COMPARISON OF A NEW BOOTSTRAPPING METHOD WITH PARAMETRIC APPROACH CONSIDERING STOCHASTIC LEAD TIME

Guilherme Corteletti Pedrosa
Departamento de Engenharia de Produção - UFMG
Av. Antônio Carlos, 6627 – Pampulha – Belo Horizonte – Minas Gerais
guilhermepedrosa@yahoo.com.br

Samuel Vieira Conceição
Departamento de Engenharia de Produção - UFMG
Av. Antônio Carlos, 6627 – Pampulha – Belo Horizonte – Minas Gerais
svieira@dep.ufmg.br

Douglas Moura Miranda
Departamento de Engenharia de Produção - UFMG
Av. Antônio Carlos, 6627 – Pampulha – Belo Horizonte – Minas Gerais
douglas.miranda@gmail.com

ABSTRACT

Inventory management has been recognized as one of the most important functions, which often has a great impact on their overall performance. One important task in the inventory management is the forecast. When the products are characterized as fast-moving, we have in the literature a good approach to predict its demand known as exponential smooth. However, when the demand pattern has a large proportion of zero values (slow-moving), the forecast task becomes a trick task. This demand pattern is common among spare parts which are responsible for a large portion of inventory costs. Thus an accurate management will bring good advantages for the company. In this paper, we developed a stochastic version of the bootstrap method of Viswanathan e Zhou (2008) and compare with the Croston method using stochastic lead time and generated demand data. The computational results show that the bootstrap method performs better, under our analysis criterions.

KEYWORDS. Forecast. Spare Parts. Inventory Control.

Main area. EST - Estatística IND - PO na Indústria

1. Introduction

An accurate forecasting of spare parts is essential to many companies due to high inventory costs associated to these items. Spare parts are held in stock to support maintenance operations and to protect against equipment failure.

Intermittent demand patterns are common among spare parts, i.e. random demand with a large proportion of zero values SILVER et al. (1971). Due their specific nature, normally very slow-moving, an accurate forecast becomes a tricky task BOYLAN and SYNTETOS (2010). In practice, intermittent demand occurs when there are many small customers and a few large customers or when the frequency of many customer requests varies.

In recent years, the area of intermittent demand forecasting has received much attention and significant advancements have been made in the field. Academics have dedicated their effort to improve the accuracy of forecasting procedures of these items. CROSTON (1972) proved the use of Single exponential smoothing (SES) technique proposed by BROWN (1959), was inappropriate for use on items with intermittent demand. He proposed a new method called Croston forecasting technique, which could handle the difficulties of intermittent demand. Since then, some authors have proposed a modification of Croston's method such as LEVEN and SEGERSTEDT (2004) and others.

Another method efficient to deal with intermittent demand items is the bootstrapping. The method proposed by WILLEMAIN (2004) is well accepted and their method produces more accurate forecasts of the distribution of demand over a fixed lead time than exponential smoothing and Croston's method.

This paper is organized as follows. In Section 2 we briefly present a literature review of the demand classification, forecasting technique and stochastic lead time. Section 3 we describe the methodology adopted. The results are presented in Section 4. Concluding remarks and directions are presented in Section 5.

2. Theoretical background

2.1 Demand Classification

Demand pattern classification is when the time series vary systematically according to their inherent variability GHOBBAR and FRIEND (2003). There are four main categories and they are distinct by their variability. Defining these categories we have: intermittent is when it appears randomly with many time periods having no demand; erratic demand pattern is characterized by highly variable demand size; lumpy demand is both intermittent and erratic; slow-moving items have intermittent demand with each demand size equal to one item or very few items.

SYNTETOS and BOYLAN (2005) proposing values for p (interval inter positive demands) and the squared coefficient of variation of demand sizes (CV^2) to be used as the boundaries of lumpiness. It can also distinguish different categories of intermittent demand as erratic, lumpy, slow and fast. The recommended cut-off points shown in the figure 1 were 1.32 and 0.49 for p and CV^2, respectively.

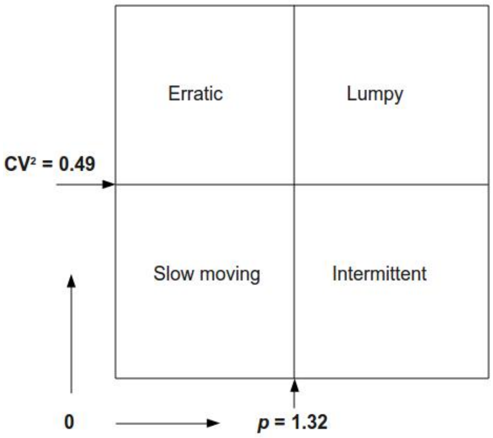

Figure 1: Items demand patterns SYNTETOS and BOYLAN (2005)

2.2 Exponential Smoothing

Exponential smoothing is a very popular technique that can be used to produce a smoothed time series data or to make forecasts. It is often applied to financial market, economic data and sales forecasting for inventory control BROWN (1959). Several empirical studies have been done through exponential models for forecasting of spare parts from different kinds. The simplest form of exponential smoothing is given by:

$$M_t = \alpha X_t + (1-\alpha) M_{t-1} \quad (1)$$

$$V = \frac{1}{T} \sum_t^T (X_t - M_{t-1})^2 \quad (2)$$

Where X_t is the real demand for an item in period t=1…T, M_t is the exponential smoothing estimated of mean demand for period t and α is the smoothing constant between 0 and 1. V is the variance of the demand.

2.3 Croston Method

CROSTON (1972) proposed a method for intermittent demand forecasting, because the exponential smoothing was not appropriate for this kind of demand. This method applies separately exponential smoothing forecasts on the size of the demands and the interval between nonzero demands. The forecast is only updated when there is a demand. If there is no demand in period t, then the smoothed estimate of the mean size of a nonzero demand $\hat{Z}_t$ and the estimate of inter-arrival time $\hat{P}_t$ are not changed. Let Q_t be the time interval since last demand and D_t the value of positive demand. Then the Croston's method can be described as follows:

$$Z_t = \alpha D_t + (1-\alpha) Z_{t-1} \quad (3)$$

$$P_t = \alpha Q_t + (1-\alpha) P_{t-1} \quad (4)$$

$$E(Y_{t)} = Z_t / P_t \quad (5)$$

SYNTETOS and BOYLAN (2001) showed that the original Croston method leads to a positive biased estimate of demand per unit time. They also propose a modified method with an unbiased estimator (eq. 6), forecast of the demand rate in period t, that is added to the original method. The forecast updates are the same as for the original Croston.

$$E(Y_{t)} = \left(1 - \frac{\alpha}{2}\right) Z_t / P_t \quad (6)$$

2.4 Bootstrap

Classical bootstrapping EFRON (1979) involves consecutive sampling, with replacement, from an available data set, to construct an empirical distribution of the data under concern. A large number of replications (say 10,000) are typically used BOYLAN and SYNTETOS (2010). The two main drawbacks of classical bootstrapping are that any potential autocorrelation of the data is not taken into account and the values generated in the reconstructed empirical distribution may not differ from the observations in the original sample.

To circumvent these drawbacks, WILLEIMAN (2004) proposed the best known bootstrapping based method for determining the safety stock. This procedure captures better the autocorrelation between the occurrences of demand, especially for intermittent demand with high proportion of zero values. It utilizes the Markov model to evaluate the probabilities of empirical transitions between the null and non-null demand for different items in order to estimate the information about demand during the lead-time.

Initially, the method evaluates the empirical transition probabilities between the zero/nonzero demands (Markov process) for different items. Based on these transition probabilities, a sequence of zero/nonzero values are generated for the entire forecast horizon. The values of the nonzero forecasts are calculated using the jittering process. It permits greater variation around larger demands.

Let X^* be a historical demand value selected randomly and Z the values of the inverse of the standard normal distribution also randomly selected. Let S be a nonzero value, so the forecast demand is given by:

$$S = \begin{cases} INT\{X^* + Z\sqrt{X^*} & ,if\ S > 0 \\ X^* & ,otherwise \end{cases} \quad (7)$$

Recently VISWANATHAN and ZHOU (2008) developed an improved bootstrapping based method and showed through computational experiments that this is superior to the method by WILLEMAIN (2004) both in the context of computer generated demand data and industrial data. To generate the positive demand arrivals they used the historical distribution of the inter-demand intervals (or intervals between non-zero demands) instead of a two-state Markov model. This change is responsible for improving the proposed method VISWANATHAN and ZHOU (2008).

2.5 Stochastic lead time

The stochastic of the lead-time may occur due various situations as contract changes, expediting polices transportation mode changes, etc. These dynamic variations will be encountered more frequently in futures RIEZEBOS (2006). According to BRADLEY and ROBINSON (2005), these instruments are increasingly used in modern supply chain management in order to increase flexibility.

BRADLEY and ROBINSON (2005) considered the standard periodic review inventory model, where at the start of each period, an order is placed that returns the inventory position to some base-stock level S. To characterize the lead time, they used the Poisson and Uniform Distributions.

3. Methodology

We use two different approaches to estimate the reorder point of a system when facing intermittent demand with stochastic lead time: a parametric approach, based on a modification of the Croston method and a stochastic version of the method proposed by VISWANATHAN and ZHOU (2008).

3.1. Modification of the Croston method

We use a modification of the Croston method to forecast the mean demand per period of the spare parts. The modification we use to forecast the mean was proposed by SYNTETOS and BOYLAN (2001), equation 6 and the variance we use the formula proposed in section 2.2., equation 2, as in WILLEIMAN (2004).

The variance and the mean of demand during the lead time are given by:

$$E(D_{lt}) = E(D)E(LT) \quad (8)$$

$$Var(D_{lt}) = Var(D)E(LT) + \left(E(D)\right)^2 Var(LT) \quad (9)$$

There are many different statistical distributions to choose from when working with inventory control. The use of the Gamma distribution is justified by the work of BURGIN (1975) and the practical analysis conducted by LEVEN and SEGERSTEDT (2004). The latter states that the findings of BURGIN (1975) were that the Gamma distribution was better suited for representing the demand of different items than the normal distribution. We estimate the order-up-to-level with gamma distribution choosing the smallest s_t that satisfies $\sum_0^{s_t} \Phi_D(x) \geq CSL$, where CSL is the cycle service level and $\Phi_D(x)$ is the distribution probability function.

3.2. A proposed modified version of the Bootstrap method

The method we propose in this paper to analyze the impact of stochastic lead time on inventory systems performance were based on the bootstrap method developed by WILLEIMAIN (2004), more precisely we use the adaptation of ZHOU and VISWANATHAN (2008) which proved to be more precise on the lead time demand estimates. This method uses the historical distribution of the inter-demand intervals to generate the positive demand arrivals, instead of using a two-state Markov chain as proposed by WILLEIMAN (2004). The pseudo code of the method is summarized below, based on the table presented in ZHOU and VISWANATHAN, (2008) and according to FRICKER JR and ROBBIN (2000) where they use a stochastic lead time in the bootstrap procedure. The authors proposed a generation of the value of lead time according to a statistical distribution for each step of the bootstrap procedure. So the pseudo-code with those features is shown next. The main difference from the work of ZHOU and VISWANATHAN (2008) is on step 2, where we don't use a fixed value for the lead time but a value randomly sampled from a pre-specified distribution.

1st : Generate histogram of the historical demand data (including both demand size and demand interval data) in a chosen time bucket according to some known probability distribution function.

2nd : Generate a lead time period according to some known probability distribution function.

3rd : Randomly generate demand interval according to the corresponding histogram. Update the time horizon, which is used to count the time passing by.

4th : If the time horizon is equal to or less than the lead time, randomly generate demand size according to the demand size interval histogram. Then go to 3rd step. Else, sum the generated demand sizes over the lead time and get one predicted value of the lead time demand. Then go to 5th step.

5th : Repeat steps 2 – 4 many times.

6th : Sort and generate the resulting distribution of the lead time demand.

7th : Using the CDF of the lead time demand and a pre-specified level of service, to compute the order-up-to-level.

3.3. Simulation procedure

To compare the bootstrapping method with stochastic lead time with the parametric approach we used simulated (or randomly generated) demand data. The demand stream is generated by separately generating the demand intervals (or inter-arrival times) and the demand sizes (or nonzero demands).

We use three different demand interval distributions with three different parameters, and three different demand size distributions with three different parameters to generate the stream of demands. We choose the same distributions and parameters used on work of

VISWANATHAN and ZHOU (2010). To test the effect of the stochastic lead time on the forecast methods, we use 4 lead time distribution.

For each stream, we use one statistic distribution to generate the demand interval and for each stream, we use one statistic distribution to generate the demand interval and other statistic distribution to generate the values of positive demands. The nine interval demands distributions and the nine demand size distribution used to generate the streams with their respective coefficient of variation are presented on table 2. We generated eighty one streams of demand using all the possible combinations of pairs of demand; interval of demands x demand size. With a quickly analyze we can note according SYNTETOS and BOYLAN (2005) that the streams of demands are characterized as intermittent demand and lumpy demand.

Demand Interval	CV	Demand size	CV
Uniform(2,9)	0.37	Uniform(2,23)	0.485
Uniform(3,8)	0.26	Uniform(5,20)	0.364
Uniform(4,7)	0.16	Uniform(8,17)	0.208
2+Exp(3)	0.6	Normal(20,16)	0.2
2.5+Exp(2.5)	0.5	Normal(20,9)	0.15
3+Exp(2)	0.4	Normal(20,4)	0.1
2+Gamma(0.25,1)	0.667	Lognormal(2.0,0.4)	0.701
2+Gamma(0.5,2)	0.471	Lognormal(2.0,0.3)	0.591
2+Gamma(1,4)	0.333	Lognormal(2.0,0.2)	0.471

Table 1: Used distribution functions and their CV's

For the tests of the methods we generate in MATLAB 7.0 a stream of demand with 2000 points of positive demand. We use the generated stream of demand until the 1000th point of positive demand as historical demands and calculate the order-up-to-level with the bootstrap and the parametric method. The simulation tests run with the stream of demands from 1001th demand positive until the 2000th point. The stream of demand generated is very long to avoid the influence of the size of generated demand on the tests. The time unit chosen in our simulation process is day.

In the calculation of all methods we also used four different lead time distributions. The distributions that we use were fix lead time of 30 days, Uniform (20, 40), Normal (30, 8) and Exponential (30) all with mean 30. For spare parts, the lead time for replenishment is long, so we considered median of one month as reasonable value.

It is not reasonable to think of a supplier with a great variability on its lead time, except for some specific cases due to the nature of the process, because great randomness implies more uncertainty and less attractiveness for the companies interested in buying its products. We decided to consider the variability of the lead time using distributions in which we could control the standard deviation to be small and oscillating around a mean value, like the normal distribution. The exponential distribution were used to analyze the behavior of our model when subject to a more extreme situation, where we could have lead time generated values with a bigger variability.

The inventory system used to analyze the impact of stochastic lead time was the order-up-to-level model as in CROSTON (1972), LEVEN and SEGERSTEDT (2004), and EAVES and KINGSMAN (2004) which is a continuous review system where quantity of items are ordered to reach the level of inventory S as soon as the inventory reaches the reorder point S-1. Our model admitted backlog.

The measure to compare the performance of the different methods was the total inventory related cost composed of holding cost, penalty cost and a fixed ordering cost. We also use the service cycle per level CSL and the fill rate FR as measures of the quality of the method. It is assumed that an order is placed immediately after the occurrence of a demand. Orders do not cross.

The values considered for the cost parameters are also consistent with the work of

VISWANATHAN and ZHOU (2010), who states that the total cost can "realistically reflect the effect of both under-forecasting and over-forecasting. In reality, under-forecasting results in shortages or penalty cost, which is normally more serious than over-forecasting, which only results in excess inventory or holding cost".

The steps of the simulation process can be summarized as:

1. Any orders placed LT periods before is received, and the stock levels are updated.
2. Demand occurs and is satisfied immediately, if stock is available.
3. If the stock level is under the order-up-to-level an order is placed with Q = order-up-to-level- inventory level + backlog.

4. Results

In order to evaluate the order-up-to-level achieved from the five methods and how they behave against a stochastic lead time, we simulate the order-up-to-level calculated for each method using the same stream of demand.

Value settings	
Demand size	Uniform,lognormal,normal
Demand interval	Uniform,gamma,exponential
Panalty cost to holding cost ratio	5
Cycle service level – CSL	80% , 85% , 90% , 95%
Smoothing parameter	0.1

Table 2: Summary of major parameters in the simulation study.

On the table 2 we present a summary of all tests. For every test we use a smoothing parameter equal 0.1 for the size of demand and the size of interval between positive demands for the modified Croston method as on ZHOU and VISWANATHAN (2010). We also use a penalty ratio between stock out and hold cost of 5 for all tests for each service level.

The table 3 shows the results of each method considering the average of the measures for all lead time distributions (except lead time exponentially distributed) for the service levels 80%, 85%, 90% and 95%. We excluded the LT=Exponential (30) because the measures for this distribution are outliers.

SL=0.95	AIL	FR	CSL	TC
boot/emp	57	98.05%	97.08%	253248
Croston/gamma	55	94.65%	92.55%	255021
SL=0.90	AIL	FR	CSL	TC
boot/emp	43	94.30%	91.86%	194130
Croston/gamma	38	87.97%	84.02%	182858
SL=0.85	AIL	FR	CSL	TC
boot/emp	34	89.41%	85.44%	158570
Croston/gamma	29	80.51%	74.94%	143805
SL=0.80	AIL	FR	CSL	TC
boot/emp	28	83.75%	78.27%	134082
Croston/gamma	22	72.74%	65.85%	119083

Table 3: Average measures for service level

As The table 3 shows for the service level 95% the FR and CSL for the method based on bootstrap forecast achieve averages bigger than the pre-specified service level. The method based in the Croston method achieved FR and CSL close but inferior to the pre-specified service level.

For service levels 80%, 85% and 90% the FR of bootstrap method on simulation results are bigger than the pre-specified service level as happens with the CSL measures but the measures of CSL are closer to the pre-specified service level. The results of the simulation of FR and CSL for the Croston methods get worse when the service level decreases, achieving values below the pre-specified service level. The total costs depends for the 95% pre-specified service level is equivalent, but for the others pre-specified service levels the Croston method presents lower costs.

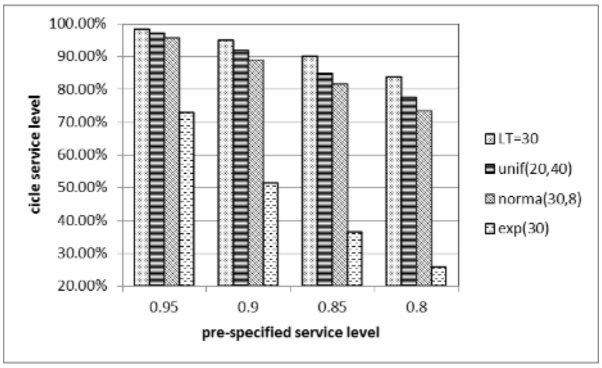

Figure 2: Cycle service level for the bootstrap method

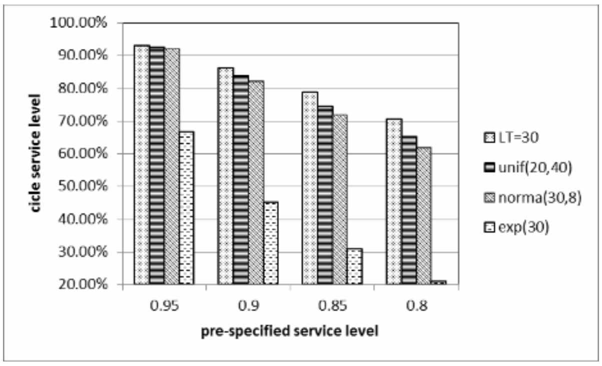

Figure 3: Cycle service level for the Croston method

Os The figures 2 and 3 show a summary of the cycle service level of all tests for all lead time distributions for bootstrap method and Croston method respectively. The bootstrap method presents CSL for all lead time distributions bigger than CSL of the Croston method.

The figures 2 and 3 show the effect of the variance of the lead time distribution. The figure shows for all pre-specified service level that all CSL achieving in the simulation results decreases with the increases of the variance of the lead time distribution. The behavior of the other measures are similar to the behavior shown on figures 2 and 3.

5. Conclusions

As In this paper, we have proposed and tested a version of the bootstrap method based on previous work of VISWANATHAN and ZHOU (2008). Also we have proposed and tested the Croston method using stochastic lead time by generating demand data. The computational results show that the method with bootstrap forecast perform better than method with Croston forecast, under an analysis using total cost, Fill rate, Customer service level and average inventory level as performance parameters.

Our method is easy to implement and proved to be an efficient way for determining reorder points in situations where stochastic demand and lead time are relevant factors.

6. References

Boylan, J. E. & Syntetos, A. A. (2010), Spare parts management: a review of forecasting research and extensions, *IMA Journal of Management Mathematics*, 21, 227–237.

Bradley, J. e Robinson, L. (2005), Improved base-stock approximations for independent stochastic lead times with order crossover. *Manufacturing and Service Operations Management*, 7, 319–329.

Brown, R. G. *Statistical forecasting for inventory control.* McGraw-Hill, New York,1959.

Burgin, T. (1975), The gamma distribution and inventory control, *Operations Research Quarterly,* 26, 507–525.

Croston, J. D. (1972), Forecasting and stock control for intermittent demands, *Research Quarterly,* 23, 289–303.

Eaves, A. & Kingsman, B. (2004), Forecasting for the ordering and stock-holding of spare parts, *Journal of the Operational Research Society,* 55, 431–437.

Efron, B. (1979), Bootstrap methods: another look at the jackknife, *Annal of Statistics,* 7, 1–26.

Fricker Jr, R. e Robbin, M. *Retooling for the logistics revolution designing marine corps inventories to support the warfighter.* RAND, Santa Monica, California, 2000.

Ghobbar, A. e Friend, C. (2003), Evaluation of forecasting methods for intermittent parts demand in the field of aviation: a predictive model. *Computers & Operations Research,* 30, 2097-2114.

Levén, E. e Segerstedt, A. (2004), Inventory control with a modified croston procedure and erlang distribution. *International Journal of Production Economics,* 90, 361-367.

Riezebos, J. (2006) Inventory order crossovers. *International Journal of Production Economics,* 104 666-675.

Silver, E. & Pyke, D. & Peterson, R. *Inventory Management and Production Planning and Scheduling,* John Wiley & Sons, New York, London, Sidney, third Edition, (1971).

Syntetos, A. A. & Boylan, J. E. (2001), On the bias of intermittent demand estimates, *International Journal of Production Economics*, 71, 457-466.

Syntetos, A. A. & Boylan, J. E. (2005), The accuracy of intermittent demand estimates, *International Journal of Production Economics*, 21, 303-314.

Viswanathan, S. e Zhou, C. (2008), A new bootstrapping based method for forecasting and safety stock determination for intermittent demand items. *Working paper*, Nanyang Business School, Nanyang Technological University, Singapore.

Viswanathan, S. e Zhou, C. (2010), Comparison of a new bootstrapping method with parametric approaches for safety stock determination in service parts inventory, *International Journal of Production Economics*, doi:10.1016/j.ijpe.2010.09.021.

Willemain, T. R. & Smart, C. N. & Schwarz, H. F. (2004), A new approach to forecasting intermittent demand for service parts inventories, *International Journal of Forecasting*, 20, 373-387.