Measuring the efficiency of assembled printed circuit boards with undesirable outputs using Two-stage Data Envelopment Analysis

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ABSTRACT

Conventional applications of data envelopment analysis generally treat the Decision-Making Unit (DMU) as a black box in which the internal processes are not examined in detail. The efficiency of decision processes that can be divided into two stages has been measured for the whole process as well as for each stage independently by using the conventional data envelopment analysis (DEA) methodology in order to identify the causes of inefficiency. The purpose of this paper is to assess the efficiency of different types of assembled printed circuit boards (PCBs), known as cards, in the presence of undesirable outputs under the weak disposability assumption. The PCB assembling production process is generally optimized to ensure very low levels of errors (defects) so as to produce the higher quality product. In our case the three types of errors – machine error, manual errors and other errors that occur in the defective assembled PCBs are the undesirable outputs. The entire production process has been viewed as two-stage assembly process and respective process efficiency has been analyzed using two-stage DEA. This analysis could help the management in identifying the areas of inefficiency and formulating suitable strategies to improve each level of the assembly process.

KEYWORDS. PCB. Undesirable outputs. Data envelopment analysis.

1. Introduction

Over the years, PCB production has evolved from a labor-intensive activity to a highly automated one, characterized by steady innovations at the level of design and manufacturing processes. Today's multi-layered, highly complex printed circuit boards are expected to continue the trend toward increased complexity. The increasing complexity of PCB inevitably leads to higher failure rates in the assembly process. The PCB assembly consists in placing (inserting, mounting) a number of electronic components of prespecified types at prespecified locations on a raw board. Several hundred components of a few distinct types (resistors, capacitors, transistors, integrated circuits, etc.) are being placed on each board. The PCB assembly and production processes are generally optimized to ensure very low levels of defects so as to produce the highest quality product. Each assembled PCB goes through a certain number of processes. However, the process of assembling often gets influenced by certain factors, which make some of the assembled PCBs to be defective. Defects arise in most instances due to incorrect or incomplete designs and poorly executed fabrication and assembly. There are eleven types of defects contributing to a defective assembled PCB. They are identified as wrong component assembled (WCA), reversal component (RC), component missing (CM), wrong cut done / cut not done (WCD/CND), pattern cut (PC), pin bend in IC’s (PB), dry soldering (DS), not cleaned (NC), wrong strapping done (WSD), not mounted properly (NMP), and solder short (SS). Charles and Dutta (2006) and Charles and Kavitha (2008), classified the above said eleven defects into three kinds of errors, viz., *machine errors* (DS & SS), *manual errors* (WCA, RC, CM, WCD/CND, PB & WSD) and *other errors* (PC, NC, NMP). The defective PCB is considered as undesirable output as it has to be reprocessed after identifying the defects which in turn results in increase in cost and time. The key to successfully produced PCBs is experience and know how to screen even the slightest possibility of a defect. Thus, it is very important to access the efficiency of different *types of PCBs* (hereafter cards) in the presence of undesirable outputs as it can provide a framework to assess the quality of individual card in each stage of production process and work out appropriate interventions to prevent failures in the assembly process.

In this paper, we propose the DEA model which accounts for not only two stage DEA problem but also the undesirability in outputs in each stage of the production process. Since Charnes, Cooper, and Rhodes (1978), data envelopment analysis (DEA) has been widely applied to measure the relative efficiency of a set of decision making units (DMUs) which apply the same inputs to produce the same outputs. The results indicate how efficient each DMU has performed as compared to other DMUs in converting inputs to outputs. An issue which is of greater concern to the inefficient DMUs is what factors causes the inefficiency, although it is obvious that either reducing inputs or increasing outputs will improve their performance (Liu and Wang, 2009). Several studies have devoted to breaking down the overall efficiency into components so that the sources of inefficiency can be identified. One such type of decomposition emphasizes the stages of the production process. The complicated production process is divided into sub-process, in that some intermediate products are the outputs of a sub-process on the one hand and the inputs of another sub-process on the other hand. Seiford and Zhu (1999) divide a commercial bank’s production process into the stages of profitability and marketability. Their model for calculating the efficiencies of the sub-processes does not reflect any relationship between components and the whole system. Different from the study of Seiford and Zhu (1999) and Kao and Hwang (2008) take the series relationship of the two sub-processes into account in measuring the efficiencies. They show that the overall efficiency is the product of the efficiencies of the two sub-processes, and the efficiencies calculated from the relational two-stage DEA approach are more meaningful than those calculated from the independent two-stage DEA approach. However, the two stage DEA problem discussed above deal with the case where we only have desirable outputs. But the evaluation of efficiency for different types of PCBs could be a special case where we have two stage assembly processes and each process involves both desirable as well as undesirable outputs.

The following section discuss two stage DEA problem with desirable outputs, followed by proposed model incorporating the undesirable output in the two stage system. Then, an illustrative numerical example has been provided for better understanding of the model. Finally, the paper concludes with the future scope of the study.

2. Two Stage DEA Model:

Data Envelopment Analysis (DEA), as initiated and developed by Charnes et al. (1978) is a nonparametric method for identifying efficient production frontiers and evaluating the relative efficiency of Decision Making Units (DMUs), each of which is an entity responsible for converting multiple inputs into multiple outputs. The basic concept of DEA is to consider the input elements and the output elements of all the DMUs, to express in terms of the linear combinations of input and output items, and then to indicate efficiency by using the ratio of the input and the output units.

Kao and Hwang (2008) posit that a production process is composed of a series of two sub-processes. The whole process uses m inputs x_{i0}, $i \in M = \{1..m\}$ to produce s outputs y_{r0}, $r \in S = \{1..s\}$. Different from the conventional one-stage production process, the production process is composed of two sub-processes with q intermediate products Z_{p0}, $p \in Q = \{1..q\}$ (Kao & Hwang 2008). Moreover, the intermediate products Z_{p0} are the outputs of stage 1 as well as the inputs of stage 2. The primal model in line with Kao and Hwang (2008) Liu and Wang (2009) is as follows:

$$\begin{aligned}
&E_0 = Min \sum_{i\in M} v_i x_{i0} \\
&s.t. \sum_{r\in S} u_r y_{r0} = 1 \\
&\sum_{i\in M} v_i x_{ij} - \sum_{r\in S} u_r y_{rj} \ge 0, \quad j \in N \\
&\sum_{i\in M} v_i x_{ij} - \sum_{p\in Q} w_p z_{pj} \ge 0, \quad j \in N \\
&\sum_{p\in Q} w_p z_{pj} - \sum_{r\in S} u_r y_{rj} \ge 0, \quad j \in N \\
&u_r, v_i, w_p \ge \varepsilon, r \in S, i \in M, p \in Q,
\end{aligned} \tag{1}$$

where u_r is the multiplier (weight) given to the rth output, v_j is the multiplier (weight) given to the jth input, w_p is the multiplier (weight) given to the pth intermediate output, there are n DMUs, and ε is a small non-Archimedean number (Charnes et al., 1984). The dual form of the above two stage system is as follows:

$$\begin{aligned}
&E_0 = Max\ \phi \\
&s.t. \sum_{j\in N} y_{rj}\lambda_j^{(3)} + \sum_{j\in N} y_{rj}\lambda_j^{(2)} \ge y_{r0}\phi, \quad r \in S, \\
&-\sum_{j\in N} z_{pj}\lambda_j^{(1)} + \sum_{j\in N} z_{pj}\lambda_j^{(2)} \le 0, \quad p \in Q, \\
&\sum_{j\in N} x_{ij}\lambda_j^{(3)} + \sum_{j\in N} x_{ij}\lambda_j^{(1)} \le x_{i0}, \quad i \in M, \\
&\lambda_j^{(1)}, \lambda_j^{(2)}, \lambda_j^{(3)} \ge 0, j \in N,
\end{aligned} \tag{2}$$

where $\lambda_j^{(1)}$ is jth intensity value correspond to input x and intermediate output z, $\lambda_j^{(2)}$ is jth intensity value correspond to intermediate output z and output y, $\lambda_j^{(3)}$ is jth intensity value correspond to input x and output y. It is to be noted that in System (1), $\sum_{i\in M} v_i x_{ij} - \sum_{r\in S} u_r y_{rj} \geq 0,$ is redundant (Chen et al., 2009) and hence the corresponding dual intensity variable $\lambda_j^{(3)}$ is redundant. The optimal multipliers solved from System (1) may not be unique; consequently, the decomposition of $E_0 = E_0^1 \text{x} E_0^2$ would not be unique (Kao and Hwang, 2008). To find the set of multipliers which produces the largest E_0^1 while maintaining the overall efficiency score at E_0 calculated from System (1), in line with Kao and Hwang (2008) we can express the following mathematical program:

$$
\begin{aligned}
& E_0^1 = Min \sum_{i\in M} v_i x_{i0} \\
& s.t. \sum_{r\in S} u_r y_{r0} = 1 \\
& \sum_{i\in M} v_i x_{ij} - E_0 \sum_{r\in S} u_r y_{rj} = 0, \\
& \sum_{i\in M} v_i x_{ij} - \sum_{p\in Q} w_p z_{pj} \geq 0, \quad j \in N \\
& \sum_{p\in Q} w_p z_{pj} - \sum_{r\in S} u_r y_{rj} \geq 0, \quad j \in N \\
& u_r, v_i, w_p \geq \varepsilon, r \in S, i \in M, p \in Q.
\end{aligned} \tag{3}
$$

To apply system (3), one needs to calculate the overall efficiency E_0 first.

3. Two Stage DEA Model with Undesirable Output

Let us consider the two stage system wherein process uses m inputs x_{i0}, $i \in M = \{1..m\}$ to produce s_d desirable outputs y_{Dr0}, $r \in S_D = \{1..s_d\}$ and s_u undesirable outputs y_{Ur0}, $r \in S_U = \{1..s_u\}$. The production process is composed of two sub-processes with q_d desirable intermediate outputs z_{Dp0}, $p \in Q_D = \{1..q_d\}$ and q_u undesirable intermediate outputs z_{Up0}, $p \in Q_U = \{1..q_u\}$. The intermediate outputs z_{Dp0} are the outputs of stage 1 as well as the inputs of stage 2. The following systems represent the dual model of the two stage production process under the weak disposability assumption on every stage of the undesirable output without measuring the overall efficiency in advance:

$$
\begin{aligned}
& E = Max\ \phi_3 \\
& s.t. \sum_{j\in N} x_{ij}\lambda_j^{(1)} \leq x_{i0}, \quad i \in M, \\
& \sum_{j\in N} z_{Dpj}\lambda_j^{(2)} \leq z_{Dp0}, \quad p \in Q_D, \\
& \sum_{j\in N} z_{Dpj}\lambda_j^{(1)} \geq z_{Dp0}\phi_1, \quad p \in Q_D,
\end{aligned}
$$

$$
\begin{aligned}
&\sum_{j\in N} z_{Upj}\lambda_j^{(1)} = z_{Up0}\phi_1^{-1}, \quad p\in Q_U,\\
&\sum_{j\in N} y_{Drj}\lambda_j^{(2)} \ge y_{Dr0}\phi_2, \quad r\in S_D,\\
&\sum_{j\in N} y_{Urj}\lambda_j^{(2)} = y_{Ur0}\phi_2^{-1}, \quad r\in S_U,\\
&\phi_3=\phi_1\phi_2\\
&\lambda_j^{(1)},\lambda_j^{(2)}\ge 0,\ j\in N.
\end{aligned}
\tag{4}
$$

where $\lambda_j^{(1)}$ is jth intensity value correspond to input x and intermediate output z, $\lambda_j^{(2)}$ is jth intensity value correspond to intermediate output z and output y. Systems (4) is basically non-linear in nature due to the involvement of ϕ_1^{-1},ϕ_2^{-1}, and $\phi_1\phi_2$ at the constraint level. ϕ_1^{-1}, and ϕ_2^{-1} can be approximated in line with Fare et al. (1989) ie., $\sum_{j\in N} z_{Upj}\lambda_j^{(1)} = z_{Up0}\phi_1^{-1}$ can be approximated by linear expression $\sum_{j\in N} z_{Upj}\lambda_j^{(1)} = z_{Up0}(2-\phi_1)$ around $\phi_1 = 1$ and $\phi_3=\phi_1\phi_2$ can be approximated around its maximal value at 1 as $\phi_3=\phi_1+\phi_2-1$. The efficiencies of the whole process is the product of the efficiencies of two sub-processes $E_0 = 1/\phi_1\phi_2$.

4. Illustrative PCB Problem

We have taken a hypothetical example of PCB manufacturing company which produces different types of PCBs (cards) in two sub-production process as shown in Figure 1. In Stage 1, input x (*number of raw PCBs*) produces desirable output, z_D (*error free PCBs*) and undesirable output, z_U (*different types of defects during the assembly process I*). The desirable output of Stage I goes to Stage II as desirable input, which in turn produces the desirable output, y_D (*final error free PCBs*) and undesirable output, y_U (*different types of defects during the assembly process II*).

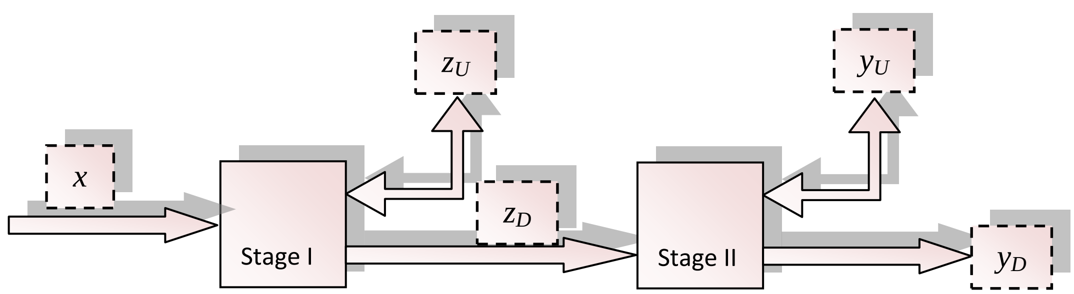

Figure 1: Two stage system with inputs x, outputs y, and intermediate inputs/outputs *z*

Table 1 reports the input and outputs of 10 cards for illustration of our proposed two stage DEA model as given in System (4).

Table 1: Inputs and Outputs for 2 Stage PCB Problems

		Assembly Process I		Assembly Process II	
DMU	Input	D1	U1	D2	U2
1	100	80	20	75	5
2	100	80	20	74	6
3	100	80	20	70	10
4	100	80	20	40	40
5	100	80	20	60	20
6	100	50	50	48	2
7	100	50	50	45	5
8	100	50	50	40	10
9	100	50	50	30	20
10	100	50	50	15	35

Note: D and U respectively indicates desirable and undesirable outputs

System (4) is used to evaluate the efficiency of the assembly process I and II respectively and the results are reported in Table 2.

Table 2: Efficiency of DMUs

DMU	ϕ_1^{-1}	ϕ_2^{-1}	$\phi_1^{-1}\phi_2^{-1}$
1	1.000	1.000	1.000
2	1.000	1.000	1.000
3	1.000	1.000	1.000
4	1.000	0.521	0.521
5	1.000	1.000	1.000
6	0.625	1.000	0.625
7	0.625	1.000	0.625
8	0.625	1.000	0.625
9	0.625	1.000	0.625
10	0.625	0.509	0.318

As it can be seen from Table 1 and Table 2, the first 5 DMUs are technically efficient and the rest of the five DMUs are inefficient with efficiency value as 0.625 each in the assembly process I. All DMUs, except DMU 4 and DMU 10 are found to be efficient in the assembly process II. The above two DMUs are extreme observations in terms of production of undesirable outputs. They produce the undesirable outputs at least as much as the number of desirable outputs (see Table 1).

The overall efficiency of DMUs is reported in the last column of the Table 2 as the product of efficiencies of DMUs in the assembly process I and the assembly process II. The DMUs which are overall efficient are DMU 1, DMU 2, DMU 3 and DMU 5. The remaining six DMUs are inefficient because of their inefficiency in the assembly process I (DMU 6, DMU 7, DMU 8 and DMU 9) or in the assembly process II (DMU 4) or in both the assembly processes (DMU 10).

DEA establishes a best practice group of units for each inefficient unit and quantifies the amount of potential improvement. In simple terms DEA indicates the level of resources savings and/or services improvements possible for each inefficient unit if it is to achieve the level of efficiency of the best practice units. In order to remain overall efficient, an inefficient DMU needs to reach the level of efficiency at par with its peers in both the assembly processes. Table 3 reports the potential improvements (%) of inputs and outputs of each inefficient unit.

Table 3: Potential Improvement (in %) of Inputs and Outputs for inefficient DMUs

DMU	Assembly Process I			Assembly Process II		
	I_1	D_1	U_1	I_2	D_2	U_2
4	0.00	0.00	0.00	0.00	92.00	-92.00
6	0.00	60.00	-60.00	0.00	0.00	0.00
7	0.00	60.00	-60.00	0.00	0.00	0.00
8	0.00	60.00	-60.00	0.00	0.00	0.00
9	0.00	60.00	-60.00	0.00	0.00	0.00
10	0.00	60.00	-60.00	-38.60	96.49	-96.49

Given the current level of inputs, each of the DMUs (6 – 9) needs to expand desirable output by 60% as well as reduce the undesirable output by 60% in the assembly process I in order to achieve the overall level of efficiency at par with its peers. Similarly, given the current level of input, DMU 4 is required to expand desirable output by 92% and shrink the undesirable output by 92% in the assembly process II in order to achieve overall efficiency at par with its peers. However, DMU 10 requires improvements in both the assembly processes in order to be overall efficient. Given the current level of inputs, the desirable outputs should be expanded by 60% and 96.5% respectively in the assembly process I and II and undesirable outputs should be contracted by the same percentage respectively in the above two processes.

5. Conclusion

In this paper, we propose the DEA model which accounts for not only two stage DEA problems but also the undesirability in outputs in each stage of the production process as the special case of manufacturing system like PCB assembly process. The model has been proposed in line with two stage DEA model proposed by Kao and Hwang (2008) Liu and Wang (2009). As illustrative example of PCB problem shows the results are consistent in both the stages which in turn help us to obtain the overall efficiency as the product of efficiencies of the sub-processes. In order to remain overall efficient, an inefficient DMU needs to expand (reduce) the desirable (undesirable) outputs at given level of input(s), in the assembly process I or in the assembly process II or in both depending on the case. The paper can be further extended to address multi stage aspect of the discussed problem and it will be also quite interesting to design a multi stage DEA model as a product of sub processes efficiency verses a sum of the sub processes efficiency.

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