A FUZZY APPROACH TO PROSPECT THEORY

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ABSTRACT

The aim of this chapter is to revisit an experiment of Kahneman and Tversky (1992) to arrive at conclusions about Prospect theory and the ways of human thinking, but using a fuzzy approach, especially the compensatory one. New results shall be proved and others well-known shall be changed or confirmed. The study comprises the examination of logical predicates like those expressed by the following sentences: "if a scenario is probable then it is 'convenient'", "there exist probable and 'convenient' scenarios" and "all the scenarios are probable and 'convenient'". In this chapter, a scenario is a premium, which is associated with a probability. An implication operator upon a set of five and a one-parameter family of compensatory systems will be selected for representing these predicates.

KEYWORDS: Prospect Theory, Compensatory Fuzzy Logic, Behavioural Economics

Introduction

Prospect theory has been well accepted by Decision Theory community. This success is due to its right and simple answer to the question: actually how human beings make decisions under uncertainty? (Kahneman and Tversky, 1979). The expected utility theory, another classic, can't deal with situations where the subjectivity of persons is relevant and, hence, objectivity is not the only factor to be taken into account (French, 1986).

Prospect theory is a consequence of many experiments carried out by Kahneman and Tversky about the attitude of human being under uncertainty situations. They maintained the concept of *lottery*, used for computing expected utility functions, which consists of a set of premiums often representing money quantities, positive if they are gains or negative if they are losses, while being associated with the probability of occurrence, such that the probabilities of all the premiums sum one. They studied the shape, slope and other characteristics of a function, named *value function* that measures the risk attitude and preferences of persons. In this context, lotteries are called *prospects*.

On the other hand, Fuzzy logic is a multi-valued logic, with a wide range of applications (Dubois et al., 2007). Some of their essential properties are their facilities to model the "vagueness" proper to the natural language and the uncertainty. These properties are arguments to justify the relevance of searching for nexuses between Fuzzy logic and Prospect theory. Also, fuzzy logic has been a useful tool for modeling preferences.

The notion of t-norm and t-conorm doesn't seem to be adequate to solve problems in decision making; however, it is the most extended approach of all, even though empirical studies prove that some compensatory operators are closest to represent real human thinking than any t-norm or t-conorm system (Mizumoto, 1989).

The insufficient study of compensatory operators in fuzzy literature (Detyniecki, 2001), usually provokes that the concept of operator prevails over the concept of integrated operators' system. Maybe, the only exceptionin the literature is *Compensatory Fuzzy Logic* (CFL) (Espin et al., 2011). The CFL consists of a set of axioms, some of them inspired in logic and others in Decision theory, which are grouped in a coherent way. It is a quartet of continuous operators (c, d, o, n) of, respectively, a conjunction operator, a disjunction operator, a fuzzy strict order operator and a negation operator.

The conjunction operator of the CFL could be defined with formulas of the quasi-arithmetic means and the disjunction operator could be their duals. CFL is a recommendable tool to be used in *Soft-computing*, which is the classification given by Zadeh (1998) to all the branches of Artificial Intelligence opposites to *hard-computing*, such that a good or approximate solution is accepted, even if it is not optimal, and fuzzy logic is one of their bases.

CFL is designed to calculate using complex sentences expressed in natural language, and not the so usually exclusive employment of simple linguistic variables. The conception of this new tool is to reaffirm the Zadeh's idea to compute with words rather than with numbers (Zadeh, 2002). This characteristic can be used to link CFL with Artificial Intelligence branches like *Knowledge Engineering*, the *Expert System*'s methodology (Bucharan and Shortliffe, 1984).

The aim of this chapter is to revisit an experiment of Kahneman and Tversky (1992) to arrive at conclusions about Prospect theory and the ways of human thinking, but using a fuzzy approach, especially the compensatory one. New results shall be proved and others well-known shall be changed or confirmed. The study comprises the examination of logical predicates like those expressed by the following sentences: "if a scenario is probable then it is 'convenient'", "there exist probable and 'convenient' scenarios" and "all the scenarios are probable and 'convenient'". In this chapter, a scenario is a premium, which is associated with a probability. An implication operator upon a set of five and a one-parameter family of compensatory systems will be selected for representing these predicates.

The chapter is structured as follows: next section, called *Preliminaries*, is divided in two subsections, the first of them explains the basic concepts of Prospect theory and the second subsection exposes some notions about CFL, including the introduction of a compensatory one-parameter family. The third section describes the experiment of Kahneman and Tversky that shall be used in the chapter; some other notions like implication operators that will be useful are included. This section finishes with the description of a fuzzy approach to Prospect theory. The fourth section describes the analysis of the results. The fifth and sixth sections are the conclusion of the chapter and references, respectively.

1. Preliminaries

1.1. Prospect theory

A *prospect* in Prospect theory, as a *lottery* in Utility theory, is represented by $L = (x_1, p_1; x_2, p_2; \ldots; x_n, p_n)$, where p_i is the probability to obtain the potential outcome or premium x_i and $\sum_{i=1}^{n} p_i = 1$.

The detailed manner to measure the prospects can be found in (Kahneman and Tversky, 1979), it is basically $V(L) = \sum_{i=1}^{n} \pi(p_i) v(x_i)$.

$\pi(p)$ is called the *weighting function* or *decision weight*, which maps over the probabilities and $v(x)$ is called the *value function*, which maps over the outcomes or premiums.

Let us note that probabilities aren't used directly in the final valorization of the prospect, because they don't influence objectively the result, but subjectively, according to a function $\pi(p)$ defined by the decision maker. Usually, $\pi(p)$ is a assumed by individual decision makers as non-linear weights, which are concave over certain interval [0, b] and convex over the interval [b, 1], where $0<b<1$.

The value function has the characteristics summarized below, according to empirical results:

1. There exists a *reference point* that is valued as indifferent by people. The other points are assumed like deviations from this point; therefore, people think in terms of gains and losses.

2. The function is concave over gains and convex over losses. That is to say, it is an s-shaped or sigmoidal function.

3. It is steeper for losses than for gains. This is because people experience losses more intensively than gains.

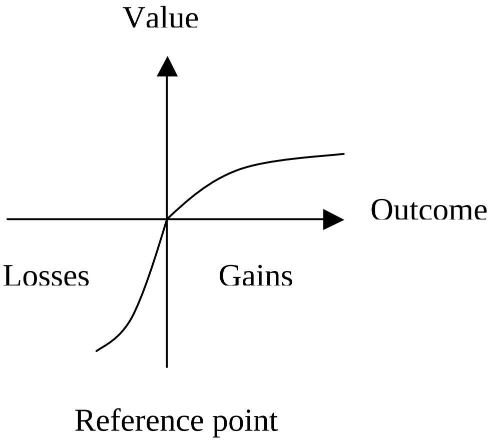

Figure 1 A hypothetical value function

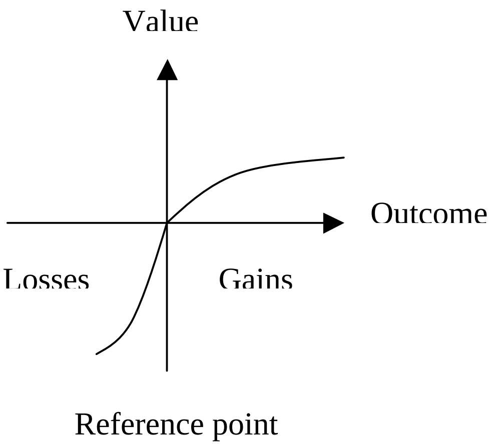

Figure 1 A hypothetical value function

In brief, people are risk-averse for gains and risk-seeking for losses.

1.2. Compensatory Fuzzy Logic

A CFL system is a quartet (*c*,*d*,*o*,*n*) of operators of conjunction, disjunction, fuzzy strict order and negation, respectively (Espin et al., 2011).

c and *d* map vectors of $[0,1]^n$ into [0,1], *o* is a mapping from $[0,1]^2$ into [0,1], and *n* is a unary operator of [0,1] into [0,1].

The following axiomatic must be satisfied:

i. Compensation Axiom: $\min(x_1, x_2, ..., x_n) \le c(x_1, x_2, ..., x_n) \le \max(x_1, x_2, ..., x_n)$

ii. Commutativity or Symmetry Axiom: $c(x_1,x_2,...,x_i,...,x_j,...,x_n)= c(x_1,x_2,...,x_j,...,x_i,...,x_n)$

iii. Strict Growth Axiom: If $x_1=y_1$, $x_2=y_2$, ..., $x_{i-1}=y_{i-1}$, $x_{i+1}=y_{i+1}$,...., $x_n=y_n$ are unequal to zero, and $x_i>y_i$ then $c(x_1,x_2,...,x_n)>c(y_1,y_2,...,y_n)$

iv. Veto Axiom: If $x_i=0$ for one i, then $c(\mathbf{x}) = 0$.

v. Fuzzy Reciprocity Axiom: $o(\mathbf{x},\mathbf{y}) = n[o(\mathbf{y},\mathbf{x})]$

vi. Fuzzy Transitivity Axiom: If $o(\mathbf{x},\mathbf{y}) \geq 0.5$ and $o(\mathbf{y},\mathbf{z}) \geq 0.5$, then $o(\mathbf{x}, \mathbf{z}) \geq \max(o(\mathbf{x},\mathbf{y}),o(\mathbf{y},\mathbf{z}))$.

vii. De Morgan's Laws:

$n(c(x_1, x_2,..., x_n)) = d(n(x_1), n(x_2),..., n(x_n))$ and
$n(d(x_1, x_2,..., x_n)) = c(n(x_1), n(x_2),..., n(x_n))$

A family of CFL systems may be obtained from the quasi-arithmetic means, with the following formula below (Mitrinovic, 1993):

$$M_f(x_1, x_2,..., x_n) = f^{-1}\left(\frac{1}{n}\sum_{i=1}^{n} f(x_i)\right) \qquad (1)$$

Where f(x) is a continuous and strictly monotonic function of one real variable.

In this chapter the one-parameter family with formula:

$$M_f(x_1, x_2,..., x_n) = \left(\frac{1}{n}\sum_{i=1}^{n} x_i^p\right)^{\frac{1}{p}} \qquad (2)$$

Where $p \in (-\infty, 0]$ satisfies the axiom of compensation, if the conjunction is defined as follows:

$$c(x_1, x_2,..., x_n) = \left(\frac{1}{n}\sum_{i=1}^{n} x_i^p\right)^{\frac{1}{p}} \qquad (3)$$

The disjunction is defined as the dual of the conjunction, that is to say:

$$d(x_1, x_2,..., x_n) = \left(\frac{1}{n}\sum_{i=1}^{n} (1 - x_i)^p\right)^{\frac{1}{p}} \qquad (4)$$

The fuzzy negation is:

$n(x) = 1-x$ (5)

The fuzzy strict order is:

$o(\mathbf{x}, \mathbf{y}) = 0.5[c(\mathbf{x}) - c(\mathbf{y})] + 0.5$ (6)

Let us remark that this is a special case of the formula (1), where $f(x) = x^p$, whenever p<0. If p = 0, f(x) = ln(x), which corresponds to the geometric mean, and for p = -1 (3) represents the harmonic mean. Even though the limit $p = -\infty$ is not included in the definition above, its limit value exists and it is $c(x_1, x_2,..., x_n) = \min(x_1, x_2,..., x_n)$. Also, $\lim_{x_i \to 0} M_f(x_1, x_2,..., x_n) = 0$ for all p, therefore,

$M_f(x_1, x_2,..., x_n)$ may be redefined as 0, if some x_i=0.

An interesting result is that for a fixed vector $(x_1, x_2,..., x_n)$, if s<p then in (2) the inequality $\left(\frac{1}{n}\sum_{i=1}^{n} x_i^s\right)^{\frac{1}{s}} \leq \left(\frac{1}{n}\sum_{i=1}^{n} x_i^p\right)^{\frac{1}{p}}$ is fulfilled (Mitrinovic, 1993). Therefore, the parameter p indicates the behavior toward preferences; the decision maker is more 'pessimistic' if he/she measures its preferences using a smaller parameter p.

Other formulas of the CFL are as follows below:

$$\underset{x\in U}{\forall} p(\mathbf{x}) = \underset{x\in U}{\wedge} p(\mathbf{x}) = M_f(x_1, x_2, \ldots, x_n) = f^{-1}\left(\frac{1}{n}\sum_{i=1}^{n} f(x_i)\right) \quad (7)$$

$$\underset{x\in U}{\exists} p(\mathbf{x}) = \underset{x\in U}{\vee} p(\mathbf{x}) = d(x_1, x_2, \ldots, x_n) = 1 - f^{-1}\left(\frac{1}{n}\sum_{i=1}^{n} f(1-x_i)\right) \quad (8)$$

If U is a continuous set, formulas (7) and (8) become, respectively, (9) and (10):

$$\forall \mathbf{x}\, p(\mathbf{x}) = f^{-1}\left(\frac{\int_X f(p(\mathbf{x}))dx}{\int_X dx}\right) \quad (9)$$

$$\exists \mathbf{x}\, p(\mathbf{x}) = 1 - f^{-1}\left(\frac{\int_X f(1-p(\mathbf{x}))dx}{\int_X dx}\right) \quad (10)$$

Where p(**x**) is a formula of the propositional calculus in CFL.

This formula is valid in the CFL if it satisfies the condition (11), below:

$$f^{-1}\left(\frac{\int_{[0,1]^n} f(p(\mathbf{x}))dx}{\int_{[0,1]^n} dx}\right) > \frac{1}{2} \quad (11)$$

2. *The experiments*

This section begins with a useful resume of fuzzy implications.
In fuzzy literature the classification of implication operators is usually defined using other operators, like conjunction, disjunction and negation, but they are always based on t-norm and t-conorm paradigm. In this chapter, these concepts will be extended to any fuzzy system, including the compensatory ones. Here, when it would be necessary, the operators will preserve their exact definition, even if they don't correspond to any classification and taking into account that often the definition of an implication operator is associated with a specific t-norm and t-conorm.

The criteria for selecting implication operators for our purposes are the following:

1. The operator satisfies the truth-value table of the bivalent classical logic, when the truth-values calculus is restricted only to the set {0, 1}. Briefly, the truth-value of the formula x→y is 1 if x = 0 or x = y = 1, and is 0 if x = 1 and y = 0.

2. The operator must be a continuous function with regard to both arguments or it has a finite number of removable discontinuities.

The reason for imposing condition 1 is that this must be a natural extension of the mathematical logic. Whereas condition 2 guarantees the 'sensitiveness' of the composed predicates, that is to say, any change in the simple predicates will be reflected in the final results of their corresponding composed predicates.

Some classifications appeared in the literature are:

- S-implication (Dubois et al., 2007): $I_S(x,y) = d(n(x),y)$, where d and n are the disjunction and negation operators, respectively.
- R-implication (Dubois et al., 2007): $I_R(x,y) = \sup\{z\in[0,1]: c(x,z) \le y\}$, where c is the conjunction operator.

- QM-implication (Trillas et al., 2000), which is also known as QL-implication (Dubois et al., 2007): $I_{QL}(x,y) = d(n(x),c(x,y))$
- A-implication (Turksen et al., 1998): The operator satisfies a group of axioms, which implicitly associate it with the conjunction, disjunction and negation operators. For example, the Law of Importation $(x \wedge y \rightarrow z) \leftrightarrow (x \rightarrow (y \rightarrow z))$ is one of its axioms, where the symbol $\leftrightarrow$ is the logic equivalence.

The implication operators that have appeared in the literature, satisfy the two conditions expressed above, and their classifications are:

- Reichenbach implication (S-implication): $x \rightarrow y = 1 - x + xy$
- Klir-Yuan implication (a variation of the above case without a classification): $x \rightarrow y = 1 - x + x^2 y$
- Natural implication (S-implication), see (Espin et al., 2011): $x \rightarrow y = d(n(x),y)$
- Zadeh implication (QL-implication): $x \rightarrow y = d(n(x),c(x,y))$
- Yager implication (A-implication): $x \rightarrow y = y^x$

The formula of the equivalence is defined as: $x \leftrightarrow y = (x \rightarrow y) \wedge (y \rightarrow x)$. It is valid for any implication operator and any conjunction operator.

Other classifications can be found in (Jayaram, 2008).

This chapter shall revisit an experiment of Tversky and Kahneman appeared in (Tversky and Kahneman, 1992). Some results are included in the table 1 as a way of illustration:

Premium 1	Premium 2	Probability 1	Probability 2	Equivalent
0	50	0.9	0.1	9
0	50	0.5	0.5	21
0	50	0.1	0.9	37
0	-50	0.9	0.1	-8
0	-50	0.5	0.5	-21
0	-50	0.1	0.9	-37
0	100	0.95	0.05	14
0	100	0.75	0.25	25
0	100	0.5	0.5	36
0	100	0.25	0.75	52
0	100	0.05	0.95	78
0	100	0.95	0.05	-8
0	100	0.75	0.25	-23.5
0	100	0.5	0.5	-42
0	100	0.25	0.75	-63
0	100	0.05	0.95	-84
0	200	0.99	0.01	10
0	200	0.9	0.1	20
0	200	0.5	0.5	76
0	200	0.1	0.9	131
0	200	0.01	0.99	188

0	-200	0.99	0.01	-3
0	-200	0.9	0.1	-23
0	-200	0.5	0.5	-89
0	-200	0.1	0.9	-155
0	-200	0.01	0.99	-190
0	400	0.99	0.01	12
0	400	0.01	0.99	377
0	-400	0.99	0.01	-14
0	-400	0.01	0.99	-380
50	100	0.9	0.1	59
50	100	0.5	0.5	71
50	100	0.1	0.9	83
-50	-100	0.9	0.1	-59
-50	-100	0.5	0.5	-71
-50	-100	0.1	0.9	-85
50	150	0.95	0.05	64
50	150	0.75	0.25	72.5
50	150	0.5	0.5	86
50	150	0.25	0.75	102
50	150	0.05	0.95	128
-50	-150	0.95	0.05	-60
-50	-150	0.75	0.25	-71
-50	-150	0.5	0.5	-92
-50	-150	0.25	0.75	-113
-50	-150	0.05	0.95	-132
100	200	0.95	0.05	118
100	200	0.75	0.25	130
100	200	0.5	0.5	141
100	200	0.25	0.75	162
100	200	0.05	0.95	178
-100	-200	0.95	0.05	-112
-100	-200	0.75	0.25	-121
-100	-200	0.5	0.5	-142
-100	-200	0.25	0.75	-158
-100	-200	0.05	0.95	-179

Table 1 Results of an experiment of Tversky and Kahneman.

Columns 1, 2, 3 and 4 of table 1 represent prospects of two alternatives and the ultimate column summarizes equivalent values of their acceptance.

The data in table 1 will be interpreted with fuzzy models. Sigmoid is the membership function that will be used, according to the recommendation appeared in (Dubois and Prade, 1984).

The sigmoid function formula is:

$$\mathrm{sigm}(x,\alpha,\gamma) = \frac{1}{1+e^{-\alpha(x-\gamma)}} \qquad (12)$$

Let us note that sigm(γ,α,γ) = 0.5 and it is s-shaped, but different from function in figure 2.

Here a ‘scenario’ is a premium associated with a probability and it will be classified with the term ‘convenient’.

Three predicates will be calculated using fuzzy variables:

1. “If the scenario is probable then it is convenient”.

2. “All the scenarios are probable and convenient”. This statement measures the risk-aversion tendency by the decision makers.

3. “There exist probable and convenient scenarios”. This statement measures the risk-seeking tendency by the decision makers.

It is converted in an optimization (maximization) problem which will be detailed below in order of apparition:

1. The first proposition is divided in the following two: “If all the scenarios are probable then they are convenient” and “If there are probable scenarios then they are convenient”.

The maximization problems are respectively:

Max $\wedge^{56}_{i=1}$ $((u_p(p_i)\to u_x(x_i)) \wedge (u_p(1-p_i)\to u_x(y_i))) \leftrightarrow \mathrm{sigm}(eq_i,\alpha_{eq},\gamma_{eq}))$, where u_p and u_x are the sigmoid functions $\mathrm{sigm}(p,\alpha_p,\gamma_p)$ and $\mathrm{sigm}(x,\alpha_x,\gamma_x)$, representing respectively the predicates ‘the scenario is probable’ and ‘the scenario is convenient’. $\mathrm{sigm}(eq_i,\alpha_{eq},\gamma_{eq})$ is the sigmoid function of the equivalent values.

Besides, the second problem is: Max $\wedge^{56}_{i=1}$ $((u_p(p_i) \to u_x(x_i)) \vee (u_p(1-p_i) \to u_x(y_i))) \leftrightarrow \mathrm{sig}(eq_i,\alpha_{eq},\gamma_{eq}))$.

2. The maximization problem is:

Max $\wedge^{56}_{i=1}$ $((u_p(p_i) \wedge u_x(x_i)) \wedge (u_p(1-p_i) \wedge u_x(y_i))) \leftrightarrow \mathrm{sig}(eq_i,\alpha_{eq},\gamma_{eq}))$.

3. The maximization problem is:

Max $\wedge^{56}_{i=1}$ $((u_p(p_i) \wedge u_x(x_i)) \vee (u_p(1-p_i) \wedge u_x(y_i))) \leftrightarrow \mathrm{sig}(eq_i,\alpha_{eq},\gamma_{eq}))$.

Let us note that each optimization problem depends on a CFL system. The one-parameter family of formulas 3,4,5,6 will be one of the parameter to be estimated. Also, each problem derives in five cases, where the implication operator is applied from the five proposed in the beginning of the section. The other parameters to estimate are the alphas and gammas of the sigmoid function.

Some heuristic restrictions of the alphas and gammas that will be applied are:

1. All the alphas are strictly equal to 0. This condition guarantees that sigmoid is an increasing function and not a constant one, such as the case where it is equal 0.

2. The values of gammas are between the minimum and the maximum data in table 1, where they do not represent the equivalent values.

3. In case of the equivalent values of the last column in table 1, the gamma will be restricted between 0 and 76. As a result of the Prospect theory, it is well-known that people don’t accept non-positive values with indifference; taking into account that gamma is the value which represents indifference (0.5). 76 is 20% of the absolute value of the maximum number in the last column in table 1, which has been selected heuristically.

The optimization will be based on the genetic algorithm coded in MATLAB.

3. Results

The parameters estimated were summarized in the tables below:

Estimated parameters	Reinchenbach	Yager	Klir-Yuan	Natural	Zadeh
α_x	64	0.88085938	64	57	128
γ_x	1	30.6367188	0	1	129
α_p	11.0376854	2.12890625	230	19.8595638	21.6115036
γ_p	0.02615738	1	0	0.10683823	0.4031105
α_{eq}	45	0.09375	65	32	74.8601074
γ_{eq}	56	60.7246094	1	57	73
P	0	0	0	0	0
Maximum truth-value	0.93791284	0.85109059	0.87150398	0.88978479	0.79259532

Table 2 Estimated parameters for problem 1. "If all the scenarios are probable then they are convenient".

Estimated parameters	Reinchenbach	Yager	Klir-Yuan	Natural	Zadeh
α_x	32	0.0703125	64	31	128
γ_x	1.5	317.71875	1	1	1
α_p	230	7.5625	6.76686478	97	17.2717075
γ_p	0	1	0	1	0
α_{eq}	0.03500748	0	25	0	65
γ_{eq}	0	1	16	1	17
P	0	0	0	0	-1.9073E-06
Maximum truth-value	0.85970284	0.7147067	0.8335026	0.5411961	0.76028923

Table 3 Estimated parameters for problem 1. "If there are probable scenarios then they are convenient".

Estimated parameters	Reinchenbach	Yager	Klir-Yuan	Natural	Zadeh
α_x	5.52869034	0.734375	6.02235603	7.0930481	12.0120811
γ_x	1	14.8125	1	1	1
α_p	230	230	230	230	230
γ_p	0	0	0	0	0
α_{eq}	62	0.09375	30	24	24
γ_{eq}	53	58.9453125	57	57	57
P	0	0	0	0	0
Maximum truth-value	0.90420119	0.81944258	0.89626517	0.83563393	0.87025163

Table 4 Estimated parameters for problem 2.

Estimated parameters	Reinchenbach	Yager	Klir-Yuan	Natural	Zadeh
α_x	97	0.3203	97	128	97
γ_x	1	18.6406	1	65	1
α_p	8.4261	6.7734	8.17059708	15.8297119	14.9041805
γ_p	0.354	0	0.24069786	0.58897972	0.82479858
α_{eq}	229.2813	0.0781	48	129	74.8599014
γ_{eq}	20.375	0	17	73	73
P	0	0	0	0	0
Maximum truth-value	0.9046	0.7803	0.87730427	0.83112339	0.76983507

Table 5 Estimated parameters for problem 3.

These results allow arriving to some conclusions:

- All the predicates show better results with the Reichenbach implication.
- People measure preferences with Geometric Mean (p=0).

- With Reichenbach implication, the values of indifference for the scenarios are equal or slightly bigger than 1.
- The probabilities are measured with small slopes and $\gamma_p>0$, for problems: "If all the scenarios are probable then they are convenient" and "There exist probable and convenient scenarios" (risk-seeking), see tables 2 and 5. Besides, the probabilities for: "If there are probable scenarios then they are convenient" and "There exist probable and convenient scenarios" (risk-aversion), have big slopes and the minimum of their values is 0.5 for the probability 0, see tables 3 and 4.

Other experiments made by authors, show that the shape of the membership function, like in figure 2, doesn't contribute to better results of the truth-values in the maximizations.

4. *Concluding Remarks*

This chapter makes a fuzzy approach to Prospect theory by using the compensatory fuzzy logic. A one-parameter family of Compensatory Fuzzy Logic and five implication operators selected are used to obtain the maximization of four objective functions with the genetic algorithm coded in MATLAB. This approach is a revisit to a 1992 experiment of Kahneman and Tversky.
The family of CFL depends on a parameter p, equal to or less than 0 and they are based on the formula of the quasi-arithmetic mean. On the other hand, Reichenbach implication, Yager implication, Klir-Yuan implication, Natural implication and Zadeh implication are selected because they generalize the truth table of the bivalent logic, when they are restricted to values 0 or 1. Also, they are continuous or they have at most a finite number of removable discontinuities.
According to the empirical results, the Reichenbach implication and the Geometric Mean are closest to the people's way of thinking. The sigmoid membership functions of some predicates, like "the scenario is convenient" or "the scenario is probable" are found to be included in the composed predicates like "If the scenario is probable then it is convenient", "All the scenarios are probable and convenient" or "There exist probable and convenient scenarios".
1 or 1.5 are the values of indifference for the premiums, according to Reichenbach implication results. The membership function for probabilities changes for each predicate.
The sigmoid functions were used for modelling every predicate, including those related with probabilities, even though its shape differs from the function illustrated in figure 2.

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