TACTICAL PLANNING OF THE OIL SUPPLY CHAIN: OPTIMIZATION UNDER UNCERTAINTY

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ABSTRACT

The oil industry is increasingly interested in improving the planning of their operations due to the dynamic nature of this business. Decisions made at the oil chain differ in the activity range (*spatial integration*) and planning horizon (*temporal integration*). This paper purpose is to address the spatial integration under uncertainty in the oil chain at the tactical planning level and proposes a mathematical model to maximize the profit of this chain. The model is formulated as two-stage stochastic program, where uncertainty is incorporated in price and demand parameters. An industrial scale study from the Brazilian industry was conducted. The Expected Value of Perfect Information (EVPI) and the Value of the Stochastic Solution (VSS) – 1.55% and 13.76% of the wait-and-see solution – indicated the benefit of incorporating uncertainty in the planning and demonstrates the effectiveness of the proposed approach. The centralized coordination (*spatial integrated*) provided a better utilization of the available resources.

KEYWORDS. Two-stage stochastic optimization. Tactical planning. Oil chain.

P&G –OR in the Oil & Gas area.

1. Introduction

The uncertain nature and high economic incentives of the refining business are driving forces for improvements in the refinery planning process. This requires a high level of decision-making not only on a single facility scale but also on an enterprise-wide scale (Chopra and Meindl, 2004). Planning for this kind of operation should be carried out centrally, thus allowing for proper integration among all operating facilities and, consequently, an efficient utilization of available resources (Al-Qahtani and Elkamel, 2008). Decisions made at the oil chain differ mainly in the range of activities coordinated in the supply chain (*spatial integration*) and in the coordination of decisions across different time scales (temporal integration). The understanding of such integration benefits has attracted attention in the research area of supply chain planning (Kogut and Kulatilaka, 1994; Huchzermeier and Cohen, 1996; Smith and McCardle, 1998; Harrison and Van Mieghem, 1999; Cachon, 2002). Planning is basically an activity in which production targets are set and market forecasts, resource availability, and inventories are considered. In general, planning is categorized into three time frames: strategic (long term), tactical (medium term), and operational (short term). Strategic planning determines the structure of the supply chain. Tactical planning is concerned with decisions such as the assignment of production targets to facilities and the transportation from facilities to distribution centers. On the other hand, operational planning determines the assignment of tasks to units at each facility, considering resource and time constraints (Maravelias and Sung, 2009). This paper covers the tactical planning level and addresses de problem of spatial integration at the oil chain.

Although planning in the oil industry was traditionally developed with well established deterministic models, these models have been extended to include uncertainties in parameters. In fact, Ben-Tal and Nemirovski (2000) stress that optimal solutions of deterministic models may become infeasible even if the nominal data is only slightly perturbed. This idea is supported by Sen and Higle (1999), who affirmed that under uncertainty the deterministic formulation, in which uncertain random variables are replaced by their expected values, may not provide a solution that is feasible with respect to the random variables. Thus, uncertainties are inevitable in mathematical modeling and also in enforcing the planning model to realistic solutions.

Uncertainties can be categorized as short-term, mid-term, or long-term. Short-term uncertainties refer to unforeseen factors in internal processes such as operational variations and equipment failures (Subrahmanyam *et al.*, 1994). Alternatively, long-term uncertainties represent external factors, such as supply, demand, and price fluctuations, that impact the planning process over a long period of time. Mid-term uncertainties include both short-term and long-term uncertainties (Gupta and Maranas, 2003). Jonsbraten (1998) classified uncertainties as external (exogenous) uncertainties and internal (endogenous) uncertainties, according to the point-of-view of process operations. As indicated by the name, external uncertainties are exerted by outside factors that impact the process. On the other hand, internal uncertainties arise from deficiencies in the complete knowledge of the process. Thus, mid-term and long-term uncertainties can also be classified as external uncertainties, whereas short-term uncertainties are internal uncertainties.

In regard of tactical models, Liu and Sahinidis (1996) developed a two-stage stochastic model and a fuzzy model for process planning under uncertainty. A method was proposed for comparing the two approaches. Overall, the comparison favored stochastic programming. Escudero *et al.* (1999) worked in the supply, transformation, and distribution planning problem that accounted for uncertainties in demands, supply costs, and product prices. As the deterministic treatment for the problem provided unsatisfactory results, they applied the two-stage scenario analysis based on a partial recourse approach. Dempster *et al.* (2000) formulated the tactical planning problem for an oil consortium as a dynamic recourse problem. A deterministic multi-period linear model was used as basis for implementing the stochastic programming formulation. Hsieh and Chiang (2001) developed a manufacturing-to-sale planning system and adopted fuzzy theory for dealing with demand and cost uncertainties. Li *et al.* (2004) proposed a probabilistic programming model to deal with demand and supply uncertainties in the tactical problem. Kim *et al.* (2008) worked on the collaboration among refineries manufacturing

multiple fuel products at different locations. Khor *et al.* (2008) treated the problem of medium-term planning of a refinery operation by using stochastic programming (a two-stage model) and stochastic robust programming. Al-Othman *et al.* (2008) have proposed a two-stage stochastic model for multiple time periods to optimize the supply chain of an oil company installed in a country that produces crude oil. Finally, Guyonnet *et al.* (2009) considered oil uploading and product distribution problems in their tactical formulation. Despite of these significant contributions, only four (4) of these nine (9) works represent actual applications and most of them still present very simplified models that exclude important aspects of a real refinery operation, such as constraints for the specification of final products.

Therefore, the refinery tactical planning problem under uncertainty is still an open issue, which is relevant for both mathematical modeling and actual applications. In this context, the main contribution of this work is to tackle an important topic from both theory and practice stand points, investigating the spatial integration of the oil supply chain through a two-stage stochastic model characterized by price and demand uncertainties. The purpose of the present paper is, then, to address the problem of integration and coordination under uncertainty in the oil supply chain at the tactical level. A tactical programming model is proposed with the objective of maximizing the expected total profit over a given time horizon. Uncertainties in demand for refined products, oil prices, and product prices (exogenous uncertainty factors) account for economic risk at the tactical level. The problem has been formulated as two-stage stochastic linear program with a finite number of realizations. In this approach, decision variables are cast into two groups, first and second stage variables (Dantzig, 1955). The first stage variables are decided prior to the actual realization of the random parameters. Once the uncertain events have unfolded, further operational adjustments can be made through values of the second stage.

The remainder of this paper is organized as follows. An overview of the refining industry is presented in section 2. Section 3 presents the tactical model for oil chain planning under uncertainty. Section 4 offers results and discussions in the context of a case study using real data from the Brazilian oil industry. Finally, some conclusions are drawn in section 5.

2. Refining Industry Overview

The oil chain covers stages from oil exploitation to product distribution including a complex logistic network and several transformation processes that take place in refineries. The petroleum supply chain is illustrated in Figure 1. The activities that comprise the oil chain are divided into three major segments: upstream, midstream, and downstream. The upstream segment includes the exploitation and oil production. The midstream is an intermediate segment and consists of the refining activity which includes the transportation of oil from the production site to refineries. The logistical tasks necessary to move the refined products from the refinery to the consumer points are in the downstream segment.

Petroleum may be produced in exploitation fields of the company itself or be supplied from international sources. The domestic oil is sent by pipeline or oil tankers to terminals from where the oil meets the demand of the refineries or is exported. Crude oil obtained from international sources is transported by pipeline or oil tankers to the terminals. The domestic terminals are in charge of receiving and forwarding oils and refined products, whereas international terminals represent points of offer and demand for foreign oils and products. The oil terminals are then connected to refineries through a pipeline network. Crude oil is converted to refined products at refineries which can be connected to each other in order to take advantage of each refinery design within the network. A planning model for oil refineries must allow for the proper selection of oil blending and consider an appropriate manipulation of intermediary streams to obtain the final products in the desired quantities and qualities. The refined products can be moved along the logistic network by road, water, rail, and pipeline modes. Crude oil and refined products are often transported to distribution centers through pipelines. From this level on, products can be transported either through pipelines or trucks, depending on consumer demand points. In some cases, products are also transported through vessels or by train.
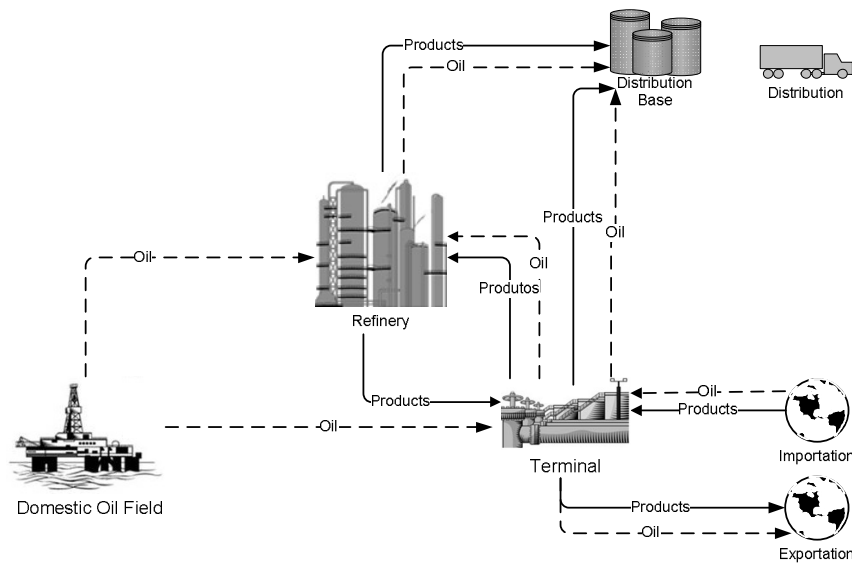
Figure 1. Oil supply chain (adapted from Ribas *et al.*, 2010)

As a result of the complexity of the oil chain, planning of the oil chain must be aided by decision-making systems, especially those that employ mathematical programming – for example, RPMS - Refinery and Petrochemical Modeling System (Bonner and Moore, 1979), OMEGA - Optimization Method for the Estimation of Gasoline Attributes (Dewitt *et al.*, 1989), and PIMS - Process Industry Modeling System (Bechtel, 1993). In this way, mathematical programming plays a crucial role to assist the decision-making process in the oil supply chain.

3. Tactical Planning of the Oil Supply Chain under Uncertainty

The tactical planning model proposed in this paper maximizes the total revenue of the supply chain allocating the production of the different products to the various refineries in each time period, while taking into account inventory holding costs and transportation costs. The optimization model is based on a scenario analysis approach, and is linear. Following Pongsakdi *et al.* (2006), many nonlinear features were simplified in order to gain computation speed, which allows the decision-maker to better explore the uncertainty issues in the model.

The associated prices, costs, and demands are assumed to be externally imposed in the planning. The modeling considers a fixed market, i.e. the model ensures the total fulfillment of the market demand. It is assumed that the physical settings in the supply chain have already been established, the configuration of the chain is given, and the number of facilities at each stage is known. It is also assumed a discrete planning horizon divided into a finite number of periods. The models have been formulated as two-stage stochastic programs with fixed recourse (Dantzig, 1955). Uncertainties are discretely represented by *SC* possible realization scenarios and modeled as a scenario tree. A scenario is a path from the root to a leaf of the tree. The probability that the

sc-th scenario will occur is represented by p_{sc} ($p_{sc} \geq 0, \sum_{sc=1}^{SC} p_{sc} = 1$, $sc \in SC$). Based on these

assumptions, the stochastic models this paper proposes can be represented as follows:

$$\underset{x}{Max} \left\{ z(x) = c^T x + \sum_{sc \in SC} p_{sc} q_{sc}^T y_{sc} \right\} \text{ subject to } Ax \leq b,$$ **(1)**

$$Wy_{sc} \leq h_{sc} - Tx, \ x \geq 0, \ y_{sc} \geq 0 \ \ sc \in SC$$

First-stage decisions are assumed to be made before the realization of random variables (here-and-now decisions), represented by a vector *x*, while second-stage decisions, denoted by

y_{sc} , are made under complete information about the realization of *sc* .

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The objective function in Equation (1) contains a deterministic term $c^T x$, which models the oil purchase decisions concerned with oil supply by long-term contracts. The second term of Equation (1) contains the expected value of the second-stage objective $\sum_{sc \in SC} p_{sc} q_{sc}^T y_{sc}$ which models the stochastic operational profit due to the first-stage decision. A set of deterministic inequalities ($Ax \leq b$) is used to model decisions related to oil purchase. Stochastic constraints ($Wy_{sc} \leq h_{sc} - Tx$) are used to represent refinery operation and to model all operative relations between the inputs (or different petroleum types) and the outputs (or final products) and the necessary network flows through the installed transportation network. Uncertainty is introduced through the product prices, oil prices, and market demand for final products.

In order to properly evaluate the added-value of including uncertainty in the problem parameters, the models can be evaluated using the Expected Value of Perfect Information (EVPI) and the Value of the Stochastic Solution (VSS) (Birge and Louveaux, 1997). The EVPI measures the loss of profit due to the presence of uncertainty which is also the measure of the maximum amount the decision maker is willing to pay in order to get accurate information on the future. As stated by the constraint (2), the EVPI results show the expected profit difference between the solution obtained by the agent able to make the perfect prediction (*wait-and-see - WS*) and the one obtained by the agent that solved the problem under uncertainty (*recourse problem - RP*).

$$EVPI = |WS - RP| \tag{2}$$

A solution based on perfect information would yield optimal first stage decisions for each realization of the random parameters (Madansky, 1960). So, assuming that the uncertainty is represented by a finite number of scenarios and that ζ is a random variable set of scenarios, the problem associated with each scenario of ξ can be defined as:

$$\begin{aligned} \underset{x}{Max}\, z(x, \xi) &= c^T x + \max\{q^T y \mid Wy \leq h - Tx, y \geq 0\} \\ X &= \{x : Ax \leq b, x \geq 0\} \end{aligned} \tag{3}$$

It is assumed that for all ξ there is at least one feasible solution $x \in R^n$. Let $x^*(\xi)$ an optimal solution to the problem (3) and $z(x^*(\xi), \xi)$ the optimal objective function value for a scenario ξ. The wait-and-see solution corresponds to the optimal value when the future realization of ξ is known, i.e., the decision maker can wait and see the future before deciding. The expected value of the wait-and-see solution is:

$$WS = E_\xi \left[\max_{x \in X} z(x, \xi) \right] = E_\xi \left[z(x^*(\xi), \xi) \right] \tag{4}$$

The recourse problem (RP) solution is also known as here-and-now decision, because the solution the first stage is decided without knowing the future realization of ξ, i.e., at the decision epoch the future scenario is known only probabilistically. So, the RP corresponds to the two-stage problem defined by the model (1) and can be written as:

$$WS = E_\xi \left[\max_{x \in X} z(x, \xi) \right] = E_\xi \left[z(x^*(\xi), \xi) \right] \tag{5}$$

The VSS, on the other hand, is defined by the difference between the stochastic solution (RP) and the average solution of the expected value problem (replacing the random events by their means - EEV) - constraint (6). The VSS can be interpreted either as the benefit expected by the agent that has taken uncertainty into account or as the loss expected by the agent that opted

for deterministic modeling using the average stochastic parameters ($E[\xi] = \overline{\xi}$).

$$VSS = \left| RP - EEV \right| \tag{6}$$

In order to quantify the VSS, first it is necessary to calculate the expected value solution (EV) which is defined by the solution of the problem to the expected scenario (expected value of ξ). Let $\overline{\xi} = E[\xi]$ and $x^*\left(\overline{\xi}\right)$ the optimal solution to EV, so:

$$EV = \max_x z\left(x, \overline{\xi}\right) \tag{7}$$

Then by fixing the first stage variables from the EV problem, the expectation of EV (EEV) can be obtained by allowing the optimization problem to choose the second stage variables with respect to different realizations:

$$EEV = E_\xi \left[z\left(x^*\left(\overline{\xi}\right), \xi\right) \right] \tag{8}$$

The model formulation proposed in this work is presented in the next section.

3.1. Tactical planning model

This section presents the stochastic formulation for tactical planning of oil refineries. This formulation is adapted from the model proposed by Ribas *et al.* (2010) by excluding all elements related to investment decisions which must only be considered in a strategic planning model. The proposed linear programming model aims to maximize the expected profit of the oil chain and consider the following factors that affect the domestic supply: configuration of refining park; refinery operations and transportation costs; import of oil; import and export of refined products; requirements for refined products defined by regulatory organizations; production of crude oil; domestic consumption of refined products; and prices of oil and refined.

The model decisions on oil refining determines the oil blending to each refinery, the production level at each process unit, and the operational mode for each unit at each period to meet the demand and respect the quality standards on the refined products. An operational mode is characterized by a set of operation patterns to prioritize the production of a specific product set. With respect to the logistic network, the model must define the minimum cost flow combination for the refinery supply and the refined products distribution. Definitions of parameters, sets, and variables are provided in Tables 1 and 2 which are followed by mathematical formulation.

Table 1. Parameters of the tactical model

Parameters			
Operational cost	$OC_{r,u}^n$	Oil field production	$FP_{i1,o}^n$
Transportation capacity	CT_{at}	Own consumption	$CP_{r,u,po}$
Transportation cost	TC_{at}	Minimum proportion	$PRPL_{r,u,pi,c}^n$
Distillation unit yield	$YDU_{r,u,c,o,po}$	Maximum proportion	$PRPU_{r,u,pi,c}^n$
Process unit yield	$YPU_{r,u,c,pi,po}$	Oil price - internal distribution	$OPBR_{r,o}^n$
Minimum capacity	$UCL_{r,u}$	**Stochastic parameters**	
Maximum capacity	$UCU_{r,u}$	Probability of scenario sc^t	P^{sc^t}
Sulfur quantity - entry product	SIO_{pi}^n	Domestic product demand	$PD_{b,po}^{n,sc^t}$
Maximum sulphur	$SPOU_{po}^n$	Product price - domestic market	$PPBR_{b,po}^{n,sc^t}$
Viscosity blending index	BI_p^n	Product export price	$PPE_{in,po}^{n,sc^t}$
Minimum viscosity	$VPOL_{po}^n$	Product import price	$PPI_{in,po}^{n,sc^t}$
		Oil import price	$OPI_{in,o}^{n,sc^t}$

Table 2. Sets and variables of the tactical model

Sets		Variables	
Set of nodes *(i1, i2)*	I	**First stage variables**	
Set of process units *(u, u')*	U	Oil purchase	$qocf_{r,o}^{n}$
Set of operational modes *(c)*	C	**Second stage variables**	
Set of products *(pi, po)*	PR	Blending	$b_{r,pi,po}^{n,sc^t}$
Set of oils *(o)*	O	Distillation unit load	$qdu_{r,u,c,o}^{n,sc^t}$
Set of transport modes *(m)*	M	Other process unit load	$qpu_{r,u,c,pi}^{n,sc^t}$
Set of transport arcs *(at)*	AT	Oil import	$oimp_{in,o}^{n,sc^t}$
Time periods $\{n \mid n = 1,..., NT \}$	N	Product export	$pexp_{in,po}^{n,sc^t}$
Set of tactical scenarios *(sc^t)*	SC^t	Product import	$pimp_{in,po}^{n,sc^t}$
Refinery *(r)*	$R \subset I$	Transported flow - entering the refinery	$ir_{r,po}^{n,sc^t}$
Natural gas producers *(ng)*	$NG \subset I$	Oil flow	$ot_{at,o}^{n,sc^t}$
International nodes *(in)*	$IN \subset I$	Transported flow - leaving the refinery	$or_{r,po}^{n,sc^t}$
Terminals *(tr)*	$TR \subset I$	Product flow	$pt_{at,po}^{n,sc^t}$
Bases *(b)*	$B \subset I$	Stock level of oil o at refinery r	$vot_{r,o}^{n,sc^t}$
Oil Field *(of)*	$OF \subset I$		
Transportation arcs available for transportation of *po* from *i1* to *i2* by the mode *m*	$ATA \subset AT$		

Model Formulation

Maximize

$$TM = \begin{cases} -\sum_{r \in R}\sum_{o \in O}\sum_{n \in N}\left(OPBR_{r,o}^{n}qocf_{r,o}^{n}\right) \\ + \sum_{sc^t \in SC^t}P^{sc^t}\left\{ \sum_{b \in B}\sum_{po \in P}\sum_{n \in N}\left(PPBR_{b,po}^{n,sc^t}PD_{b,po}^{n,sc^t}\right) \right. \\ \qquad + \sum_{n \in N}\sum_{in \in I}\sum_{po \in P}\left(PPE_{in,po}^{n,sc^t}pexp_{in,po}^{n,sc^t} - PPI_{in,po}^{n,sc^t}pimp_{in,po}^{n,sc^t}\right) \\ \qquad - \sum_{n \in N}\sum_{in \in I}\sum_{o \in O}\left(OPI_{in,o}^{n,sc^t}oimp_{in,o}^{n,sc^t}\right) \\ \qquad - \sum_{n \in N}\sum_{r \in R}\sum_{u \in U}\left(OC_{r,u}^{n}\sum_{o \in O}\sum_{c \in C}qdu_{r,u,o,c}^{n,sc^t}\right) - \sum_{n \in N}\sum_{r \in R}\sum_{u \in U}\left(OC_{r,u}^{n}\sum_{pi \in P}\sum_{c \in C}qpu_{r,u,pi,c}^{n,sc^t}\right) \\ \qquad - \sum_{at \in AT}\sum_{n \in N}\sum_{po \in P}pt_{at,po}^{n,sc^t}TC_{at} - \sum_{at \in AT}\sum_{n \in N}\sum_{o \in O}ot_{at,o}^{n,sc^t}TC_{at}\right\} \end{cases} \quad (9)$$

Refining balance

$$qocf_{r,o}^{n} + vot_{r,o}^{n-1,sc^t} = \sum_{u \in U}\sum_{c \in C}\left(qdu_{r,u,c,o}^{n,sc^t}\right) + vot_{r,o}^{n,sc^t} \quad \forall r \in R, \forall o \in O, \forall n \in N, \forall sc^t \in SC^t \quad (10)$$

$$\sum_{u \in U}\sum_{c \in C}\sum_{o \in O}qdu_{r,u,c,o}^{n,sc^t}YDU_{r,u,c,o,po} + \sum_{u \in U}\sum_{c \in C}\sum_{pi \in P}qpu_{r,u,c,pi}^{n,sc^t}YPU_{r,u,c,pi,po} + \sum_{pi \in P}b_{r,pi,po}^{n,sc^t}$$

$$+ir_{r,po}^{n,sc^t} = \sum_{pi \in P}b_{r,po,pi}^{n,sc^t} + \sum_{u \in U}\sum_{c \in C}qpu_{r,u,c,po}^{n,sc^t} + \sum_{u \in U}\sum_{c \in C}\sum_{pi \in P}qpu_{r,u,c,pi}^{n,sc^t}CP_{r,u,po} + or_{r,po}^{n,sc^t} \quad (11)$$

$$\forall r \in R, \forall po \in PR, \forall n \in N, \forall sc^t \in SC^t$$

Refining operation constraints

$$PRPL^n_{r,u,pi,c} \sum_{pi \in P} qpu^{n,sc^t}_{r,u,pi,c} \le qpu^{n,sc^t}_{r,u,pi,c} \le PRPU^n_{r,u,pi,c} \sum_{pi \in P} qpu^{n,sc^t}_{r,u,pi,c}$$

$$\tag{12}$$

$$\forall r \in R, \forall u \in U, \forall pi \in PR, \forall c \in C, \forall n \in N, \forall sc^t \in SC^t$$

$$UCL_{r,u} \le \sum_{o \in O} \sum_{c \in C} qdu^{n,sc^t}_{r,u,o,c} + \sum_{pi \in P} \sum_{c \in C} qpu^{n,sc^t}_{r,u,pi,c} \le UCU_{r,u} \quad \forall r \in R, \forall u \in U, \forall n \in N, \forall sc^t \in SC^t \tag{13}$$

Environmental legislation requirements

$$\sum_{u \in U} \sum_{pi \in P} \sum_{c \in C} qpu^{n,sc^t}_{r,u,c,pi} SIO^n_{pi} YPU_{r,u,c,pi,po} \le \left(\sum_{u \in U} \sum_{c \in C} \sum_{pi \in P} qpu^{n,sc^t}_{r,u,c,pi} YPU_{r,u,c,pi,po} \right) SPOU^n_{po}$$

$$\tag{14}$$

$$\forall r \in R, \forall po \in PR, \forall n \in N, \forall sc^t \in SC^t$$

$$\sum_{pi \in P} \left(b^{n,sc^t}_{r,pi,po} BI^n_{pi} \right) + \left(\sum_{u \in U} \sum_{c \in C} \sum_{o \in O} qdu^{n,sc^t}_{r,u,c,o} YDU_{r,u,c,o,po} + \sum_{u \in U} \sum_{c \in C} \sum_{pi \in P} qpu^{n,sc^t}_{r,u,c,pi} YPU_{r,u,c,pi,po} \right) BI^n_{po}$$

$$\le VPOL^n_{po} \left(\sum_{pi \in P} b^{n,sc^t}_{r,pi,po} + \sum_{u \in U} \sum_{c \in C} \sum_{o \in O} qdu^{n,sc^t}_{r,u,c,o} YDU_{r,u,c,o,po} + \sum_{u \in U} \sum_{c \in C} \sum_{pi \in P} qpu^{n,sc^t}_{r,u,c,pi} YPU_{r,u,c,pi,po} \right)$$

$$\tag{15}$$

$$\forall r \in R, \forall po \in PR, \forall n \in N, \forall sc^t \in SC^t$$

Logistic balance

$$\sum_{(at,i2,m) \in ATA} pt^{n,sc^t}_{at,po} + or^{n,sc^t}_{i1,po} = PD^{n,sc^t}_{i1,po} + \sum_{(at,i2,m) \in ATA} pt^{n,sc^t}_{at,po} + ir^{n,sc^t}_{i1,po}$$

$$\tag{16}$$

$$\forall i1 \in R \bigcup TR \bigcup OF, \forall po \in PR, \forall n \in N, \forall sc^t \in SC^t$$

$$\sum_{(at,i2,m) \in ATA} ot^{n,sc^t}_{at,o} + FP^n_{i1,o} = qocf^n_{i1,o} + \sum_{(at,i2,m) \in ATA} ot^{n,sc^t}_{at,o}$$

$$\tag{17}$$

$$\forall i1 \in R \bigcup B \bigcup TR \bigcup NG \bigcup OF, \forall o \in O, \forall n \in N, \forall sc^t \in SC^t$$

Logistic capacity constraints

$$\sum_{po \in P} pt^{n,sc^t}_{at,po} + \sum_{o \in O} ot^{n,sc^t}_{at,o} \le CT_{at} \qquad \forall at \in AT, \forall n \in N, \forall sc^t \in SC^t \tag{18}$$

$$qocf^n_{r,o}, b^{n,sc^t}_{r,pi,po}, dfr^{n,sc^t}_{r,u,c,o}, pfr^{n,sc^t}_{r,u,c,po}, oimp^{n,sc^t}_{in,o}, pexp^{n,sc^t}_{in,po}, pimp^{n,sc^t}_{in,po}, ir^{n,sc^t}_{r,po}, ot^{n,sc^t}_{at,o}, or^{n,sc^t}_{r,po}, pt^{n,sc^t}_{at,po} \in \Re^+ \tag{19}$$

The objective function (9) maximizes the expected tactical margin. This margin includes the revenue from the product sales and the product exports, minus the raw material costs, the oil and product imports, the refining operation costs, and the transportation costs. The oil purchase ($qocf^n_{r,o}$) represents the first stage decisions. The second-stage decisions are the amount of product and oil transported ($pt^{n,sc^t}_{at,po}$ and $ot^{n,sc^t}_{at,o}$), the amount of oil imported ($oimp^{n,sc^t}_{in,o}$), and the amount of imported product and exported product ($pimp^{n,sc^t}_{in,po}$ and $pexp^{n,sc^t}_{in,po}$).

Equation (10) and (11) represent the oil balance and the product balance, respectively. For them, the sum of the entry flows must be equal to the sum of the output flows. Equation (12) establishes the proportion between the entry flows (pi) and the total process unit (u) loading. The maximum and minimum capacities of the process unit u in period n are limited by equation (13).

Equations (14) and (15) limit the sulfur content (SIO^n_{pi}) and the viscosity (BI^n_{pi}) of the final products. Final product properties must be within a range established by environmental regulations. Property calculations yield a set of nonlinear constraints (Moro and Pinto, 2004) where the nonlinear terms arise from the multiplication between the products' properties and their

volumes. These terms can be linearized by estimating the properties of intermediate products. At the tactical level it is possible to estimate the sulfur content $SIO_{pi,n}$ and the viscosity $BI_{pi,n}$ of the intermediate products with sufficient accuracy, making the constraint that controls the final products' properties linear. The tactical model controls only these two properties because they are the ones that affect most tactical decisions such as oil purchase and oil blending.

Logistic balance constraints (equations 16 and 17) determine that the sum of the input flows must be equal to the sum of the output flows for each node (*i*), product (*po*) or oil (*o*), period of time (*n*) and scenario (*sc*^t). **ATA** represents the set of transportation arcs (*at*) for a product (*po*) from an origin node (*i1*) to a destination node (*i2*) by a transportation mode (*m*). Equation (18) limits the maximum volume transported by the transportation arc (*at*) in the period *n*. Finally, constraints (19) define the non-negativity of the variables.

4. Numerical Example

An industrial scale study using real data from the Brazilian industry was used to evaluate the performance of the proposed model in optimizing large-scale problems. The refining system includes 3 refineries (named R1, R2, and R3) and represents a general system that can be found in many industrial sites around the world. The refineries are coordinated centrally, the feedstock oil supply is shared, and the refineries collaborate to meet a given market demand. The refineries are supplied by 8 groups of national oils produced in 2 exploitation fields, and 1 group of foreign oils. The refineries process up to 50 intermediate products to produce 10 final products associated to the local market demand. The logistic network includes 2 domestic and 4 international terminals, 2 distribution bases, and 73 transportation arcs relative to road, water, rail, and pipeline modes. The time horizon in the tactical level covers 6 monthly periods.

R1 is a small and low complex refinery which focuses on the production of lubricants, asphalt, and fuel oils. This refinery is supplied by 1 group of oil (group A). R2 can also be considered a low complexity refinery that aims at the production of solvents and fuels and processes 3 groups of oils (groups C, D, and E). Finally, R3 is a medium complexity refinery and processes 7 groups of oils (groups A, B, C, E, F, G, and H) with the focus on the production of naphtha, but also has significant production of jet fuel, diesel, and gasoline.

The method used to create the scenarios of the stochastic model was based on data collection and direct contraposition of primary (data obtained from the oil Brazilian oil industry) and secondary research (historical economic data available online). Developing methodologies for scenario generation is beyond the scope of this paper, and the interested reader can refer to the work by Kouwenberg (2001), for example. As it is essential to test the proposed models, the scenario generation with the associated probabilities was arbitrated in consistency with the real problem and validated with experts of the oil industry. Table 3 shows the probability of each possible realization of the stochastic uncertainty. The demand for refined products, oil prices, and product prices are mid-term uncertainties which are considered in the tactical planning.

Table 3. Probabilities of the stochastic parameters

Model	Stochastic Parameter	Realizations	Probability
Tactical	Demand	High	25%
		Base	*50%*
		Low	25%
	Price	High	25%
		Base	*50%*
		Low	25%

Each stochastic parameter at the tactical level (price and demand) has three possible realizations (high, medium, and low). Assuming that the random variables are independent, these two parameters were combined to create the nine scenarios presented in Figure 2. For a given stochastic parameter, it is assumed complete dependence for all products - for example, one scenario of high demand for one product implies in high demand to the other products. Similar

pattern is presented to oil and product prices. The base case (scenario 5) used data from the current planning system of Brazilian refineries. This system addresses only a deterministic case which was used to generate the base case. The other scenarios were constructed based on the expertise of employees of the industry under study.
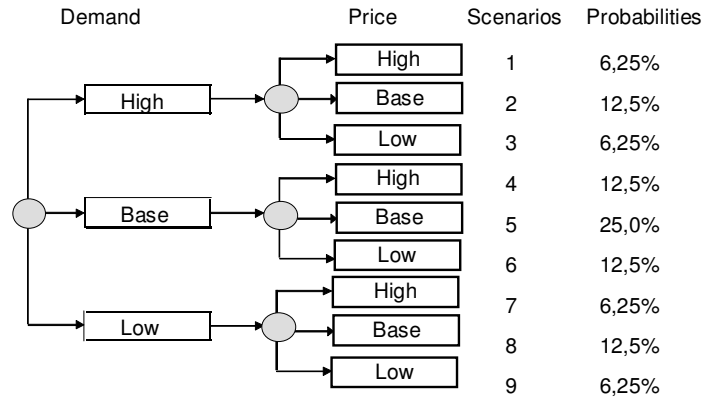

Figure 2. Scenario tree

4.1. Computational results and discussion

The model was implemented in the Advanced Integrated Multidimensional Modeling Software – AIMMS and solved using the CPLEX 12.1. Table 4 summarizes the model statistics:

Table 4. Model Statistics

#Variables	#Constraints	#Non zeros	Solving time (s)	E[margin] (million $)
96,899	119,105	218,286	0.78	707.9

As shown in Figure 3a, the best results for the model were found in the scenarios with high prices. This finding indicates the model's sensitivity to the uncertain parameters and that the prices uncertainty had a greater impact on total profit than the demands uncertainty had.
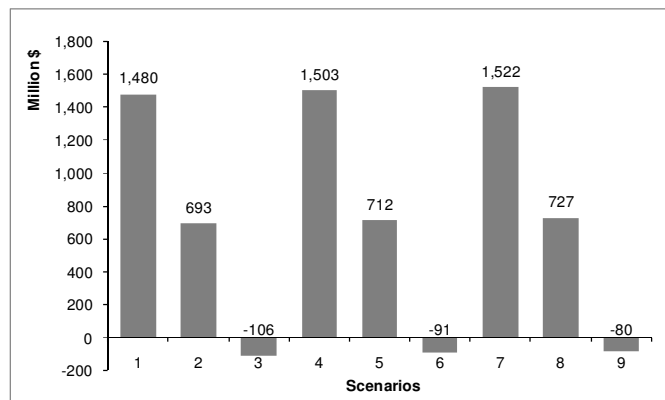

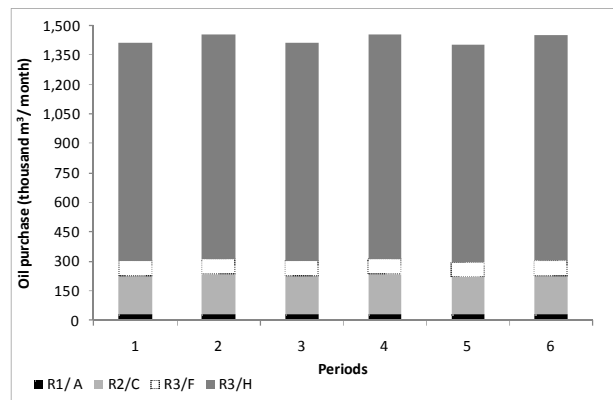**Figure 3a.** Tactical margin solutions by scenarios

Figure 3b. Tactical oil purchase decisions

Figure 3. Tactical model solutions

The tactical oil purchase decisions for the 6 periods of planning (*n*), defined by the first stage variable $qocf_{r,o}^{n}$, are presented at Figure 3b. The legend represents the refinery/ group of oil allocated to the refinery. Refinery R3 is responsible by 83.90% of the total oil purchases

presented in Figure 4. Two groups of oils are allocated to R3. This refinery processes the entire amount available of the oil F and completes the maximum capacity level with oil (H). In addition, 13.76% of total oil quantity is allocated to the oil family C at refinery R2. Finally, the tactical model attributes the last 2.34% of oil to the family A for the lubricant production at R1.

In this numerical study, the EVPI reached a maximum of 1.55% of the wait-and-see (WS) solution – 11.16 million $. The EVPI result shows the difference between the solution of the problem in which the oil purchase decisions are postponed until that the uncertainty is unfolded (WS) and the solution of the stochastic problem (recourse problem - RP). The lower the EVPI, the better the stochastic models accommodate uncertainties as the stochastic objective function value was not so far from the result obtained by the WS solution. So, the result indicates the benefit of incorporating uncertainty in the different model parameters of the oil chain. However, since acquiring perfect information is not viable, the value of the stochastic solution (VSS) can be considered as a more realistic result. The VSS result, 98.95 million $ (13.76% of the WS solution), shows that the stochastic model provided a good solution, as an expressive gain was obtained by the inclusion of uncertainty into the problem.

5. Conclusions

The purpose of this paper was to discuss the problem of spatial integration and coordination in the tactical planning level of the oil chain. A stochastic mathematical programming model was developed to improve the tactical planning of oil refineries considering uncertainties in demand and prices. The model was applied to an actual refining system in Brazil. The values obtained for the Expected Value of Perfect Information (EVPI) and the Value of the Stochastic Solution (VSS) – 1.55% and 13.76% of the wait-and-see solution, respectively – indicated the benefit of incorporating uncertainty in the dominant random parameters of the tactical planning. The optimization results are suitable to the real planning activities of the oil chain. The centralized coordination (*spatial integrated*) and the shared feedstock oil supply provided a better utilization of the available resources in meeting a given market demand.

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