a uniform distribution in the intervals shown in Table 1. The machine capacity is $K^{II} = 867.48$ minutes, which is considered to be loose capacity.

Parameters	Intervals
h_j	[0.006, 0.009]
g_j	[15, 18,9]
a_j^{II}	[0.03, 0.06]
b_{ij}^{II}	[4; 30]
s_{ij}^{II} - Classes 1 and 3	0.5 $\times b_{ij}$
s_{ij}^{II} - Class 2	0.1 $\times b_{ij}$
d_{jt}	[746, 12,958]
r_{lj}	[0.237, 0.290]

Table 1: Intervals for random data (Defalque *et al.* (2010))

The optimal solution for all 30 instances of models P1S1MTS and P1S1MTS-DL was found. The average computational time did not exceed 0.10 seconds. Due to space limitation in the tables, from now on we will refer to models P1S1MTS and P1S1MTS-DL as TS and DL respectively. Table 2 shows the results for a set of 15 instances of the three models (5 from Class 1, 4 from Class 2 and 6 from Class 3). For these instances, Table 2 shows the linear relaxation value (LR), the value of the optimal solution (Z), the number of nodes necessary to prove optimality (NN) and the node where the solution was found (SN). The results for the other 15 instances can be found in Carretero (2011).

The use of the lifted subtour inequalities in formulation DL did not improve the value of the linear relaxation. However, as the results in Table 2 show, it did affect the solution process. For formulation DL, in 11 instances, the total number of nodes necessary to prove optimality was smaller or equal to the number of nodes necessary with the original formulation. For only one instance, the number of nodes increased more than 50%. The optimal solution was found in the root node in four instances of each formulation (the same instances as for models TS and DL). From these results, we can expect formulation DL to have similar or better behavior than the original formulation.

I	LR	Z	NN		SN	
			TS	DL	TS	DL
a1	250451	269211	27	18	27	17
a2	269054	278729	45	28	40	20
a3	430373	446301	10	10	9	9
a4	391788	421772	12	19	0	0
a5	154880	187150	15	16	13	13
b2	277278	287911	45	46	37	38
b5	344598	370688	1	1	0	0
b6	389732	408012	32	32	30	30
b10	162517	189858	25	25	24	24
c1	429335	466601	23	23	20	20
c2	330025	372326	22	20	0	0
c3	312395	339595	20	17	10	9
c5	353439	385602	5	6	3	5
c8	137678	178670	21	20	19	19
c9	351468	387116	47	47	0	0

Table 2: Optimal solution for random instances

CPLEX 10.0 solved the problems by a branch and cut method. Several types of cutting planes were generated during the solution process. Due to space limitation, Table 3 only shows, for each instance of each model, the number of cutting planes generated for four types: implied Bounds (implied), flow cover (flow), mixed integer rounding (mir), and Gomory(Gom) inequalities. The last column in Table 3 (Total) shows the total number of cutting planes generated.

	implied		flow		mir		Gom		Total	
I	TS	DL	TS	DL	TS	DL	TS	DL	TS	DL
a1	5	4	8	9	2	2	8	8	23	23
a2	10	8	6	6	2	2	9	9	27	25
a3	7	8	10	8	3	3	4	4	24	23
a4	7	13	9	13	2	2	5	5	23	33
a5	3	3	8	9	7	7	9	9	27	28
b2	4	12	9	7	3	3	8	8	24	30
b5	5	6	12	10	4	4	8	8	29	28
b6	7	11	19	18	2	2	8	8	36	39
b10	6	9	8	6	3	3	8	8	25	26
c1	12	13	12	11	6	6	8	8	38	38
c2	7	13	11	11	5	5	13	13	36	42
c3	8	12	11	8	12	11	17	15	48	46
c5	1	8	11	10	7	7	18	16	37	41
c8	8	10	13	15	7	7	12	13	40	45
c9	11	16	17	14	9	9	12	12	49	51

Table 3: Cutting planes for random instances

The total number of cutting planes generated for each instance (Table 3) is similar for the two models. However, the number of implied cuts generated for instances of formulation DL is higher than the ones generated for TS in 12 instances. More details of the computational study using random instances can be found in Carretero (2011).

4.2 Results from real data

Ten instances of the proposed formulation DL were generated using the same real data used in Ferreira *et al.* (2010) and Defalque *et al.* (2010). Two instances, S1 and S6, differ by their demand and initial inventory values. The other 8 instances were obtained from these two by reducing or increasing the costs of changeover and inventory. All the instances consider $J = 27$ items, $L = 10$ syrups, $T = 5$ periods. The tank capacity is $K^I = 84000$ liters. The machine capacity is $K^{II} = 6840$ minutes for all but the first period which has $K^{II} = 2280$ of capacity. More details of theses instances can be found in Ferreira *et al.* (2010)

The maximum total execution time for solving each instance of the TS and DL models by CPLEX was set to three hours. No instance was solved to optimality. Table 4 shows the linear relaxation value (LR), the value of the best solution (ZB), the associated GAP as given by Cplex, the number of nodes examined (NE) and the node where the best solution was found (SN) for the instances of the original model TS and the formulation DL.

LR		ZB		GAP (%)		NE		SN	
I	LR	TS	DL	TS	DL	TS	DL	TS	DL
S1	1793	55695	54984	31.13	27.12	2572701	1992479	2546300	1897500
S2	2114	62404	60236	31.23	35.09	2430053	1925901	2237900	1880000
S3	4229	70472	76522	32.97	36.87	2671452	2611559	2467200	1632000
S4	2114	24148	23923	27.77	25.16	2959653	2032036	2869200	1979200
S5	4229	29148	28977	22.47	19.82	2980671	2552008	2755700	2449600
S6	1209	57212	57212	14.01	21.67	2039220	1688801	681000	1450000
S7	1256	68768	67422	27.54	23.85	2648122	2252726	2533400	2094500
S8	2418	66593	64302	17.47	15.36	2736483	2180101	2508200	2168700
S9	2512	32637	32537	18.05	17.62	2977021	2288662	2743700	2211300
S10	1256	26417	27231	16.80	18.96	2563825	1950434	2364000	1799100

Table 4: Best solution for real instances

The results shown in Table 4 indicate that the use of the lifted inequalities improved the original formulation TS. For eight instances it was possible to find a solution with a smaller cost than the solution given by the original formulation. However, the integrality GAP was smaller for only six instances of DL. For all instances, the total number of nodes examined during the three hours was smaller for formulation DL then for the original formulation TS. In all, but one instance, the best solution was found examining less nodes using formulation DL.

	implied		flow		mir		gom		Total	
I	TS	DL	TS	DL	TS	DL	TS	DL	TS	DL
S1	497	1372	359	370	264	55	68	66	1287	2090
S2	531	1313	395	381	374	61	65	65	1518	2056
S3	531	1185	365	354	316	51	58	61	1407	2019
S4	526	1268	317	333	248	55	61	67	1255	2012
S5	462	1237	311	303	393	45	56	67	1346	1990
S6	500	1513	402	400	222	46	73	75	1333	2232
S7	472	1459	378	396	340	44	74	75	1423	2230
S8	471	1442	391	391	295	34	76	62	1384	2214
S9	512	1364	358	330	319	39	86	89	1423	2115
S10	492	1392	362	369	274	32	81	76	1318	2108

Table 5: Cutting planes for real instances

Note in Table 5 that the total number of cutting planes generated for the instances of DL is higher than the ones generated for TS. The number of implied bounds for DL is far higher than the number of this type of cut generated for the original model TS. In contrast, fewer mir cuts were generated for DL. The number of flow cover and Gomory cuts are similar for both models. Since the integer GAP of both formulations TS and DL is higher than 14%, there is scope for further research into the effect of other types of valid inequalities in the solution process of the original and the proposed reformulations.

5 Conclusions and further work

In this paper we investigated the effect of lifted valid inequalities in the solution process of a lotscheduling model applied to soft drink production planning. The proposed valid inequalities was used to obtain *a priori* reformulations of a model given in the literature. The computational study was conducted using instances generated from random and real data, and showed the benefits the proposed reformulation. With formulation DL, it was possible to improve the solution process of both random and real data instances. However, the integrality GAP associated with the real instances (between 14% and 36%) shows scope for further research. Other types of valid inequalities (*e.g.* Sherali and Driscoll (2002)) might be useful as cutting planes in a branch-and-cut algorithm and/or to obtain *a priori* reformulations of the lot scheduling problem. Other reformulations strategies, for example exploring different formulations for the ATSP problem (Oncan *et al.* (2009)) and/or formulations for the scheduling problem (Aldowaisan *et al.* (1999)), might also be useful to improve the solution process of this problem.

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