FUZZY TODIM FOR GROUP DECISION MAKING

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RESUMO

Muitos métodos de tomada de decisão multicritério (do inglês, *multi-criteria decision making*, abreviada por MCDM) têm sido propostos para lidar com problemas de tomada de decisão incertos. A maioria deles tem como base números nebulosos e não são capazes de lidar com risco no processo de tomada de decisão. Nos últimos anos, alguns métodos MCDM baseados na teoria da propensão para lidar com problemas MCDM têm sido desenvolvidos. Neste artigo, nós estendemos o *Fuzzy* TODIM para tomada de decisão em grupo, para que seja possível abordar o problema que envolve um grupo de tomadores de decisão. Um estudo de caso envolvendo derramamento de óleo no mar ilustra a aplicação do novo método. Os resultados mostram a viabilidade do método.

Palavras chave: Tomadores de decisão multicritério, tomada de decisão em grupo, lógica nebulosa

ADM – Apoio à Decisão Multicritério

ABSTRACT

Many multi-criteria decision making (MCDM) methods have been proposed to handle uncertain decision making problems. Most of them are based on fuzzy numbers and they are not able to cope with risk in decision making. In recent years, some MCDM methods based on prospect theory to handle risk MCDM problems have been developed. In this paper, we propose the Fuzzy TODIM for group decision making, so it is possible to tackle a problem that involves a group of decision makers. A case study involving oil spill in the sea illustrates the application of the novel method. The results show the feasibility of the fuzzy TODIM framework.

Keywords: Multi-criteria decision making (MCDM), group decision-making, fuzzy logic

MCDM – Multi-criteria Decision Making

1. Introduction

Complex decision processes may be considered difficult to solve most due to the involved uncertainties, associated risks and inherent complexities of multi-criteria decision making (MCDM) problems (Fenton & Wang, 2006). One of these techniques, proposed by Gomes & Lima (1992), is known as TODIM (an acronym in Portuguese for Iterative Multi-criteria Decision Making). The TODIM method has been applied to rental evaluation of residential properties (Gomes & Rangel, 2009) among others applications with good performance.

It is difficult to treat uncertain data and human opinions using conventional multi-criteria analysis. This motivated the search for new techniques for decision support that are able to handle uncertainties in an effective manner. The theory of fuzzy sets and fuzzy logic developed by Zadeh (1965) has demonstrated suitable to model uncertainty or lack of knowledge when applied to a variety of problems in science and engineering.

The process of building a model for multiple criteria decision making consists of alternatives and criteria, which forms the decision matrix. For real world-problems the decision matrix is affected by uncertainty and may be modeled using fuzzy number. A fuzzy number (Dubois & Prade, 1980) can be seen as an extension of an interval with varied grade of membership. This means that each value in the interval has associated a real number that indicates its compatibility with the vague statement associated with a fuzzy number. Fuzzy numbers have their own rules of operation. In the last decades many MCDM methods using fuzzy logic to describe uncertain data have been developed (Zimmermann 1991).

The objective of this work is to develop a tool to aid a group of decision makers to find the best alternative given the preference of each group member over the criteria and the importance weights assigned to each of the decision makers. The rest of this article is organized as follows: in section 2 we develop the fuzzy TODIM for group decision making to deal with preference of the decision makers. In section 3, simulation results are shown in order to illustrate the feasibility of the approach. In section 4, conclusions are given.

2. The Proposed Method – Fuzzy TODIM for group decision making

The group decision-making framework proposed by Zhang & Lu (2003) integrates the following properties: decision makers may have different weights; decision makers can express fuzzy preferences for alternative solution; decision makers can give different judgments on selection criteria; and to each group member (decision maker) is assigned a weighting. The final group decision is made through aggregating group members' preferences on alternative under their weights and judgments on selection criteria. The majority of group decision making methods use utility aggregation to derive a consensus preference. In a previous work, it was developed a Fuzzy TOPSIS for group decision making (Krohling & Campanharo, 2011). Based on that work, here we extend the recently developed Fuzzy TODIM (Krohling & de Souza, 2012) for group decision making. So, it is possible to take into account the preferences of the decision makers over the fuzzy matrices.

For the group decision making, we have a group of decision makers (members). So, a group G consists of L members (DM) that participate in the decision-making process as given by $G = \{M_1, M_2, \dots, M_L\}$. As we have a group of L decision makers, the weight vector with respect to each group member is described by $W^l = \left(w_{1,}^l w_2^l \dots, w_n^l\right)$ with $l = 1, 2, \dots, L$ where each w_j^l represents $\sum_{j=1}^{n} w_j^l = 1$ by the group member M_l, which satisfies $0 \leq w_l^j \leq 1$. We assume also that each group member (DM) has a degree of importance described by $0 \leq \propto_l \leq 1$, $\sum_{l=1}^{L} \propto_l = 1$.

The fuzzy TODIM method is applied to the fuzzy decision matrix with the assigned weights for each of the decision makers. The results are then aggregated to create a new decision matrix, with the results from the previous method. The TODIM method is then applied over the aggregated decision matrix with the assigned importance weights to each of the decision makers. We now have the ranking of each alternative through the final normalized values obtained from the application of the TODIM method. The proposed method is illustrated in Fig. 1.

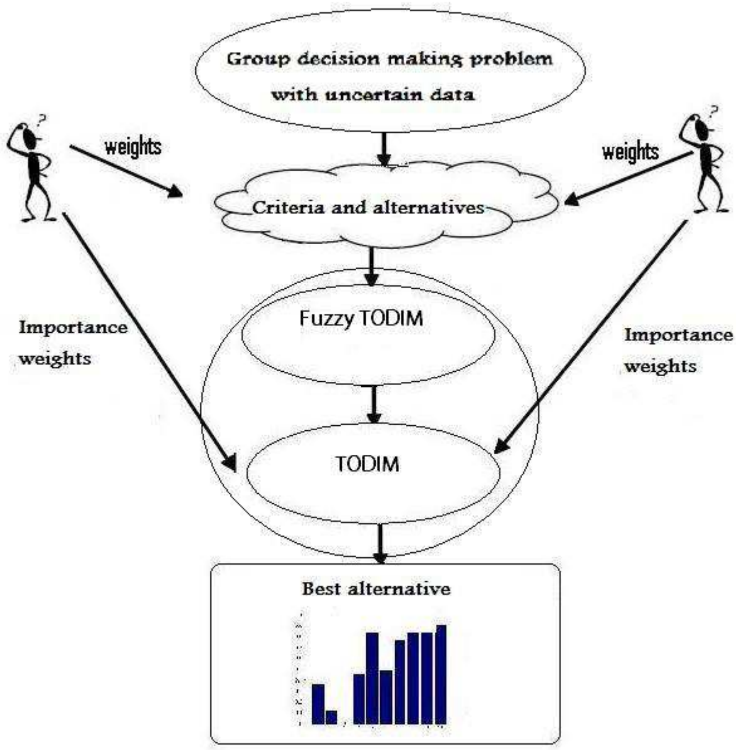

Figure 1. Illustration of the Fuzzy TODIM Method for Group Decision Making.

The steps to calculate the best alternatives are described in the following:

Step 1: The criteria are normally classified into two types: *benefit* and *cost*. The fuzzy-decision matrix $\tilde{A} = \left[\tilde{x}_{ij}\right]_{mxn}$ with $i = 1,...,m$, and $j = 1,...,n$ is normalized which results the correspondent fuzzy-decision matrices $\tilde{R} = \left[\tilde{r}_{ij}\right]_{mxn}$.

$$r_{ij}^{k} = \frac{\max(a_{ij}^{4}) - a_{ij}^{k}}{\max_i(a_{ij}^{4}) - \min_i a_{ij}^{1}} \quad \text{with } k\text{=1,2,3,4} \qquad \text{for cost criteria}$$

$$r_{ij}^{k} = \frac{a_{ij}^{k} - \min(a_{ij}^{1})}{\max_i(a_{ij}^{4}) - \min_i a_{ij}^{1}} \quad \text{with } k\text{=1,2,3,4} \qquad \text{for benefit criteria} \tag{1}$$

Step 2: Calculate the dominance of each alternative $\tilde{A}_i$ over each alternative $\tilde{A}_j$ of the group members' decision matrixes $l = 1,...,L$ using the following expression:

$$\delta_l(\tilde{A}_i, \tilde{A}_j) = \sum_{c=1}^{m} {}^{l}\phi_c(\tilde{A}_i, \tilde{A}_j) \qquad \forall (i, j) \tag{2}$$

where

$${}^{l}\phi_c(\tilde{A}_i, \tilde{A}_j) = \begin{cases} \sqrt{\dfrac{{}^{l}w_{rc}}{\sum_{c=1}^{m} {}^{l}w_{rc}}} \cdot d(\tilde{x}_{ic}, \tilde{x}_{jc}) & \text{if } [m(\tilde{x}_{ic}) - m(\tilde{x}_{jc})] > 0 \\ 0, & \text{if } [m(\tilde{x}_{ic}) - m(\tilde{x}_{jc})] = 0 \\ \dfrac{-1}{\theta} \sqrt{\dfrac{\left(\sum_{c=1}^{m} {}^{l}w_{rc}\right)}{{}^{l}w_{rc}}} \cdot d(\tilde{x}_{ic}, \tilde{x}_{jc}) & \text{if } [m(\tilde{x}_{ic}) - m(\tilde{x}_{jc})] < 0 \end{cases} \tag{3}$$

The term ${}^{l}\phi_c(\tilde{A}_i, \tilde{A}_j)$ represents the contribution of the criterion c to the function $\delta_l(\tilde{A}_i, \tilde{A}_j)$ when comparing the alternative i with alternative j. The parameter θ represents the attenuation factor of the losses, which can be tuned according to the problem at hand. In equation (3) $m_{(\tilde{x}_{ic})}$ and $m_{(\tilde{x}_{jc})}$ stands for the defuzzified values (Wang & Lee, 2009) of the fuzzy number $\tilde{x}_{ic}$ and $\tilde{x}_{jc}$, respectively. The term $d(\tilde{x}_{ic}, \tilde{x}_{jc})$ designates the distance between the two fuzzy numbers $\tilde{x}_{ic}$ and $\tilde{x}_{jc}$, as defined in Mahdavi *et al.*, 2008. Three cases can occur in Equation (3): i) if the value $m_{(\tilde{x}_{ic})} - m_{(\tilde{x}_{jc})}$ is positive, it represents a gain; ii) if the value $m_{(\tilde{x}_{ic})} - m_{(\tilde{x}_{jc})}$ is nil; and iii) if the value $m_{(\tilde{x}_{ic})} - m_{(\tilde{x}_{jc})}$ is negative, it represents a loss.

Step 3: Calculate the global value of the alternative i by means of normalizing the final matrix of dominance according to the following expression:

$${}^{l}\xi_i = \frac{\sum \delta_l(i, j) - \min \sum \delta_l(i, j)}{\max \sum \delta_l(i, j) - \min \sum \delta_l(i, j)} \tag{4}$$

The final matrix of dominance for each group member is then aggregated to form a new crisp decision matrix as given by:

$$C = \begin{pmatrix} {}^{1}\xi(A_1) & \cdots & {}^{L}\xi(A_1) \\ \vdots & \ddots & \vdots \\ {}^{1}\xi(A_m) & \cdots & {}^{L}\xi(A_m) \end{pmatrix} \tag{5}$$

From this stage on our method continues by applying the standard TODIM method to the decision matrix in equation 5 in order to identify the matrix of dominance and its alternatives' ranking of the alternatives.

Step 4: For the decision matrix C, we now have associated an importance weight to each group member α_l, for $l = 1,...,L$. Within the decision matrix C, we calculate the dominance of each alternative A_i over each alternative A_j using the following expression:

$$\delta_G(A_i, A_j) = \sum_{c=1}^{L} \phi_l(A_i, A_j) \qquad \forall (i, j) \tag{6}$$

where

$$\phi_l(A_i, A_j) = \begin{cases} \sqrt{\dfrac{\alpha_l (x_{ic} - x_{jc})}{\sum_{c=1}^{L} \alpha_l}} & \text{if } (x_{ic} - x_{jc}) > 0 \\ 0 & \text{if } (x_{ic} - x_{jc}) = 0 \\ \dfrac{-1}{\theta} \sqrt{\dfrac{\left(\sum_{c=1}^{m} \alpha_l\right)(x_{jc} - x_{ic})}{\alpha_l}} & \text{if } (x_{ic} - x_{jc}) < 0 \end{cases} \tag{7}$$

The term $\phi_l(A_i, A_j)$ represents the contribution of the group member l to the function $\delta_G(A_i, A_j)$ when comparing the alternative i with alternative j. The parameter θ represents the attenuation factor of the losses, which can be tuned according to the problem at hand. In expression 3) it can occur 3 cases: i) if the value $(x_{ic} - x_{jc})$ is positive, it represents a gain; ii) if the value $(x_{ic} - x_{jc})$ is nil; and iii) if the value $(x_{ic} - x_{jc})$ is negative, it represent a loss. The final matrix of dominance is obtained by summing up the partial matrices of dominance for each group member.

Step 5: Calculate the global value of the alternative i by normalizing the final matrix of dominance according to the following expression:

$$\xi_G = \frac{\sum \delta_G(i, j) - \min \sum \delta_G(i, j)}{\max \sum \delta_G(i, j) - \min \sum \delta_G(i, j)} \tag{8}$$

Ordering the values ξ_G provides the rank of each alternative. The best alternatives are those that have higher value ξ_G.

3. Experimental Results

Case study – Decision making in case oil spill in the sea

In this study, different combat strategies based on an accident with oil spill in the sea are simulated. This way, we can build various scenarios of responses, which can be selected according to criteria such as oil that reaches the coast or oil collected. The rating of the alternatives in terms of these criteria contributes to form the decision matrix. Through simulation results, the consequences of using different combat strategies for each specified criterion can be evaluated. Thus, the type of impact is necessary to provide means to assess the consequences of a decision in each possible scenario. Our focus here is the development of a fuzzy TODIM for group decision making and its application to a relevant problem in crisis management in order to help to select the best combat alternatives.

The decision matrix *A* in Table 1 is composed of 10 alternatives and 2 criteria. According to our notation, the criteria C_1 *oil at the coast* (OC) is a *cost* criterion and the criterion C_2 *oil intercepted* (OI) is a *benefit* criterion. In this study, simulations data as given in Table 1 are affected by uncertainty because the simulation of oil spots depends on several factors such as quantity and type of oil spilled, location of spill, weather and ocean conditions, among others. This issue is especially difficult to be treated due to the dynamic nature of the marine environment concerning variables that change over time. For a detailed description of how the data have been obtained the reader is referred to Krohling & Campanharo (2011).

Table 1. Decision matrix for the oil spill case.

Alternatives	*Oil at the coast* (OC) m^3 (x 10^3)	*Oil intercepted* (OI) m^3 (x 10^3)
A1	8.627	5.223
A2	9.838	4.023
A3	10.374	3.495
A4	8.200	5.659
A5	5.854	7.989
A6	8.108	5.790
A7	6.845	7.083
A8	5.738	8.238
A9	5.858	8.189
A10	6.269	7.808

To the original decision matrix listed in Table 1 was introduced -10%, -5%, +5%, +10% uncertainty to build up a_1, a_2, a_3, a_4, respectively in form of trapezoidal fuzzy number according to: $a_1 = m - 0.1m,\ a_2 = m - 0.05m,\ a_3 = m + 0.05m,\ a_4 = m + 0.1m,$
where *m* stands for the mean graded (the original crisp value in the Table 1) of the trapezoidal fuzzy number. The fuzzy decision matrix is shown in Table 2.

In the process of decision making for management of oil spill responses, it is evident that for each criterion as OC, and OI, the perspective of the decision makers (Environmental Agency, NGO and Oil Company) is not given the same importance. Therefore, a weight vector *W* is introduced to denote the weight for the criterion based on the preferences of each decision maker. Three levels of importance weight are assigned for each criterion: *very important, moderate and unimportant*.

For the labels *unimportant, moderate and important* are assigned the weights 0.05, 0.5 and 0.95, respectively. The preference of each decision maker is described in Table 3.

Table 2. Fuzzy decision matrix.

Alternatives	Criteria	
	Oil at the coast	Oil intercepted
A1	[7.76, 8.20, 9.06, 9.49]	[4.70, 4.96, 5.48, 5.75]
A2	[8.85, 9.35, 10.33, 10.82]	[3.62, 3.82, 4.22, 4.43]
A3	[9.34, 9.86, 10.89, 11.41]	[3.15, 3.32, 3.67, 3.84]
A4	[7.38, 7.79, 8.61, 9.02]	[5.09, 5.38, 5.94, 6.22]
A5	[5.27, 5.56, 6.15, 6.44]	[7.19, 7.59, 8.39, 8.79]
A6	[7.30, 7.70, 8.51, 8.92]	[5.21, 5.50, 6.08, 6.37]
A7	[6.16, 6.50, 7.19, 7.53]	[6.37, 6.73, 7.44, 7.79]
A8	[5.16, 5.45, 6.02, 6.31]	[7.41, 7.83, 8.65, 9.06]
A9	[5.27, 5.57, 6.15, 6.44]	[7.37, 7.78, 8.60, 9.01]
A10	[5.64, 5.96, 6.58, 6.90]	[7.03, 7.42, 8.20, 8.59]

Table 3. Preference of each decision maker over criteria (Krohling & Campanharo, 2011).

Decision Makers (DM)	Oil at the coast	Oil intercepted
DM1: Enviromental Agency	*Moderate*	*Moderate*
DM2: Oil Company	*Unimportant*	*Very Important*
DM3: NGO	*Very Important*	*Unimportant*

The fuzzy decision matrix shown in Table 2 is now normalized (Krohling & de Souza, 2012) and then the method F-TODIM is applied to the normalized fuzzy decision matrix, with different weights, according to the preference of each decision maker over the criteria(see Table 3). The results are now aggregated to form the matrix shown in Table 4.

Table 4. Results obtained by the application of the Fuzzy TODIM providing the aggregated final matrix of dominance.

	${}^{1}\xi_i$	${}^{2}\xi_i$	${}^{3}\xi_i$
A_1	0.3430	0.3690	0.3651
A_2	0.1089	0.1199	0.1188
A_3	0	0	0
A_4	0.4489	0.4801	0.4752
A_5	0.9495	0.9631	0.9446
A_6	0.4812	0.5114	0.5105
A_7	0.7177	0.7349	0.7323
A_8	1	1	1
A_9	0.9755	0.9751	0.9800
A_{10}	0.8645	0.8671	0.8771

Next, the TODIM method is applied to the aggregated decision matrix given in table 4, with different importance weights, as shown in Table 5.

Table 5. Results obtained by the application of TODIM with importance weights $\propto_1 = (0.33, 0.33, 0.33), \propto_2 = (0.1, 0.8, 0.1), \propto_3 = (0.4, 0.2, 0.4)$.

	α_1	α_2	α_3	ranking
A_1	0.3441	0.3550	0.3460	8
A_2	0.1186	0.1228	0.1193	9
A_3	0	0	0	10
A_4	0.4671	0.4801	0.4695	7
A_5	0.9343	0.9371	0.9355	3
A_6	0.5127	0.5255	0.5149	6
A_7	0.7074	0.7161	0.7088	5
A_8	1	1	1	**1**
A_9	0.9700	0.9727	0.9704	2
A_{10}	0.8416	0.8491	0.8422	4

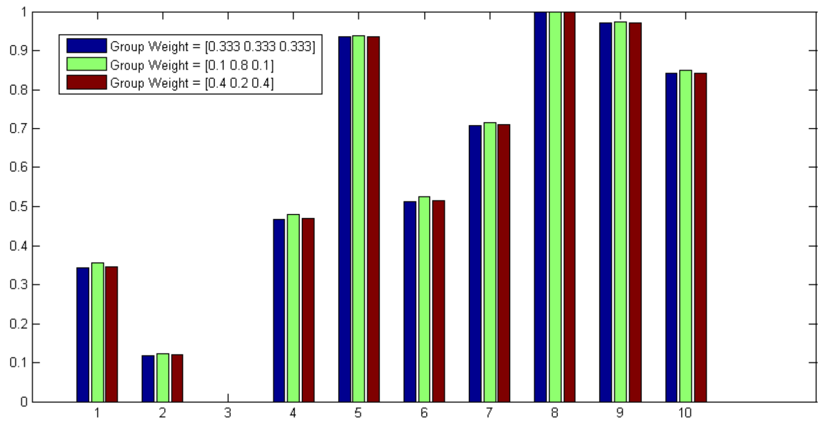

Figure 2. Plot showing the ranking of the alternatives for three different importance weights.

The final ranking obtained is in agreement with that obtained by the Fuzzy TOPSIS for group decision making (Krohling & Campanharo, 2011). According to the results, the best alternative is Alternative 8 for the three importance weights. However, the uncertainty of the decision matrix may affect the final ordering of the alternatives (Krohling & de Souza, 2012).

The method can be applied to other MCDM problems with a finite number of alternatives, criteria and decision makers, on which the change of the importance weight might imply on different ranking of the alternatives.

4. Conclusions

In this paper, based on previous work by Krohling & Campanharo (2011), which is based on Fuzzy TOPSIS for group decision making, we develop a Fuzzy TODIM for group decision making. This approach takes into account the uncertainty of the decision matrices, risk behavior and the preference of the decision makers to find the best alternative in a multi-criteria decision making problem.

In this study we applied the method for a case study involving an accident in the oil field of Jubarte, in the south coast of Espírito Santo state, Brazil. The results indicated the effectiveness of the proposed method in dealing with uncertain problems that involve several decision makers with different preferences. The method is currently being expanded to other applications.

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