SINGLE-STAGE MULTI-PRODUCT PRODUCTION AND INVENTORY SYSTEMS: AN ITERATIVE ALGORITHM BASED ON DYNAMIC SCHEDULING AND FIXED PITCH PRODUCTION

Euclydes da Cunha Neto
National Institute of Technology
Rio de Janeiro, RJ - Brazil
euclydes.cunha@int.gov.br

Virgílio José Martins Ferreira Filho
Industrial Engineering Program, Federal University of Rio de Janeiro
Rio de Janeiro, RJ - Brazil
virgilio@ufrj.br

Edilson Fernandes de Arruda
Industrial Engineering Program, Federal University of Rio de Janeiro
Rio de Janeiro, RJ - Brazil
efarruda@pep.ufrj.br

ABSTRACT

The paper presents a model and an algorithm for solving a class of stochastic economic lot sizing problems (SELSP). SELSP is important because it is very common in both manufacturing and process industries. The algorithm is related to the manufacturing strategy which consists of dynamic scheduling and fixed pitch production. These strategies are rather unexplored in the literature; yet, they are promising since they give flexibility to the solution.

The algorithm returns a near-optimal solution composed of all the parameters needed to operate the production system. In order to do so, it iteratively combines two procedures: (i) a method of successive approximations which chooses progressively better values for the production pitch and the lot sizes, and (ii) a discrete stochastic simulation routine to find the order points.

The algorithm proved to be fast enough to allow its frequent use in real world problems, on computers in current use.

KEYWORDS: SELSP, lot sizing, scheduling.

1. Introduction

The present paper deals with production and inventory systems of multiple products processed in a single stage, with a single machine, one product at a time, while the system is subject to the presence of significant setup time. This problem is extensively treated in literature, being identified as SELSP (Stochastic Economic Lot Sizing and Scheduling Problem), or just ELSP (Economic Lot Sizing and Scheduling Problem) in its deterministic version.

Its importance stems from the fact that it is very common in both process and manufacturing industries. In process industries, it is central to planning and controlling the production of various segments, as for example: petrochemical, chemical, pharmaceutical and food industries. In manufacturing, it frequently appears in production steps such as: stamping, plastic and metal molding, machining centers, assembly cells, etc.

More recently, interest in SELSP has been renovated and expanded due to the popularization of a management approach known as *lean manufacturing,* which seeks to organize the industrial plants as chains composed of consecutive modules of production and inventory, of which a large amount are real instances of SELSP.

The methods of solution in the deterministic version of the problem (ELSP) try to find a production cycle comprising a fixed sequence of production and fixed lot sizes, so that: (i) it can be repeated indefinitely, (ii) it fully meets the demand (iii) it respects the capacity and (iv) it incurs in the lowest possible long term storage and setup costs.

In order to solve the stochastic problem (SELSP), there are several different strategies. Several methods were proposed that complement ELSP with rules that make the production sequence and / or lot sizes more flexible, in response to the current state of the system, represented by the balance for each product in inventory and the current machine setup. Generally, the solution must contain a set of rules capable of indicating, at each moment, if the machine must be kept in its current state (continuing with the same product in production or continuing idle), or if its status should be changed to begin the production of another product or if it should start the idle period.

This article aims at presenting a solution to a class of SELSP restricted by the following characteristics:

a) The demand is known, with exponential and stationary probability function;
b) The service level (i.e. the percentage of time in which demand was fully met during inventory lead time with regards to the total number of lots requested) is pre-defined and equal for all products;
c) Demand that is not readily met is pending for service as soon as possible (backlog);
d) Production and setup unit times of each product are deterministic, known and independent of the production sequence;
e) The production capacity is fixed, known and previously contracted for fixed cost, regardless of percentage of use;
f) There is no significant setup cost (besides the loss of production time);
g) The cost of stocking is directly proportional to the maximum total coverage of the inventory of products (i.e. the ratio between the highest possible level of inventory of finished products plus work-in-process inventory of each product divided by its rate of demand);
h) The raw material needed to produce one lot is integrally provided to the production process when the lot production is requested;
i) The products of a lot are delivered to stock all at once at the end of the lot production;
j) The planning horizon is indefinite or infinite.

The remainder of this paper is organized as follows: the next section presents the theoretical context of SELSP, limited to the references which are most relevant to the work. Section 3 explains and justifies the rules of control that make up the production strategy adopted. Section 4 formulates a mathematical model to calculate the parameters needed to operate the system through the chosen strategy. Section 5 presents the structure of the algorithm that was implemented for solution of the model. Section 6 shows the output of the algorithm to three instances of the problem. In section 7, some conclusions are discussed and possible developments are pointed out.

2. Theoretical background

Management methods that establish a priori rules or strategies might be discarding the best solutions. The best ones can be found, in principle, by fully flexible methods, i.e. methods whose decisions are made dynamically, depending on the current state of the system. Nevertheless, the latest revision of the state of the art in SELSP by Winands et al. (2010) showed that most proposed solutions pre-set static rules, which are kept fixed regardless of the state of the system.

Only two fully flexible solutions proposed for SELSP were found in the literature. In one of them, Qiu and Loulou (1995) modeled the SELSP as a semi-Markov stochastic process and solved it using dynamic programming. Thus, a near-optimal solution and an estimated maximum deviation in relation to optimum were obtained. Their proposal, however, is limited by the following: (a) it may only be used for 2 or 3 products, due to the exponential growth in the state space of the stochastic process, and (b) the difficulty to operate, and even communicate to the user, the policy to be pursued when we have more than two products.

The other fully flexible solution was presented by Paternina-Arboleda and Das (2005), who developed an optimization method based on multi-agent simulation, using an artificial intelligence procedure called *reinforcement learning* to "teach" the agents to take the decision. According to the authors, this method is efficient enough to be used with many products. As the difficulty to inform the policy to the user persists, the authors proposed to use *data mining* to extract and communicate only the most relevant part of the decision rules.

With so few results regarding flexible methods, Winands et al. (2010) strongly suggested that further research target particularly on dynamic scheduling strategies based on intuitive near-optimal rules which can be used in problems with many products.

Following this point of view, Segerstedt (Segerstedt, 1999) and Nilsson (Nilsson and Segerstedt, 2008) have developed and perfected dynamic scheduling methods of production, exploring the idea of producing first the product which is closest to running out of stock. Furthermore, Brander et al. (2005) showed through simulations that the queuing discipline has a more decisive impact on the quality of solutions than the lot size. They postulated that flexibility in the sequence was more important than the lot sizes in solutions for the SELSP.

Another important contribution to the development of flexible methods has been given by Zipkin (Zipkin, 1986). He proposed the connection between classical models of inventory and queuing replenishment orders for the solution of SELSP. His work showed that these problems are convex as long as certain restrictions are met. Among these, the most 'uncomfortable' one is that the production time for a product lot should be approximately independent of the lot quantity.

3. Control rules proposed

Following the guidelines recommended by Winands et al. (2010), we adopted a set of control rules to solve the SELSP described in section 1, which are, at the same time, flexible and easy to operate. The control rules (to be used together) are:

3.1. Production dynamic scheduling

Similarly to Segerstedt’s proposal (Segerstedt, 1999), products will be prioritized for production according to their time coverage. ‘Time coverage’ is the result of dividing the balance stock for its demand rate.

Thus, every time it is possible to start the production of a new lot size, the product with the shortest time coverage will be chosen among those whose inventory is below the ‘order point’. If inventory is above the ‘order point’ for all products, then the machine must be idle. Due to this dynamic prioritization rule and to cost assumptions, the order points of several products should cover about the same demand time, subject to rounding errors.

The 'order point', on the one hand, is part of the inventory management policy (rule '3.2' below). It works as a production halt point, which is essential to avoid overproduction, since any feasible solution for the problem requires slack capacity.

This scheduling rule will be identified as *FSFP* (acronym for 'first stockout, first to be produced'). This choice has the following advantages:

- It is dynamic and highly flexible.
- It is intuitive and communicates in a rather simple way with the user when compared to the fully flexible methods, e.g. (Qiu and Loulou, 1995) and (Paternina-Arboleda and Das, 2005).

3.2. Inventory control policies (s, Q)

Tracking inventory through policies (s, *Q)* means that, when the inventory of product i reaches the order point $s_{i,}$ a replenishment order of size Q_i is immediately triggered. This choice is justified due to the following:

- Ease of control, understanding and practical adoption of the method.
- Guarantee that the percentage of time actually consumed in setups will not be affected by the current demand, keeping the level planned at the time of calculating lot sizes.
- As Brander et al. (2005) have shown through simulations, dynamic scheduling rules are sufficient and more effective to respond to demand fluctuations than lot sizes defined dynamically. That is, with the use of a dynamic sequence, the fixed lot does not overload the solution.

3.3. Fixed production pitch

It is said that the plant uses a 'fixed production pitch' (or 'production pitch' or simply 'pitch'), when the scheduled time for the execution of a production lot (including setup time) is constant, regardless of what product is made.

The interest in policies based on fixed pitch production stems from two facts: (i) many industries already operate in this way on a daily basis, and (ii) the pitch is part of the increasingly popular *lean manufacturing* methodology.

The adoption of a fixed pitch surely means absorbing some extra cost, besides the minimum cost. Depending on the instance, this cost can be more or less significant. However, the following organizational advantages can be obtained, which may be often compensating:

- Easier daily control, making room for a more predictable and rhythmic production, and more reliable plans.
- Possibility of rationalizing and reducing the unit setup time and cost due to its need to become perfectly predictable (once at each pitch). Lower setup time and cost, in turn, enable smaller economic lots, which allow greater flexibility, smaller stocks and smaller final costs.

4. Model for calculation of parameters

In order to fully solve the SELSP for the best production strategy of the kind described in the previous section, it is necessary to determine the following variables: the pitch, the lot sizes and the order point for each product.

This section formulates a model by coupling typical inventory model equations with typical queue model expressions. As mentioned in section 2, this approach was used originally by Zipkin (Zipkin, 1986) and has the advantage to result in convex models.

4.1. Nomenclature

a) Sets:

i index that indicates an inventory item;
N total amount of different products in the system.

b) Data entry:

d_i demand for product i (random variable with exponential probability function in units per time unit);

$\bar{d}_i$ average demand for product i (in units per unit of time);

o_i operation time required to produce a single unit of product i (in units per unit of time);

a_i setup time for product i (in units of time);

K service level required by the market (common to all products - in percentage).

c) Decision variables:

Q_i fixed batch size for product i (units);

s_i order point of product i (units).

d) Auxiliary variables:

P length of production pitch (in unit of time);

A number of setups required per time unit (in times per time unit);

ρ utilization factor of capacity for all products, including the setup times (in percentage);

Tq waiting time of an order in the queue (random variable, in unit of time).

4.2. The model

a) Objective function:

Given the set of assumptions related to cost, the aim of minimizing the cost of operation can be expressed as an objective function that pursues the lowest total maximum coverage time. Then we have:

$$Z = min \ \sum_{i=1}^{N} \frac{s_i}{\bar{d}_i} + \sum_{i=1}^{N} \frac{Q_i}{\bar{d}_i} \tag{1}$$

In equation (1), the first sum comprises the safety stock and the work in process (to the extent that, on average, it corresponds to the amount of material that is below the order point), whereas the second sum is related to the highest level of the inventory cycle.

b) Capacity constraint:

Insofar as the contracted capacity remains fixed throughout the planning horizon, and this is infinite, the total amount of time available is divided into three parts:

- Total execution time of operations: it is a fixed percentage of the contracted capacity.
- Total setup time: it is inversely proportional to the lot sizes and limited by the difference between the total time contracted and the total time of operations.
- Idle time: time that is neither employed in operations or in setup, which is necessarily greater than zero due to random fluctuations in demand.

Constraint (2) below expresses the limitation in the total time used in setups. The average total time employed in setups per unit of time is on the left side of the equation, where for each product i, the demand rate $\bar{d}_i$ divided by the lot size Q_i results in the average number of setups per time unit. On the right side of the equation, the result of the sum is the average total time spent in operations per unit of time and the result of the subtraction is the average total idle time per unit of time.

$$\sum_{I=1}^{N} \left(a_i \frac{\bar{d}_i}{Q_i} \right) < 1 - \sum_{i=1}^{N} \left(\bar{d}_i \, o_i \right) \tag{2}$$

c) Fixed-pitch production constraints:

The policy of using the same production pitch for any product imposes the set of constraints (3). Thus, all lot sizes are determined uniquely as a direct result of the chosen pitch P.

$$(Q_i \ o_i) \ + \ a_i \ = \ P \qquad \forall \ i = 1 \ldots N \qquad (3)$$

d) A constraint that fixes the number of setups required per unit of time (A) according to the pitch P:

Through constraint (4), the number of setups per unit of time A is determined from the lot sizes Q_i (indirectly by pitch P):

$$A = \sum_{i=1}^{N} \frac{\bar{d}_i}{Q_i} \qquad (4)$$

e) A constraint that relates the number of setups A with the percentage of utilization capacity ρ:

As the number of setups per unit of time A is exactly the number of non-idle pitches per unit of time (or lots to be produced per unit of time), equation (5) determines the utilization factor:

$$\rho = A \ P \qquad (5)$$

f) Assistance constraints to the service level K:

By definition from the basic theory of inventory, the order point s_i is the amount of products dedicated to meet the demand during the inventory lead time. In this case the lead time is the delay time of an order in a queue Tq plus the pitch P (which is the lot production time). Then, the set of constraints (6) require that the order points s_i are such that the service level K are met:

$$P\left(\left(T_q + P\right) d_i \le s_i \right) \ge K \qquad \forall \ i = 1 \ldots N \qquad (6)$$

g) Variables domain:

The constraints (7) establish the domain of model variables:

$$Q_i > 0; \ \ s_i \ge 0; \ \ P > 0; \ \ A > 0; \ \ 1 > \rho \ge 0; \ \ T_q \ge 0 \qquad \forall \, i = 1..N \qquad (7)$$

h) Model overview:

$$Z = min \ \sum_{i=1}^{N} \frac{s_i}{\bar{d}_i} + \sum_{i=1}^{N} \frac{Q_i}{\bar{d}_i} \qquad (1)$$

Subject to:

$$\sum_{I=1}^{N} \left(a_i \ \frac{\bar{d}_i}{Q_i} \right) \ < \ 1 \ - \ \sum_{i=1}^{N} \left(\bar{d}_i \ o_i \right) \qquad (2)$$

$$(Q_i \ o_i) \ + \ a_i \ = \ P \qquad \forall \ i = 1 \ldots N \qquad (3)$$

$$A = \sum_{i=1}^{N} \frac{\bar{d}_i}{Q_i} \qquad (4)$$

$$\rho = A \ P \qquad (5)$$

$$P\left(\left(T_q + P\right) d_i \le s_i \right) \ge K \qquad \forall \ i = 1 \ldots N \qquad (6)$$

$$Q_i > 0; \ \ s_i \ge 0; \ \ P > 0; \ \ A > 0; \ \ 1 > \rho \ge 0; \ \ T_q \ge 0 \qquad \forall \, i = 1..N \qquad (7)$$

5. Resolution of the model

In the chart below, this section presents the structure of an algorithm which solves the model described in the previous section, determining near-optimal values for the pitch, the lot sizes and order points.

It is important to notice that an exact analytical resolution of the model is difficult due to the fact that it is a non-linear one and, at the same time, contains a large-sized semi-markov stochastic process represented by equations (6). Given that, the algorithm operates on the convexity of the model, iteratively combining two procedures:

a) A method of successive approximations which chooses values progressively better for the pitch and, consequently, any lot size.
b) A discrete stochastic simulation routine to find the lowest order points that are sufficient to meet the service level *K*.

Structure of the Algorithm to Solve the Problem

(1) Make $Q_i = 1, \ \forall\, i = 1..N$ in equations (3) and assume the highest value of *P* as its initial value.
(2) Repeat it until you find a pitch *P* that is viable (it will be its lower limit):
  (2.1) Calculate the lot sizes *Qi* using equations (3).
  (2.2) Make sure the pitch *P* is feasible using equation (2).
  (2.3) If it is not feasible, increase the value of pitch *P*.
(3) While it is possible to improve *Z*, do the following:
  (3.1) Choose a new pitch *P* which is larger than its lower limit.
  (3.2) Calculate the lot sizes *Qi* with equations (3).
  (3.3) Calculate the number of setups *A* and utilization factor ρ with equations (4) and (5).
  (3.4) Establish provisional values for the order points s_i, such that all match the same time demand.
  (3.5) Simulate the operation of the system, collecting a sample of the demand during inventory lead time of each product *i*.
  (3.6) Using the sample taken in the previous step, find the order points s_i such that they satisfy equations (6) and are the smallest possible ones.
  (3.7) Calculate *Z* using equation (1) and store its lowest value so far.

On step 2.3, the pitch increment size may be small because, as the loop on step 2 is very fast, there will be no performance problem.

On step 3.1, as the model is convex, the values for *P* should be chosen according to a strategy to minimize non-linear functions of a single variable, based on successive approximations.

It is worth noting that, at each iteration of the algorithm, when execution reaches step 3.4, all variables are already determined except the order points s_i. On step 3.4, in order to enable simulation of the next step, provisional values are set to order points s_i. These are equivalent to the same arbitrary demand time. During the simulation of line 3.5, a sample of the demand is taken during the inventory lead time of each product *i*. From this sample, the order points s_i are finally determined on step 3.6 so that they meet the service level *K* (equation 6), and are as small as possible.

6. Computational experiments

Table 1 below shows three instances derived from Bomberger's proposal (Bomberger, 1966), which is one of the most used in the literature. They differ only in the values of the demands that multiply Bomberger's 'basis demand' by 2, 3 and 4, respectively. The work day has 480 minutes and the service level required by the market is 90%.

Each of these instances has been solved experimentally twice, using the method proposed in this article. For this, an algorithm based on the structure described in the previous section was coded in Pascal and compiled with Delphi 2007. In the resolution it was used a machine with an Intel Core i5 processor of 2.53 GHz, 4 GB RAM and Windows 7 operating system.

Table 1: Three instances varying only the demand rate

Products	Operation time (min./piece)	Setup time (min)	Demand 1 (pieces/day)	Demand 2 (pieces/day)	Demand 3 (pieces/day)
1	1,60	60	2	3	4
2	6,00	60	2	3	4
3	5,05	120	4	6	8
4	6,40	60	8	12	16
5	24,00	240	0,4	0,6	0,8
6	8,00	120	0,4	0,6	0,8
7	20,00	480	0,12	0,18	0,24
8	36,92	240	1,7	2,55	3,4
9	24,00	360	1,7	2,55	3,4
10	3,20	60	2	3	4

In each simulation performed by the algorithm (step 3.5 on the structure of the algorithm described in the previous section), at least 5,000 demand during lead time samples were collected for determining the order points.

The results are shown in Tables 2, 3, 4 and 5 below. Besides the parameters of the items (lot size and order point), tables 3, 4 and 5 also show indicators from simulated use of the solution in the 'outcomes' columns. We observed the service levels of each product from at least 20,000 demand during lead time samples.

Table 2: Indicators of Solutions Obtained in Two Runs for each Instance

Instance	Execution time (sec.)	Production pitch P (min.)	Maximum stock Z (days)	Operation time (%)	Setup time (%)	Slack (%)
1	28	508 / 507	520 / 520	44,1	46,3 / 47,0	9,6 / 8,9
2	34	692 / 691	569 / 569	66,2	31,2 / 31,3	2,6 / 2,5
3	78	1834 / 1834	1425 /1424	88,2	10,8 /10,8	1,0 / 1,0

Table 3: Solutions Obtained in Two Runs for Instance 1

Products	Solution				Outcomes	
	Lot size - Q_i (pieces)	Order point – s_i (pieces)	Lot Consumption Time (days)	Order point in Days (days)	Service Level (%)	Demand Served (%)
1	280 / 279	13 / 14	140 / 140	7 / 7	94 / 94	99,9 / 99,9
2	75 / 74	13 / 13	37 / 37	7 / 7	96 / 95	99,8 / 99,8
3	77 / 77	24 / 26	19 / 19	6 / 7	96 / 96	99,8 / 99,7
4	70 / 70	47 / 49	9 / 9	6 / 6	97 / 96	99,5 / 99,4
5	11 / 11	3 / 3	28 /28	8 / 8	91 / 91	99,0 / 98,9
6	48 / 48	3 / 3	121 / 121	8 /8	90 / 89	99,7 / 99,7
7	1 / 1	2 / 2	12 / 11	17 /17	91 / 90	92,5 / 91,9
8	7 / 7	11 / 11	4 / 4	6 / 6	96 / 95	98,6 / 98,3
9	6 / 6	11 / 11	4 / 4	6 / 6	95 / 95	98,1 / 97,7
10	140 / 140	13 / 14	70 / 70	7 / 7	95 / 94	99,9 / 99,9

Table 4: Solutions Obtained in Two Runs for Instance 2

Products	Solution				Outcomes	
	Lot Size - Q_i (pieces)	Order Point – s_i (pieces)	Lot Consumption Time (days)	Order Point in Days (days)	Service Level (%)	Demand Served (%)
1	395 / 394	26 / 26	132 / 131	9 / 9	89 / 90	99,8 / 99,9
2	105 / 105	26 / 25	35 / 35	9 / 8	90 / 91	99,5 / 99,5
3	113 / 113	49 / 49	19 / 19	8 / 8	90 / 91	99,1 / 99,2
4	99 / 99	95 / 97	8 / 8	8 / 8	90 / 91	98,1 / 98,3
5	19 / 19	6 / 6	31 / 31	10 / 10	86 / 88	98,7 / 98,9
6	71 / 71	6 / 6	119 / 119	10 / 10	86 / 87	99,7 / 99,7
7	11 / 11	2 / 2	59 / 59	11 / 11	82 / 83	98,0 / 98,1
8	12 / 12	21 / 22	5 / 5	8 / 9	90 / 91	96,2 / 96,6
9	14 / 14	21 / 22	5 / 5	8 / 9	89 / 90	96,4 / 96,7
10	197 / 197	26 / 26	66 / 66	9 / 9	89 / 90	99,7 / 99,7

Table 5: Solutions Obtained in Two Runs for Instance 3

Products	Solution				Outcomes	
	Lot size - Q_i (pieces)	Order Point – s_i (pieces)	Lot Consumption Time (days)	Order Point in Days (days)	Service Level (%)	Demand Served (%)
1	1109 / 1109	82 / 83	277 / 277	21/ 21	91 / 91	99,9 / 99,9
2	296 / 296	83 / 82	74 / 74	21 / 21	92 / 91	99,7 / 99,6
3	339 / 339	164 / 163	42 / 42	21 / 20	92 / 91	99,5 / 99,4
4	277 / 277	327 / 326	17 / 17	20 / 20	93 / 91	98,9 / 98,6
5	66 / 66	17 / 17	83 / 83	21 / 21	90 / 89	99,6 / 99,5
6	214 / 214	17 / 17	268 / 268	21 / 21	90 / 89	99,9 / 99,9
7	68 / 68	6 / 6	282 / 282	25 / 25	86 / 85	99,7 / 99,6
8	43 / 43	71 / 69	13 / 13	21 / 20	92 / 91	98,3 / 98,0
9	61 / 61	71 / 70	18 / 18	21 / 21	92 / 91	98,7 / 98,5
10	554 / 554	81 / 83	139 / 139	20 / 21	92 / 91	99,8 / 99,8

In tables 2, 3, 4 and 5 above, it can be noticed how the difference of each pair of values obtained as a solution to the problem instances (pitch, lot size and order point) is minimized. This fact shows that there is stability in the method developed.

On the other hand, the service levels shown in Tables 3, 4 and 5, obtained as output indicators of simulated use of the method developed, show good approximation to the service level of 90% required in advance.

7. Conclusions and developments

The computational experiments showed that the implemented algorithm is fast and provides stable results. It can often be used with real world problems, on computers in current use. These results could be achieved due to the high convergence rate presented by the stochastic simulation step performed at each iteration of the algorithm.

As a result, it can be said that the method proposed here can provide near-optimal solutions to the studied SELSP problem, delimited by the control rules laid down in section 3 of this article.

Since the proposed rules are simple and flexible, the algorithm has great potential for real use, while providing good results for the proposed the SELSP problem.

With some changes, the approach adopted also seems to be able to solve less restricted problems, where the parameters of cost and service levels can be differentiated for each product.

The implementation of the algorithm needs to be further enhanced to provide more accurate and reliable results and more efficient execution. In this regard, two important improvements are required:

a) A more accurate determination of sample sizes and confidence intervals for the stochastic simulation that is performed at each iteration;
b) The use of a more effective convergence method in controlling the iterations.

Another relevant work would be to compare the solution with different solutions that employ distinct production strategies. The comparison with totally dynamic near-optimal solutions would be particularly interesting, so that the extra costs imposed by control rules proposed in this work could be measured.

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