Application of L-Shaped decomposition techniques for the problem of supply chain investment planning under demand uncertainty

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Abstract

This paper presents the application of the L-Shaped algorithm for the problem of supply chain investment and planning under uncertainty applied to the petroleum byproducts supply chain. The uncertainty considered is related with the unknown demand levels for byproducts. For this purpose, a model was developed based on two-stage stochastic programming. It is proposed two different solution methodologies, one based on the classical L-Shaped formulation, and the other, based on a multi cut extension of it. The methods were evaluated on a real sized case study. Preliminary numerical results obtained from computational experiments are encouraging.

Keywords: Supply Chain Investment Planning, Stochastic Optimization, L-Shaped Decomposition

1 Introduction

Oil companies are global multinational organizations whose decisions involve a large number of factors related to the supply of raw materials, their processing and distribution. For companies with strongly diversified sources of petroleum supply, a long cast of products, and multiple markets, the advance planning of all activities along the supply chain is vital. Such planning includes the definition of production levels of oil (from oil fields and offshore platforms) and of petroleum products (from oil refineries), as well as the distribution among these refineries and to the final consumers of oil products. Major oil companies are characterized by integrated and verticalized activities, and the activities of refining and distributing oil byproducts are characterized by low profit margins. Therefore, techniques for decision-making optimization are frequently used in the context of the oil supply chain.

The use of optimization techniques for supply chain design and planning has already been observed in the literature since the 1970's, especially the in seminal works of Geoffrion and Graves (1974). Vidal and Goetschalckx (1997) and Beamon (1998) present an extensive literature review on supply chain models. Although the research literature on the strategic modeling of supply chains is quite rich, few studies have included uncertainty mitigation in addition to other decisions of financial scope, such as commercialization income, market considerations and investment planning. According to Sahinidis (2004), the incorporation of uncertainty into planning models using stochastic optimization remains a challenge due to the large computational requirements involved.

For nearly 50 years, companies in the oil and chemical industries have led the development and use of mixed integer linear programming to support decision making at all levels of planning. An overriding feature in the oil industry is its wide range of uncertainties, typically related to the unpredictable levels of demand for refined products, fluctuations in prices in domestic and international markets and inaccuracies in the forecasted production of oil and gas. For this reason, many works have used techniques based on mathematical programming to support decision-making under uncertainty.(Escudero et al., 1999, Dempster et al., 2000, Al-Othman et al., 2008, Khor et al., 2008)

Due to the great level of uncertainties taken into consideration, and the fact that the aforementioned problem is modeled as a mixed-integer linear program, it might become computationally infeasible to deal with great number of scenarios by solving deterministic equivalents of the stochastic problems. Therefore, a decomposition approach might turn out to be a valid alternative as solution methodology.

The first approaches using decomposition schemes for stochastic programs were presented by Van Slyke and Wets (1969), a framework based on Benders decomposition (Benders, 1962) directly applied to two-stage stochastic problems, which became known as the L-Shaped method. Birge and Louveaux (1988) present an extension of the method presented by Van Slyke and Wets (1969), exploiting the structure of two-stage stochastic problems to place several cuts at once at each major iteration.

Cutting-plane schemes has been successfully used in solving large-scale problems since the pioneering paper of Geoffrion and Graves (1974): e.g., the uncapacitated network design problem with undirected arcs (Magnanti et al., 1986), the stochastic transportation-location problems (França and Luna, 1982), the locomotive and car assignment problem (Cordeau et al., 2000, 2001), and the non-convex water resource management problem (Cai et al., 2001), to name a few.

The objective of this paper is present a mathematical model for the optimization of the supply chain investment planning problem applied to the petroleum byproducts supply chain. Uncertainties related to product demand levels are considered, thus, the stochastic programming framework is adopted as modeling approach. Furthermore, it is shown an application of two decomposition techniques based on L-Shaped decomposition (Van Slyke and Wets, 1969) as solution technique. Experiments were performed in order to evaluate the efficiency of the proposed algorithms.

The document is organized as follows: section 2 describes the proposed mathematical model; section 3 presents the L-Shaped decomposition framework, while section 4 presents the multi cut framework for the L-Shaped algorithm; computational results are shown in section 5; Section 6 draws some conclusion.

2 Mathematical Model

Petroleum byproducts supply chains are composed by several types of nodes and arcs. Nodes are different in a sense that they might represent refineries, international markets, distribution bases, and marine terminals. Arcs are the connections between the nodes, and might represent pipelines, roadways, waterways, and so forth.

The objective here is to choose, among some possible investment, which projects should be implemented in order to reach the best logistic efficiency. What we understand as the ideal logistic efficiency here is the configuration that would provide the lowest combination of costs for the chain. The system is subject to several costs. Costs are related with freight, operation, product inventory, investments, and demand shortfall. The commercialization of petroleum byproducts is also considered, since the logistic configuration must take into consideration such opportunities.

To address the problem in question, a two-stage stochastic model is proposed based on mathematical programming (Birge and Louveaux, 1997). The first-stage comprises the decisions of which projects to implement and when; the second-stage decisions are those relating to the flows of products, inventory levels, supply provided to each demand site, amounts to be internally and externally commercialized, and supply levels at sources. The purpose of the model is to provide the optimal distribution of refined products to meet the demand of distribution bases, minimizing the logistics costs of this operation and maximizing revenue for retailing such products. Meeting the demand depends on the characteristics of the network operations, refinery availability and sources of production. The production distribution is defined in conjunction with investment decisions, which are chosen from a predefined portfolio of possibilities and allocated over the planning horizon. The uncertainties in the model are related to the levels of demand for byproducts in the distribution bases, which are modeled as random variables.

The notation to be used for the presentation of the mathematical model is presented below. In order to provide greater clarity to the notation, the domains of summations will be omitted, except when the summation is evaluated only on a subset of the natural domain. When there is no mention of this fact, its domain should be considered as the original set to which the index refers.

2.1 Indexes and Sets

$i, i' \in \mathcal{B}$ - Set of basis
$j \in \mathcal{R}$ - Set of refineries
$k, k' \in \mathcal{M}$ - Set of marine terminals
$l \in \mathcal{J}$ - Set of international markets
$p \in \mathcal{P}$ - Set of products
$t \in \mathcal{T}$ - Set of periods
$\xi \in \Omega$ - Uncertainty realizations (scenarios)

2.2 Constants

2.2.1 Freight Costs

FTB_{ki}^t - Terminal-base
FRB_{ji}^t - Refinery-base
FRT_{jk}^t - Refinery-terminal
FTE_{kl}^t - Terminal-international market
FET_{lk}^t - International Market-terminal
$FBB_{ii'}^t$ - Base-base
$FTT_{kk'}^t$ - Terminal-terminal

2.2.2 Inventory Costs

EB_{ip} Base inventory cost
ET_{kp} Terminal inventory cost

2.2.3 Arc Capacities

ATB_{ki}^t - Terminal-base seasonal adjustment
ARB_{ji}^t - Refinery-base seasonal adjustment
ART_{jk}^t - Refinery-terminal seasonal adjustment
$ABB_{ii'}^t$ - Base-base seasonal adjustment
$ATT_{kk'}^t$ - Terminal-terminal seasonal adjustment
CTB_{ki} - Terminal-base before investment
$CITB_{ki}$ - Terminal-base after investment
CRB_{ji} - Refinery-base before investment
$CIRB_{ji}$ - Refinery-base after investment
CRT_{jk} - Refinery-terminal before investment
$CIRT_{jk}$ - Refinery-terminal after investment
$CBB_{ii'}$ - Base-base before investment
$CIBB_{ii'}$ - Base-base after investment
$CTT_{kk'}$ - Terminal-terminal before investment
$CITT_{kk'}$ - Terminal-terminal after investment

2.2.4 Node Capacities

TB_{ip} - Base storage capacity before investment
TIB_{ip} - Base storage capacity after investment
TT_{kp} - Terminal storage capacity before investment

TIT_{kp} - Terminal storage capacity after investment
GB^t_{ip} - Base throughput rotation
GT^t_{ip} - Terminal throughput rotation

2.2.5 Investment Costs

IRT^t_{jk} - Refinery-terminal connection
ITB^t_{ki} - Terminal-base connection
IRB^t_{ji} - Refinery-base connection
$ITT^t_{kk'}$ - Terminal-terminal connection
$IBB^t_{ii'}$ - Base-base connection
IT^t_{kp} - Terminal storage
IB^t_{ip} - Base storage

2.2.6 Market parameters

$DB^t_{ip}(\xi)$ - Product demand at the base
OR^t_{jp} - Refinery supply
OM^t_{lp} - Export limit
POM^t_{lp} - Export sales price
DM^t_{lp} - Import Limit
PDM^t_{lp} - Import purchase price
PDS_{ip} - Demand shortfall cost

2.3 Variables

2.3.1 Continuous

$e^t_{klp}(\xi)$ - Amount exported
$i^t_{lkp}(\xi)$ - Amount imported
$u^t_{jkp}(\xi)$ - Refinery-Terminal flow
$w^t_{jip}(\xi)$ - Refinery-base flow
$v^t_{kip}(\xi)$ - Terminal-base flow
$x^t_{ii'p}(\xi)$ - Flow between bases
$z^t_{kk'p}(\xi)$ - Flow between terminals
$t^t_{kp}(\xi)$ - Terminal inventory
$s^t_{ip}(\xi)$ - Base inventory
$o^t_{ip}(\xi)$ - Demand shortfall

2.3.2 Binary (Investment decisions)

υ^t_{jk} - Refinery-terminal connection
φ^t_{ki} - Terminal-base connection
ω^t_{ji} - Refinery-base connection
$\kappa^t_{kk'}$ - Connection between terminals
$\chi^t_{ii'}$ - Connection between bases
σ^t_{ip} - Base tank
τ^t_{kp} - Terminal tank

2.4 Formulation

The mathematical model for the optimization of aforementioned problem can be stated as follows:

$$\min \sum_{jkt} IRT^t_{jk}\upsilon^t_{jk} + \sum_{kit} ITB^t_{ki}\varphi^t_{ki} + \sum_{jit} IRB_{ji}\omega^t_{ji} + \sum_{kk't} ITT_{kk't}\kappa^t_{kk'} + \sum_{ii't} IBB_{ii'}\chi^t_{ii'}$$
$$+ \sum_{ipt} IB_{ip}\sigma^t_{ip} + \sum_{kpt} IT_{kt}\tau^t_{kp} + \mathcal{Q}(\upsilon,\varphi,\omega,\kappa,\chi,\sigma,\tau) \quad (1)$$

s.t.

$$\sum_t \upsilon^t_{jk} \le 1 \quad \forall j,k \quad (2)$$

$$\sum_t \varphi^t_{ki} \le 1 \quad \forall k,i \quad (3)$$

$$\sum_t \omega^t_{ji} \le 1 \quad \forall j,i \quad (4)$$

$$\sum_t \kappa^t_{kk'} \le 1 \quad \forall k,k'|k \ne k' \quad (5)$$

$$\sum_t \chi^t_{ii'} \le 1 \quad \forall i,i'|i \ne i' \quad (6)$$

$$\sum_t \sigma^t_{ip} \le 1 \quad \forall i,p \quad (7)$$

$$\sum_t \tau^t_{kp} \le 1 \quad \forall k,p \quad (8)$$

where the term $\mathcal{Q}(\upsilon,\omega,\varphi,\chi,\kappa,\sigma,\tau) = \mathbb{E}_\Omega\left[Q(\upsilon,\omega,\varphi,\chi,\kappa,\sigma,\tau,\xi)\right]$ represents the expectation evaluated over all $\xi \in \Omega$ possible realizations of the uncertain parameters of the second-stage problem, given a investment decision $(\upsilon,\omega,\varphi,\chi,\kappa,\sigma,\tau)$. Constraints (2)-(8) define that each investment can happens only once along the time horizon considered. The second-stage problem $Q(\upsilon,\omega,\varphi,\chi,\kappa,\sigma,\tau,\xi)$ can be stated as follows:

$$Q(\upsilon,\omega,\varphi,\chi,\kappa,\sigma,\tau,\xi) =$$
$$\min \sum_{kipt} FTB_{ki} v^t_{kip}(\xi) + \sum_{jipt} FRB_{ji} w^t_{jip}(\xi) + \sum_{jkpt} FRT_{jk} u^t_{jkp}(\xi) + \sum_{klpt}(FTE_{kl} - PDM_{lp}) e_{klp}(\xi)^t +$$
$$+ \sum_{lkpt}(FET_{lk} + POM_{lp}) i^t_{lkp}(\xi) + \sum_{ii'pt} FBB_{ii'} x^t_{ii'p}(\xi) + \sum_{kk'pt} FTT_{kk'} z^t_{kk'p}(\xi)$$
$$+ \sum_{ipt} EB_{ip} s^t_{ip}(\xi) + \sum_{kpt} ET_{kp} t^t_{kp}(\xi) + \sum_{ipt} PDS_{ip} o^t_{ip}(\xi) \tag{9}$$

s.t.

$$\sum_{k' \neq k} z^t_{k'kp}(\xi) + \sum_j u^t_{jkp}(\xi) + \sum_l i^t_{lkp}(\xi) + t^{t-1}_{kp}(\xi) =$$
$$\sum_i v^t_{kip}(\xi) + \sum_l e^t_{klp}(\xi) + t^t_{kp}(\xi) + \sum_{k' \neq k} z^t_{kk'p}(\xi) \quad \forall k,p,t \tag{10}$$

$$\sum_k v^t_{kip}(\xi) + \sum_j w^t_{jip}(\xi) + \sum_{i' \neq i} x^t_{i'ip}(\xi) + s^{t-1}_{ip} = D^t_{ip}(\xi) - o^t_{ip}(\xi) + s^t_{ip}(\xi) + \sum_{i' \neq i} x^t_{ii'p}(\xi) \quad \forall i,p,t \tag{11}$$

$$\sum_k u^t_{jkp}(\xi) + \sum_i w^t_{jip} \leq OR^t_{jp} \quad \forall j,p,t \tag{12}$$

$$\sum_k i^t_{lkp}(\xi) \leq OM_{lp} \quad \forall l,p,t \tag{13}$$

$$\sum_k e^t_{klp}(\xi) \leq DM_{lp} \quad \forall l,p,t \tag{14}$$

$$\sum_p w^t_{jip}(\xi) \leq ARB^t_{ji}\left[CRB_{ji} + (CIRB_{ji} - CRB_{ji}) \sum_{t' \leq t} \omega^{t'}_{ji}\right] \quad \forall j,i,t \tag{15}$$

$$\sum_p u^t_{jkp}(\xi) \leq ART^t_{jk}\left[CRT_{jk} + (CIRT_{jk} - CRT_{jk}) \sum_{t' \leq t} \upsilon^{t'}_{jk}\right] \quad \forall j,k,t \tag{16}$$

$$\sum_p v^t_{kip}(\xi) \leq ATB^t_{ki}\left[CTB_{ki} + (CITB_{ki} - CTB_{ki}) \sum_{t' \leq t} \varphi^{t'}_{ki}\right] \quad \forall k,i,t \tag{17}$$

$$\sum_p x^t_{ii'p}(\xi) \leq ABB^t_{ii'}\left[CBB_{ii'} + (CIBB_{ii'} - CBB_{ii'}) \sum_{t' \leq t} \chi^{t'}_{ii'}\right] \quad \forall i,i',t \tag{18}$$

$$\sum_p z^t_{kk'p}(\xi) \leq ATT^t_{kk'}\left[CTT_{kk'} + (CITT_{kk'} - CTT_{kk'}) \sum_{t' \leq t} \kappa^{t'}_{kk'}\right] \quad \forall k,k',t \tag{19}$$

$$s^t_{ip}(\xi) \leq TB_{ip} + (TIB_{ip} - TB_{ip}) \sum_{t' \leq t} \sigma^{t'}_{ip} \quad \forall i,p,t \tag{20}$$

$$t^t_{kp}(\xi) \leq TT_{kp} + (TIT_{kp} - TT_{kp}) \sum_{t' \leq t} \tau^{t'}_{kp} \quad \forall k,p,t \tag{21}$$

$$\sum_{k} v^{t}_{kip}(\xi) + \sum_{j} w^{t}_{jip}(\xi) + \sum_{i' \neq i} x^{t}_{i'ip}(\xi) \leq GB_{ip}TB_{ip} \quad \forall i, p, t \tag{22}$$

$$\sum_{k' \neq k} z^{t}_{k'kp}(\xi) + \sum_{j} u^{t}_{jkp}(\xi) + \sum_{l} i^{t}_{lkp}(\xi) \leq GT_{kp}TT_{kp} \quad \forall k, p, t \tag{23}$$

The objective function represents freight costs between the nodes, the net value of importation and exportation, inventory costs at terminals and bases, and the cost of the unmet demand. Equation (10) comprises the material balance for marine terminals. Equation (11) represents the material balance for bases. Constraint (12) limits the amount offered by refineries. Constraints (13) and (14) limit the amounts imported from/exported to international markets. Constraints (15) to (19) model arc capacities and the possibility of substituting its current capacity with an eventual investment subject to a seasonal adjustment due to external reasons. Constraints (20) and (21) represent the storage capacity of bases and terminals and the possibility of investing on its expansion. Constraints (22) and (23) limit the maximum throughput allowed for bases and terminals.

3 L-Shaped Algorithm

For notational simplicity, let us define $\mathrm{y} = (\upsilon, \omega, \varphi, \chi, \kappa, \sigma, \tau)$ as the first-stage decisions, and Y as the feasible set for y. The model in the previous section can be defined as an optimization model with binary first-stage variables, continuous second-stage variables and discrete random parameters. Moreover, the model has relatively complete recourse (**?**), that is for any feasible first stage solution, the second stage is feasible ($Q(\mathrm{y}, \xi) < +\infty$ for any $\mathrm{y} \in \mathrm{Y}$). Such characteristics allow us to use the Benders decomposition framework (Benders, 1962) applied to stochastic optimization, commonly known as L-Shaped Algorithm (Van Slyke and Wets, 1969). We start by noting that the *master problem* can be equivalently reformulated as follows:

$$\begin{aligned} &\min \mathcal{F}(\mathrm{y}) + M \\ &\text{s.t.} \\ &\quad (2) \text{ to } (8) \\ &\quad M \geq \mathcal{Q}(\mathrm{y}) \end{aligned}$$

Where $\mathcal{F}(\mathrm{y})$ represents the first-stage cost function. This formulation allows one to distinguish an important issue. The inequality $M \geq \mathcal{Q}(\mathrm{y})$ cannot be used computationally as a constraint, since it is not defined explicitly, but only implicitly, by a number of optimization problems. The main idea of the L-Shaped method is to relax this constraint and replace it by a number of cuts, which may be gradually added following an iterative solving process. The L-Shaped method applied to the aforementioned problem can be stated as follows:

Initialization: Define LB and UB as lower and upper bounds. Set $LB = -\infty$ and $UB = \infty$. Define B as the iteration counter and set $B = 0$. Let $\hat{\mathrm{y}}$ denote the incumbent solution.

Step 1: Solve the *master problem* and let y^B and LB be its optimal solution and optimal objective value respectively.

Step 2: For each realization $\xi \in \Omega$ solve the slave problem (9)-(23) stated before for y^B and calculate the value for $\hat{\mathcal{Q}}(\mathrm{y}^B)$ given by:

$$\hat{\mathcal{Q}}(\mathrm{y}^B) = \sum_{\xi \in \Omega} P(\xi) Q(\mathrm{y}^B, \xi)$$

Where $P(\xi)$ is the probability of realization ξ occurs. Let:

$$\mathcal{G}(\mathrm{y}^B) = \mathcal{F}(\mathrm{y}^B) + \hat{\mathcal{Q}}(\mathrm{y}^B)$$

If $\mathcal{G}(\mathrm{y}^B) < UB$ then update $UB = \mathcal{G}(\mathrm{y}^B)$ and the incumbent solution $\hat{\mathrm{y}} = \mathrm{y}^B$

Step 3: If $UB - LB \leq \epsilon$, where ϵ is a pre specified tolerance, then return the incumbent solution $\hat{\mathrm{y}}$ and the objective function value UB. Otherwise, proceed to *step 4*.

Step 4: Let $\alpha(\xi), \beta(\xi), \gamma(\xi), \delta(\xi), \zeta(\xi), \eta(\xi), \theta(\xi), \lambda(\xi), \mu(\xi), \pi(\xi), \rho(\xi), \iota(\xi), \vartheta(\xi)$, and $\phi(\xi)$ be the dual variables associated with constraints $(10) - (23)$ of the second-stage problem for each realization $\xi \in \Omega$. Generate the cut:

$$M \geq \sum_{jkt} a^t_{jkb} \sum_{t' \leq t} \upsilon^{t'}_{jk} + \sum_{jit} b^t_{jib} \sum_{t' \leq t} \omega^{t'}_{ji} + \sum_{kit} c^t_{ki} \sum_{t' \leq t} \varphi^{t'}_{ki} + \sum_{ii't} d^t_{ii'b} \sum_{t' \leq t} \chi^{t'}_{ii'} +$$
$$\sum_{kk't} e^t_{kk'b} \sum_{t' \leq t} \kappa^{t'}_{kk'} + \sum_{ipt} f^t_{ipb} \sum_{t' \leq t} \sigma^{t'}_{ip} + \sum_{kpt} g^t_{kpb} \sum_{t' \leq t} \tau^{t'}_{kp} + K_b \quad \forall b = 1, ..., B$$

where:

$$a^t_{jkB} = \sum_{\xi \in \Omega} P(\xi) \Big[ART^t_{jk}(CIRT_{jk} - CRT_{jk}) \theta^t_{jk}(\xi) \Big]$$
$$b^t_{jiB} = \sum_{\xi \in \Omega} P(\xi) \Big[ARB^t_{ji}(CIRB_{ji} - CRB_{ji}) \eta^t_{ji}(\xi) \Big]$$
$$c^t_{kiB} = \sum_{\xi \in \Omega} P(\xi) \Big[ATB^t_{ki}(CITB_{ki} - CTB_{ki}) \lambda^t_{ki}(\xi) \Big]$$
$$d^t_{ii'B} = \sum_{\xi \in \Omega} P(\xi) \Big[ABB^t_{ii'}(CIBB_{ii'} - CBB_{ii'}) \mu^t_{ii'}(\xi) \Big]$$
$$e^t_{kk'B} = \sum_{\xi \in \Omega} P(\xi) \Big[ATT^t_{kk'}(CITT_{kk'} - CTT_{kk'}) \pi^t_{kk'}(\xi) \Big]$$
$$f^t_{ipB} = \sum_{\xi \in \Omega} P(\xi) \Big[(TIB_{ip} - TB_{ip}) \rho^t_{ip}(\xi) \Big]$$
$$g^t_{kpB} = \sum_{\xi \in \Omega} P(\xi) \Big[(TIT_{kp} - TT_{kp}) \iota^t_{kp}(\xi) \Big]$$
$$K_B = \sum_{\xi \in \Omega} P(\xi) \Bigg[\sum_{ipt} D^t_{ip}(\xi) \beta^t_{ip}(\xi) + TB_{ip} \rho^t_{ip}(\xi) + \sum_{jpt} OR^t_{jp} \gamma^t_{jp}(\xi) + \sum_{lpt} OM^t_{lp} \delta^t_{lp}(\xi) + DM_{lp} \zeta^t_{lp}(\xi) +$$
$$\sum_{jit} ARB_{jit} CRB_{ji} \eta^t_{ji}(\xi) + \sum_{jkt} ART_{jkt} CRT_{jk} \theta^t_{jk}(\xi) + \sum_{kit} ATB^t_{ki} CTB_{ki} \lambda^t_{ki}(\xi) + \sum_{ii't} ABB_{ii't} CBB_{i,i'} \mu^t_{ii'}(\xi) +$$
$$\sum_{kk't} ATT_{kk't} CTT_{kk'} \pi^t_{kk'}(\xi) + \sum_{kpt} TT_{kp} \iota^t_{kp}(\xi) + \sum_{ipt} GB_{ip} TB_{ip} \vartheta^t_{ip}(\xi) + \sum_{kpt} GT_{kp} TT_{kp} \phi^t_{kp}(\xi) \Bigg]$$

and add it to the *master problem*. Update $B = B + 1$ and go to *step 1*.

4 Multi Cut L-Shaped Algorithm

The structure of stochastic programs allows one to add multiple cuts instead of a single one at each major iteration. Birge and Louveaux (1988) shown in their work that the usage of such a framework may greatly speed up convergence. The main idea behind this multi cut framework is to generate an outer linearization for all function $Q(\mathrm{y}, \xi)$, replacing the outer linearization of $\mathcal{Q}(\mathrm{y})$. The multi cut approach relies on the idea that using outer approximations of all $Q(\mathrm{y},\xi)$ send more information than the single cut on $\mathcal{Q}(\mathrm{y})$ and that, therefore, fewer iterations are needed. In fact, following Birge and Louveaux (1988), it is possible to show that the maximum number of iterations for the multi cut procedure is given by:

$$1 + |\Omega|(q^m - 1) \tag{24}$$

While the maximum number of iterations for the L-shaped algorithm is given by:

$$[1 + |\Omega|(q - 1)]^m \tag{25}$$

where q represents the total of slopes for the second-stage problem and m the number of recourse constraints. Although q might turn out to be quite complicated to calculate for real world problems, bounds (24) and (25) show that the maximum number of iterations needed for reaching the optimum grows linearly with the number of realizations for the multi cut approach, while it grows exponentially for the traditional L-Shaped approach. We start by reformulating the original *master problem* to conveniently adequate it to the multi cut framework:

$$\min \mathcal{F}(\mathrm{y}) + \sum_{\xi\in\Omega} P(\xi)M(\xi)$$

s.t.

(2) to (8)

$$M(\xi) \geq Q(\mathrm{y},\xi) \quad \forall \xi \in \Omega$$

The multi cut L-Shaped procedure can be stated as follows:
Initialization: Set lower and upper bounds $LB = -\infty$ and $UB = \infty$. Set the iteration counter $B = 0$. Let $\hat{\mathrm{y}}$ denote the incumbent solution.

Step 1: Solve the *master problem* and let y^B and LB be its optimal solution and the optimal objective value.

Step 2: For each realization $\xi \in \Omega$ solve the slave problem (9)-(23) stated before and calculate the value for $\hat{\mathcal{Q}}(\mathrm{y}^B)$ given by:

$$\hat{\mathcal{Q}}(\mathrm{y}^B) = \sum_{\xi\in\Omega} P(\xi)Q(\mathrm{y}^B,\xi)$$

Where $P(\xi)$ is the probability of realization ξ occurs. Let:

$$\mathcal{G}(\mathrm{y}^B) = \mathcal{F}(\mathrm{y}^B) + \hat{\mathcal{Q}}(\mathrm{y}^B)$$

If $\mathcal{G}(\mathrm{y}^B) < UB$ then update $UB = \mathcal{G}(\mathrm{y}^B)$ and the incumbent solution $\hat{\mathrm{y}} = \mathrm{y}^B$

Step 3: If $UB - LB \leq \epsilon$ then return the incumbent solution $\hat{\mathrm{y}}$ and the objective function value UB. Otherwise, proceed to *step 4*.

Step 4: Let $\alpha(\xi), \beta(\xi), \gamma(\xi), \delta(\xi), \zeta(\xi), \eta(\xi), \theta(\xi), \lambda(\xi), \mu(\xi), \pi(\xi), \rho(\xi), \iota(\xi), \vartheta(\xi)$, and $\phi(\xi)$ be the dual variables associated with constraints $(10) - (23)$ of the second-stage problem for each realization $\xi \in \Omega$. Generate the cuts:

$$M(\xi) \geq \sum_{jkt} a^t_{jk}(\xi)_b \sum_{t'\leq t} \upsilon^{t'}_{jk} + \sum_{jit} b^t_{jib}(\xi)_b \sum_{t'\leq t} \omega^{t'}_{ji} + \sum_{kit} c^t_{ki}(\xi)_b \sum_{t'\leq t} \varphi^{t'}_{ki} + \sum_{ii't} d^t_{ii'b}(\xi)_b \sum_{t'\leq t} \chi^{t'}_{ii'} +$$
$$\sum_{kk't} e^t_{kk'b}(\xi)_b \sum_{t'\leq t} \kappa^{t'}_{kk'} + \sum_{ipt} f^t_{ipb}(\xi)_b \sum_{t'\leq t} \sigma^{t'}_{ip} + \sum_{kpt} g^t_{kpb}(\xi)_b \sum_{t'\leq t} \tau^{t'}_{kp} + K_b(\xi)_b \quad \forall b = 1, ..., B, \forall \xi \in \Omega$$

where:

$$a^t_{jk}(\xi)_B = \left[ART^t_{jk}(CIRT_{jk} - CRT_{jk})\theta^t_{jk}(\xi)\right]$$

$$b^t_{ji}(\xi)_B = \left[ARB^t_{ji}(CIRB_{ji} - CRB_{ji})\eta^t_{ji}(\xi)\right]$$

$$c^t_{ki}(\xi)_B = \left[ATB^t_{ki}(CITB_{ki} - CTB_{ki})\lambda^t_{ki}(\xi)\right]$$

$$d^t_{ii'}(\xi)_B = \left[ABB^t_{ii'}(CIBB_{ii'} - CBB_{ii'})\mu^t_{ii'}(\xi)\right]$$

$$e^t_{kk'}(\xi)_B = \left[ATT^t_{kk'}(CITT_{kk'} - CTT_{kk'})\pi^t_{kk'}(\xi)\right]$$

$$f^t_{ip}(\xi)_B = \left[(TIB_{ip} - TB_{ip})\rho^t_{ip}(\xi)\right]$$

$$g^t_{kp}(\xi)_B = \left[(TIT_{kp} - TT_{kp})\iota^t_{kp}(\xi)\right]$$

$$K(\xi)_B = \Big[\sum_{ipt} D^t_{ip}(\xi)\beta^t_{ip}(\xi) + TB_{ip}\rho^t_{ip}(\xi) + \sum_{jpt} OR^t_{jp}\gamma^t_{jp}(\xi) + \sum_{lpt} OM^t_{lp}\delta^t_{lp}(\xi) + DM_{lp}\zeta^t_{lp}(\xi) +$$

$$\sum_{jit} ARB_{jit}CRB_{ji}\eta^t_{ji}(\xi) + \sum_{jkt} ART_{jkt}CRT_{jk}\theta^t_{jk}(\xi) + \sum_{kit} ATB^t_{ki}CTB_{ki}\lambda^t_{ki}(\xi) + \sum_{ii't} ABB_{ii't}CBB_{i,i1}\mu^t_{ii'}(\xi) +$$

$$\sum_{kk't} ATT_{kk't}CTT_{kk'}\pi^t_{kk'}(\xi) + \sum_{kpt} TT_{kp}\iota^t_{kp}(\xi) + \sum_{ipt} GB_{ip}TB_{ip}\vartheta^t_{ip}(\xi) + \sum_{kpt} GT_{kp}TT_{kp}\phi^t_{kp}(\xi)\Big]$$

Add the cuts to the *master problem*. Update $B = B + 1$ and go to *step 1*.

5 Numerical Results

In this section we describe numerical experiments using the proposed methodology for solving a realistic supply chain investment planning under demand uncertainty problem. The transport in the case study considered is primarily done using modal waterways, which are strongly affected by seasonality issues regarding the navigability of rivers. For this study, four different products were considered - diesel, gasoline, aviation fuel and fuel oil - to be distributed over 19 locations (13 bases, 3 of which have sea terminals, one refinery and two external supply locations). Waterway transportation is generally by large ferries, typically done during periods of river flooding and by smaller boats, which are able to navigate the sections during droughts, i.e., in periods of low levels of the rivers, which have high transportation costs. The portfolio of projects considered for the study consists of 28 local projects and three arc project. Such projects are considered mutually independent and can therefore be combined as needed by the problem. The planning horizon considered was 8 years, divided into a total of 32 quarterly periods.

To take into account the uncertainty in demand levels for byproducts, scenarios were generated by the following first order autoregressive model:

$$D^t_{lp} = D^{t-1}_{lp}\left[1 + \omega_p + \sigma\varepsilon\right], \quad t = 2, \dots, |T|$$

where ω_p represents the expected average growth rate for the consumption of product p over the planning horizon, σ represents the estimated maximum deviation for byproduct consumption in the region and $\varepsilon \sim N(0,1)$. The estimate of the maximum deviation was made based on the analysis of the annual consumption historical series over the last 40 years. Each scenario represents a possible byproduct demand curve for the whole time horizon considered, for each product and place.

N	#Variables	#Constraints	DE(s)	SCut(s)	MCut(s)
20	194,443	204,024	18.20	56.08	12.25
30	291,243	306,024	29.81	41.14	28.52
40	388,043	408,024	40.92	45.70	24.98
50	484,843	510,024	48.34	84.42	43.53
60	581,643	612,024	86.31	113.92	51.17
70	678,443	714,024	160.84	101.30	70.75
80	755,243	816,024	110.20	98.28	61.09
90	872,043	918,024	136.06	138.28	71.11
100	968,843	1,020,024	150.13	171.28	53.48

Table 1: Experiment Summary

The mathematical model and the scenario generation routines were implemented using AIMMS 3.10. The mathematical model was solved using CPLEX 11.2. All experiments were performed on a Pentium Quad-Core 2.6 GHz with 8 Gb RAM. Table 1 summarizes the data of the experiments performed.

The first column of table 1 represents the 9 different instances generated, with 20 up to 100 scenarios. The next two columns summarize the size of the complete model considering all scenarios at once, what is commonly known as the deterministic equivalent (Birge and Louveaux, 1997). It is worth to notice that all instance have the same number of integer variables, a total of 840 each. The following columns shows the solving time taken by each technique to reach the optimum of the instance, namely solving the complete deterministic equivalent (DE), using the classical L-Shaped framework (SCut), and using the proposed multi cut approach (MCut).

As can be seen in table 1, the multi cut approach has the smallest solution time for every instance, being up to 3 times faster than solving the determinitic equivalent and up to 5 times faster than using the single cut approach. Furthermore, it is worth to notice that the solution time for the single cut procedure is consistently higher than the solution of the deterministic equivalent among the experiments performed. This indicates that, for this particular case, it seems more efficient to simply solve the complete deterministic problem than use the L-Shaped procedure.

The results suggest that the classical L-Shaped framework performs worse when compared with directly solving the deterministic equivalent. However, when it is used the multi cut approach, it can be notice that there is a considerable improvement in the time taken to reach the optimal solution, specially for the instance with a greater number of scenarios.

6 Conclusions

This paper presents the application of a decomposition scheme for the problem of supply chain design applied to the petroleum byproducts supply chain. We propose a mathematical model that captures the impact of uncertainty on investment decisions, since the problem approached here is a mixture of logistic infrastructure investment planning problem and the stochastic transportation problem. With demand at each destination as a random variable, the objective is to minimize the sum of expected holding and shortage costs, transportation costs, fixed investment costs, and demand shortfall costs.

In order to solve the proposed model, we propose an application of the L-Shaped decomposition framework (Van Slyke and Wets, 1969) to the problem at hand, together with the application of the multi cut extension of it, proposed by Birge and Louveaux (1988).

The results suggest that the first approach performs worse than the second in terms of computational time. It is a expected, yet important, result that corroborates the theoretical bounds for the total number of necessary iterations before complete convergence of the algorithms. In a general sense, the multi cut framework performs better than simply solving the deterministic equivalent - or even than directly applying the classic L-Shaped framework - allowing one to solve instances of greater size and, thus, with a more precise representation of the random variables.

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