# THE ECONOMIC LOT-SIZING PROBLEM WITH FIXED PERIODS FOR REMANUFACTURING

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## RESUMEN

En este trabajo abordaremos el problema del tamaño del lote económico con remanufacturación (ELSR) en donde los períodos en los cuales la remanufacturación se puede llevar a cabo han sido fijados previamente. El objetivo es determinar de forma independiente las cantidades óptimas de remanufacturación de cada uno de estos períodos. Empezaremos el estudio con el caso particular de un único período con remanufacturación positiva y luego abordaremos el caso general de más de un período. Para el caso de un solo período, brindaremos un procedimiento eficiente para hallar la cantidad óptima bajo ciertos supuestos en los costos. Para el caso de más de un período demostraremos que el problema es NP-hard aun sabiendo cual es la cantidad total óptima de remanufacturación. El artículo concluye con algunas ideas con el fin de desarrollar un procedimiento efectivo para el problema.

**PALABRAS CLAVES. Remanufacturing, ELSP, Inventory Control.**

**AD & GP**

## ABSTRACT

In this paper we address the economic lot-sizing problem with remanufacturing (ELSR) assuming that the periods where remanufacturing is allowed to be positive have been fixed in advance. The goal is to determine the optimal quantities of remanufacturing for the periods fixed in an independent way. We begin considering the case of only one period fixed as positive-remanufacturing period and then the general case of more than one period fixed. For the single-period case, we are able to determine an efficient time procedure for obtaining the optimal remanufacturing quantity under certain assumptions on the costs. For the multi-period case we show that the problem is NP-hard even in the case that the total remanufacturing quantity of an optimal solution is known. The paper concludes with some guidelines for developing an effective procedure for the problem.

**KEYWORDS. Remanufacturing, ELSP, Inventory Control.**

**AD & GP**

## 1. Introduction

We address the economic lot-sizing problem (ELSP) for which the demand requests can be satisfied either by producing new items or by remanufacturing used items backed to the origin. A detailed description of the problem is as follows. The economic lot-sizing problem with remanufacturing (ELSR) refers to the problem of determining the quantities to produce, remanufacture, and dispose in each period over a finite planning horizon in order to meet the demand requirements of a single item on time, minimizing the sum of the involved costs. Used products returned by the customers are available at each period for remanufacturing. In addition, the returns can be disposed off, e.g., when there is an overstock of used products. The ELSR has been receiving an increasing academic attention from late 90s as the industry has been involved with the recovery of used products due to governmental and social pressures as well as economic opportunities. Remanufacturing can be defined as the recovery process of returned products after which the products look as good as new for the customer's point-of-view. Remanufacturing tasks often involve disassembly, cleaning, testing, part replacement or repairing, and reassembling operations. Products that are remanufactured include automotive parts, engines, tires, aviation equipment, cameras, medical instruments, furniture, toner cartridges, copiers, computers, and telecommunications equipment. Remanufacturing offers benefits for all of the parties involved. We refer the readers to de Brito and Dekker (2002), Guide (2000), Gungor and Gupta (1999), and Hormozi (2003) for details descriptions about the remanufacturing benefits.

As we know, Richter and Sombrutzki (2000) and Richter and Weber (2001) are the first to consider the ELSP with returns options, analyzing the particular case that the number of returns in the first period are sufficient to satisfy the total demand over the planning horizon. Golany et al. (2001) suggest a Network Flow formulation for the ELSR and provide an exact algorithm of $O(T^3)$ time for the case of linear cost functions. They also show that the ELSR is a NP-hard problem for the case of general concave cost functions. Yang et al. (2005) and van den Heuvel (2004) extend this last result about complexity for the cases of stationary concave cost functions and set-up and unit costs for the activities and for holding inventory, respectively. Teunter et al. (2006) consider the ELSR with joint set-up costs for the production and remanufacturing activities, and suggest an $O(T^4)$ time algorithm based on a dynamic programming approach. Piñeyro and Viera (2009) suggest and compare several inventory policies for the ELSR using a divide-and-conquer approach and a Tabu Search based on procedure. Piñeyro and Viera (2010) consider an ELSR extension with different demand streams for new and remanufactured items where in addition substitution is allowed for remanufactured items but not viceversa. Nenes et al. (2010) provide an analysis of the ELSR taking into account the quality of the returns and Helmrich et al. (2010) provide and compare different mathematical formulations for the ELSR with separate and joint set-up costs for the activities. They show that the ELSR with joint set-up costs is also NP-hard.

In this paper we consider the ELSR under the assumption that the periods where remanufacturing can be positive have been fixed in advance. This assumption is supported by academic as well as practical reasons. The analysis on the remanufacturing quantities contributes to deepen the knowledge of the characteristics of the ELSR solutions, and then it is hopefully that we can develop better solution procedures or improve the existing ones. We also note that the remanufacturing activity plays a key role in the ELSR resolution (Piñeyro and Viera, 2009). If the remanufacturing plan is known, the others plans can be determined easily by means of any of the well known procedures for solving the ELSP as the algorithm of Wagner and Whitin (1958) of $O(T^2)$ time. On the other hand, there must be real situations for which it makes sense to restrict the periods where remanufacturing can be positive, e.g., operative reasons if the machinery and workers are the same for production and remanufacturing; availability of used items only in certain periods; or economic reasons due to periods with remanufacturing at low cost.

According to our best knowledge, the problem of determining the remanufacturing quantities of the ELSR with fixed periods for positive remanufacturing was first tackled by Piñeyro and Viera (2012). The aim of this work is to extend the analysis begun in that paper. In particular, we show that the problem of determining the optimal remanufacturing quantities is NP-hard in general, even if the total remanufacturing quantity of an optimal solution is known.

The remainder of the paper is organized as follows. Section 2 introduces the mathematical notation used through the paper. The analysis of the ELSR with fixed periods for remanufacturing for the cases of only one period and more than one period fixed are considered in Section 3 and Section 4, respectively. Section 5 concludes the paper along with some directions for future research.

## 2. Mathematical formulation

We consider a dynamic and finite economic lot-sizing problem with $T$ periods, $0 < T < \infty$, with demand and returns values denoted by $D_t$ and $R_t$ for each period $t = 1,...,T$ respectively; $K_t^p$, $K_t^r$, $K_t^d$, $c_t^p$, $c_t^r$ and $c_t^d$ denote the set-up and unit costs for production, remanufacturing and final disposing in periods $t = 1,...,T$, respectively; $h_t^s$ and $h_t^u$, denote the unit cost of holding inventory for serviceable and used products in periods $t = 1,...,T$, respectively. In addition, $F \in 2^T$ denote the set of periods for which the remanufacturing is allowed to be positive, i.e., $r_t \geq 0$ if and only if $t \in F$, $r_t = 0$ otherwise. The objective is to determine the values for the decision variables $p_t$, $r_t$ and $d_t$ of producing, remanufacturing and final disposing at each period $t = 1,...,T$, respectively, and for holding inventory of serviceable and used items $y_t^s$ and $y_t^u$, respectively. The problem can be modeled as the following Mixed Integer Linear Programming (MILP) problem:

$$\min \sum_{t=1}^{T} \{K_t^p \delta_t^p + c_t^p p_t + K_t^r \delta_t^r + c_t^r r_t + K_t^d \delta_t^d + c_t^d d_t + h_t^s y_t^s + h_t^u y_t^u\} \quad (1)$$

subject to :

$$y_t^s = y_{t-1}^s + p_t + r_t - D_t \qquad \forall t = 1,2,...,T \quad (2)$$

$$y_t^u = y_{t-1}^u - r_t + R_t \qquad \forall t = 1,2,...,T \quad (3)$$

$$M\delta_t^p \geq p_t \qquad \forall t = 1,2,...,T \quad (4)$$

$$M\delta_t^r \geq r_t \qquad \forall t = 1,2,...,T \quad (5)$$

$$M\delta_t^d \geq d_t \qquad \forall t = 1,2,...,T \quad (6)$$

$$y_0^s = y_0^u = 0 \quad (7)$$

$$r_t = 0 \qquad \forall t \notin F \quad (8)$$

$$\delta_t^p, \delta_t^r, \delta_t^d \in \{0,1\} \quad p_t, r_t, d_t, y_t^s, y_t^u \geq 0 \qquad \forall t = 1,2,...,T \quad (9)$$

With $M$ a number at least as large as $\max\{D_{1T}, R_{1T}\}$, where $D_{ij}$ and $R_{ij}$ are the accumulative demand and returns between periods $i$ and $j$, with $1 \leq i \leq j \leq T$. Binary variables $\delta_t^p$, $\delta_t^r$ and $\delta_t^d$, are equal to 1 if production, remanufacturing or disposing is carried out in periods $t = 1,...,T$, or 0 otherwise, respectively. We also note that variables $y_t^s$ and $y_t^u$ can be eliminated of the model, as they can be obtained by means of the expressions $y_t^s = p_{1t} + r_{1t} - D_{1t}$

and $y_t^s = R_{1t} - r_{1t} - d_{1t}$ respectively, where $p_{ij}$, $r_{ij}$, and $d_{ij}$ denote the accumulative production, remanufacturing and disposing quantities between periods $i$ and $j$, with $1 \le i \le j \le T$, respectively.

The model above is similar to that given in Golany et al. (2001), Yang et al. (2005) and Piñeyro and Viera (2009) for the ELSR except by constraint (8) which is introduced in order to indicate the periods where remanufacturing is not allowed to be positive. The ELSR with periods fixed for remanufacturing can be considered as an extension of the traditional ELSR, if we consider the case that $|F| = T$, and then it is also NP-hard in general (van den Heuvel, 2004). As discussed in the following sections, the problem of determining the remanufacturing quantity for the case that $|F| = 1$ can be solved in polynomial time and the NP-hard result remains valid for those cases for which $1 < |F| < T$. For the analysis we consider the following condition about the costs related to the used items.

*Definition 1*. We say that the costs of the returns are at most equal than the costs of the new items when the expressions below are fulfilled by the cost components:

$$K_i^r \le K_j^p, \qquad (10.1)$$
$$c_i^r \le c_j^p, \qquad (10.2)$$
$$h_i^u \le h_j^s, \qquad (10.3)$$

for any couple of periods *i* and *j* in $1,...,T$.

Henceforth we assume that the expressions of Definition 1 are fulfilled for the ELSR-*F*.

**3. The single-period case**

In this section we address the problem of determining the remanufacturing quantity of the ESLR with only one period $i$ fixed as positive remanufacturing period, with $1 \le i \le T$. We refer to this problem as ELSR-$\{i\}$. This problem was tackled first by Piñeyro and Viera (2012) for the case that the costs satisfy the condition that it is profitable to maximize the remanufacturing of the period fixed. Here we relax this condition for the analysis. However, we also consider that remanufacturing is profitable according to the following definition, as it is common in the literature.

Consider first the case that the number of available returns is at most equal then the demand of the period, i.e., $R_i + y_{i-1}^u \le D_i$. Then, the optimal remanufacturing quantity is $r_i = R_i + y_{i-1}^u$ as we are assuming that expressions of (10) are fulfilled. On the other case, if the number of available returns is greater than the demand requirement of the period, we must determine the last period *j* within the planning horizon for which it is more profitable satisfy its demand by remanufacturing in period *i* rather than by producing in period *j* or in a previous one, with $1 \le i \le j \le T$. Piñeyro and Viera (2012) solve this last case of the ELSR-$\{i\}$ by imposing certain condition on the costs that makes profitable to maximize the remanufacturing quantity in the period fixed. Here, we provide an efficient procedure for solving the ELSR-$\{i\}$ in general, i.e., without assuming any other condition on the costs. It is based on the following result about the profitability of remanufacturing in an optimal solution of the ELSR-$\{i\}$.

*Proposition 1*. Consider an ELSR-$\{i\}$ instance. If it is profitable to meet one unit of the demand of certain period *j* by remanufacturing in *i*, then it is profitable satisfy as much as possible by remanufacturing in period *i*, with $1 \le i \le j \le T$.

*Proof*. Consider an ELSR-$\{i\}$ solution for which $r_i = D_{i(j-1)} + 1 < R_i + y^u_{i-1}$. Without loss of generality we assume that $D_j \ge 2$. We note that the remaining demand of period *j* is satisfied by producing only in certain period *t*, with $1 \le t \le j \le T$. Let us suppose that it is not profitable to increase the remanufacturing of period *i* in one unit. Then, it must be that $c^p_j + \sum_{t}^{(j-1)} h^s_\tau < c^r_j + \sum_{i}^{(j-1)} h^s_\tau + \sum_{i}^{T} h^u_\tau$. Therefore, we can reduce the cost of the current solution by reducing the remanufacturing of period *i* in one unit, i.e., $r_i = D_{i(j-1)}$ rather than $r_i = D_{i(j-1)} + 1$. This means that not even one unit of the demand of period *j* should be fulfilled through the remanufacturing of period *i*. However, as we are assuming that it is profitable to meet one unit of the demand of period *j* there must be that $c^p_j + \sum_{t}^{(j-1)} h^s_\tau \ge c^r_j + \sum_{i}^{(j-1)} h^s_\tau + \sum_{i}^{T} h^u_\tau$ and then, it is profitable to increase as much as possible the remanufacturing in period *i* in order to meet the demand of period *j*. Therefore, we can obtain a new solution with a lower cost than the original, which fulfills that either nothing or as much as possible of the demand of period *j* is satisfied by remanufacturing in period *i*.

■

Proposition 1 means that in order to determine the optimal remanufacturing quantity for the period *i* of the ELSR-$\{i\}$ we must consider the periods one by one from period *i* onwards until we find one period for which either it is not profitable to meet as much as possible of its demand by remanufacturing at period *i*, or the returns in period *i* has been exhausted. Having this result in mind we provide the following pseudocode of a procedure for solving the ELSR-$\{i\}$.

```
r_t = 0,  ∀t = 1...T
(p,d) = ELSP_solver(r)
s = (p,r,d)
α = R_i + y^u_{i-1}
t = i
c = max_int
fin = 0
while α > 0 and t ≤ T and fin = 0 do
    r_i = min(α, D_it)
    (p,d) = ELSP_solver(r)
    if cost(p,r,d) < cost(s)
        s = (p,r,d)
        α = α − r_i
        t = t + 1
    else
        fin = 1
    endif
enddo
return s
```

**Sketch of procedure for solving the ELSR-{*i*}**

In the pseudocode above $s=(p,r,d)$ makes reference to the ELSR solution *s* with a production plan *p*, remanufacturing plan *r* and final disposing plan *d*, respectively. Procedure ELSP_solver of lines 2 and 10 can be implemented by any of the well known algorithms for solving the ELSP like the $O(T^2)$ time algorithm of Wagner and Whitin (1958) or faster algorithms of $O(T\log T)$ time of Federgruen and Tzur (1991), Wagelmans et al. (1992) or Aggarwal and Park (1993). We note that the ELSP can be applied since the optimal production and final disposing plans of the ELSR can be solved independently if the remanufacturing plan is known (Piñeyro and Viera, 2009).

We analyze now the computational complexity of the procedure sketched above. First we note that we must consider at most $(T-i+1)$ periods if the period *i* is fixed as the single period with positive remanufacturing of the ELSR. Then, the worst case is $i=1$ since we must consider *T* periods. For each period under consideration we need to compute the optimal production and final disposing plans in order to obtain the ESLR solution, solving two independent ELSP instances. We also assume that the time for computing the cost of an ELSR solution can be neglected. Therefore, the ELSR-$\{i\}$ can be solved in $O(T^3)$ time if the algorithm of Wagner and Whitin (1958) is used or in $O(T^2\log T)$ time if faster algorithms are used for the ELSP like the algorithms of $O(T\log T)$ time mentioned above. We also note that the optimal single period for remanufacturing of an ELSR instance can be computed in $O(T).O(\text{ELSR-}\{i\})$ time as we must consider each one of the *T* different periods.

## 4. The multi-period case

This section is devoted to analyze the computational complexity of the problem of determining the remanufacturing quantities of the ELSR-*F*, with $1<|F|<T$, in an independently way, i.e., the periods and quantities of both the production and final disposing plans are unknown. Piñeyro and Viera (2012) show that when the periods with strictly positive remanufacturing are known in advance, the total remanufacturing quantity of an optimal solution can be obtained in linear time. They also claim that it is unlikely that we can determine the optimal remanufacturing quantity for each period by means of a polynomial time procedure. In this section we formally show that this problem is NP-hard, even if the total remanufacturing quantity is known in advance.

*Proposition 2*. The problem of determining the quantities of the remanufacturing plan in an independently way for the ELSR-*F*, with $1<|F|<T$, is NP-hard.

*Proof*. Let us consider an ELSR-*F* instance for which the inventory holding cost for serviceable items is zero. Thus, we note that the Piñeyro and Viera (2012) condition on the costs for maximizing the remanufacturing quantity in the periods fixed is complied for all periods within the planning horizon, i.e., $c_i^r \le c_j^p + \sum_{t=i}^{T} h_t^u$ is fulfilled for any couple of periods *i* and *j*, with $1\le i\le j\le T$. Also by Piñeyro and Viera (2012) we know that there is an optimal solution for which the total remanufacturing quantity $r_{1T}$ is equal to the sum of the upper bound of remanufacturing $u_i$ obtained for each period *i*, with $u_i=\min(R_i+y_i^u, D_{i(j-1)})$, $i\in F$ and $j=\text{npr}(i)$. Function npr() takes a period and returns the next period fixed as positive remanufacturing period, if it exists, or the last period within the planning horizon. Formally, this function is defined as follows:

$$\mathrm{npr}(i)=\begin{cases} j, & i,j\in F, r_t=0\ \forall t: i<t<j\le T \\ T, & i\in F, r_t=0\ \forall t: i<t\le T \\ 0, & i\notin F \end{cases} \tag{11}$$

We note that the upper bounds of remanufacturing can be computed sequentially from the first to the last period within the planning horizon. Then, the problem of determining the quantities of the remanufacturing plan of perfect cost for this particular instance of the ELSR-*F* can be formulated as the following MILP:

$$\min\sum_{t=1}^{T}\left\{K_t^r\delta_t^r+c_t^r r_t+h_t^u y_t^u\right\} \tag{12}$$

subject to:

$$y_t^u=y_{t-1}^u-r_t+u_t \qquad \forall t=1,2,\dots,T \tag{13}$$

$$M\delta_t^r\ge r_t \qquad \forall t=1,2,\dots,T \tag{14}$$

$$y_0^s=y_0^u=0 \tag{15}$$

$$u_t\le C_t^r \qquad \forall t=1,2,\dots,T \tag{16}$$

$$\delta_t^r\in\{0,1\}\ r_t, y_t^u\ge 0 \qquad \forall t=1,2,\dots,T$$

where the capacity limit $C_i^r$ for each period *i* is defined as follows:

$$C_i^r=\begin{cases} D_{i(j-1)}, & i\in F \text{ with } j=\mathrm{npr}(i) \\ 0, & i\notin F \end{cases} \tag{18}$$

We can note that the MILP formulated above is equivalent to that formulated for the Capacitated Lot-Sizing Problem (CLSP) for which the demand requirement of each period *i* is $u_i$, with $1\le i\le T$. As we know by Florian et al. (1980) the CLSP is NP-hard if the capacity pattern is arbitrary, and then the problem of determining the quantities of the remanufacturing plan of perfect cost for the ELSR-*F*, with $1<|F|<T$ is also NP-hard.

## 5. Conclusions and future research

In this paper we have addressed the ELSR with fixed periods for remanufacturing. For the case of only one period fixed as positive remanufacturing period, we derived a polynomial time procedure for obtaining the optimal remanufacturing quantity. The optimal single period for remanufacturing can be also determined in polynomial time. For the general case of more than on period fixed, we showed that the problem of determining the remanufacturing quantities is NP-hard, even if the total remanufacturing quantity is known in advance and the number of periods fixed is less than the length of the planning horizon.

Considering the NP-hard result of the problem of determining the remanufacturing quantities for the ELSR-*F*, we should put the effort in developing effective heuristic procedures for the problem. In this sense we should consider the result of Piñeyro and Viera (2012) about the total remanufacturing quantity of an optimal solution of the ELSR-*F*. Secondly, we should take into account the result about the single-period case described in this paper, which states that we must consider that the remanufacturing level in the period fixed is either nothing or as much as possible in order to meet the demand requirement of each period. However, we note that there may be cases in which this last fact is not the best decision. Therefore, more research is still needed on the problem of determining the remanufacturing quantities, along with the periods where the remanufacturing is carried out.

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