PORTFOLIO ROBUSTNESS EVALUATION: AN APPLICATION IN THE ELECTRICITY SECTOR

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ABSTRACT

Managers continually face the task of allocating resources to projects when there is not enough money to fund them all. Portfolio Robustness Evaluation (PROBE) is a multicriteria decision support system developed to help managers to perform that difficult task. This paper presents a case study developed at EDP Distribuição (EDPD), the main electricity distribution company in Portugal, in which the selected portfolio of projects must comply with budget constraints concerning different types of projects and organizational units in multiple time periods. The paper discusses the shortcomings of the selection procedure used by EDPD and the advantages of adopting PROBE. The robustness of selecting a portfolio when the net present value (NPV) of the projects varies within given uncertainty ranges is analyzed. It is also shown the extent to which the introduction of an environmental impact criterion would affect the stability of a portfolio previously selected taken only the NPV criterion into consideration.

KEYWORDS. Capital budgeting. Portfolio robustness evaluation. Resource Allocation.

1. Introduction

The issue of portfolio selection can be conceived in an extraordinarily broad scope. In this paper, the focus is directed to real assets and to capital budgeting, more precisely, to the efficient allocation of financial resources to indivisible investment projects.

The use of linear programming in capital budgeting was suggested since the mid-1950s—Gunther (1955), Lorie and Savage (1955), Markowitz and Manne (1957) and Asher (1962). In face of a number of investment projects that exceed their available resources, companies and organizations, in general, aim at finding the subset of projects (the portfolio) that maximizes the created value. Assuming that projects are indivisible, the problem leads to a binary mathematical programming formulation subject to a budget constraint (the knapsack problem—Kleinmuntz 2007), which can be extended with several other types of linear constraints. Interdependencies among projects may also lead to non-linear objective functions (Dickinson et al. 2001) and the formulation can still accommodate multiple objectives (see, e.g., Ewing et al. 2006, Golabi 1987, Ringuest and Graves 1990, Stummer and Heidenberger 2003).

As an alternative to the optimization approach, the selection of multiple projects subject to the scarcity of resources may also be made by means of the prioritization approach. Projects are prioritized by their benefit-to-cost ratios until the available budget is exhausted (Buede and Bresnick 2007, Edwards 1977, Phillips and Bana e Costa 2007, Sharpe and Keelin 1998). This approach is appealing in the context of strategic decision-aiding processes, as it permits a straightforward interaction with the decision-makers; however, it is not so suitable to handle programming and multi-period constraints. This is the case of the portfolio decision analysis presented in this paper and developed for the EDPD company (EDP Distribuição - Energia, S.A.), the main distributor of electricity in Portugal.

The paper first analyses the heuristic prioritization procedure used by EDPD managers and then describes our approach to the problem using the software PROBE (Portfolio Robustness Evaluation). PROBE enables to consider several types of linear constraints and, given the costs and benefits of the projects, identifies all (convex and non-convex) efficient portfolios by running the algorithms proposed in Lourenço et al. (*forthcoming*). It also permits to analyze the robustness of a chosen portfolio, given uncertainty ranges on the benefits of the projects, searching for (competitor) portfolios that do not increase costs and simultaneously may provide more overall benefits.

Section 2 describes the case and discusses the drawbacks of the EDPD prioritization approach. Section 3 explains, step-by-step, the PROBE approach to the case. Currently, EDPD considers only a benefit criterion (the NPV) in the selection process, but the company intends to add other evaluation criteria in the future, such as the environmental impacts of the projects. Section 4 shows the extent to which the introduction of an environmental impact criterion would affect the stability of the selected portfolio. We conclude the paper in Section 5 with some final remarks.

2. The EDPD case

EDP S.A. is the largest industrial group in Portugal. The group is a leader in the energy sector, spreading its activities across Portugal, Spain, France, Belgium, Poland, Romania, the United States and Brazil. Among EDP S.A. companies, one is devoted to the distribution of electricity in mainland Portugal, EDPD, the grounds for this case study.

Each year, EDPD selects its investment projects for the next budgetary period. The investment phase of these projects can last from one up to three years, and the operating phase can last for thirty years. Typically, there are several hundred candidate projects annually, and for this particular case 409 projects are being considered. Their total cost (76 125.44×10^3 Euros) exceeds by far the amount available for funding new projects (48 552×10^3 Euros), after deducting the commitments with ongoing investments (on projects approved in the previous two years that are still being implemented) and with compulsory investments (see Table 1). EDPD distinguishes large-scale investment projects (e.g. an electric power substation) that should not absorb more than 75% of the annual budget, from small-scale projects (e.g. a low-voltage power line) that can be assigned to different geographical zones in Mainland Portugal (Z1 to Z6 in Table 1).

Table 1. EDPD's budget (in 10^3 Euros).

	Year 1	Year 2	Year 3	Total
(A) Total budget	20 000	20 000	20 000	60 000
For large-scale projects	15 000	15 000	15 000	45 000
For small-scale projects	5 000	5 000	5 000	15 000
(B) Ongoing investments	6 359	4 451	0	10 810
On large-scale projects	6 234	4 451	0	10 685
On small-scale projects	125	0	0	125
(C) Compulsory investment (on small-scale projects)	552	86	0	638
(D) = (A) – (B) – (C) Available for new projects	13 089	15 463	20 000	48 552
For new large-scale projects	8 766	10 549	15 000	34 315
For new small-scale projects	4 323	4 914	5 000	14 237
Generic	2 500	4 914	5 000	12 414
Zone Z1	255			
Zone Z2	296			
Zone Z3	387			
Zone Z4	70			
Zone Z5	281			
Zone Z6	534			

For each project, the company computes the value of the profitability index PV/C (also called benefit-cost ratio), with PV being the project's present value and C its investment cost. After that, projects are sequentially approved by decreasing order of their ratios as long as there are available resources for their specific type, zone, and annual expenses. When resources are exhausted in one of the corresponding "slots" (each slot being defined by the respective type of project, year and zone) the project under consideration is disregarded and the analysis proceeds with the next one in the rank.

This procedure followed by EDPD involves two important aspects that must be analyzed in detail. The first concerns the definition of profitability index. Some authors (e.g. Brealey and Myers 2003, Damodaran 2001) state that it is equivalent to use the PV or the net present value ($NPV = PV – C$) as the numerator of the profitability index. In fact, $(PV – C) / C$ is equivalent to $(PV / C) – 1$. This means that when deciding the approval of a single project one can use as a decision rule $PV / C > 1$ or $NPV / C > 0$. Also, when ranking the projects based on the profitability index (a prioritization approach), the ranking will be similar in both cases, since the difference between the two ratios is merely a constant (–1). However, when using linear programming, the portfolio that maximizes *cumulative PV* may be different from the portfolio that maximizes *cumulative NPV*, for the same budgetary constraint, if they are non-convex efficient (Dantzig 1957). One should also be aware that the EDPD prioritization approach may ignore particular combinations of projects that attain higher benefits (here, corresponding to the sum of the NPVs of all the projects integrating the portfolio) for the available money, because prioritization always ignores non-convex efficient portfolios (as it is shown in Section 3.1).

3. An alternative approach to the EDPD problem

The approach suggested to, and discussed with, EDPD managers follows the alternative path described hereafter. The first suggestion consisted of splitting the problem into two sub-problems, making the analysis first for the large projects and only afterwards for the small ones, given the distinct amounts allocated to the two types of projects and their differences in scope and in magnitude of investment. Similarly to other kinds of resource allocation problems in the presence of projects with significantly different costs (Kleinmuntz and Kleinmuntz 1999), special attention was devoted to the analysis of the large-scale projects, preventing disturbing the analysis with the small-scale projects. So, the further steps of the procedure can be summarized as follows:

1. Computation of the efficient frontier for the large projects, globally for all the three years, disregarding the annual budget constraints that EDPD has in place. The idea in this phase was to inform the EDPD officers about how much the company was losing by imposing rigid budget constraints each year instead of considering a global amount for the three years period.

In fact, as reported by Lorie and Savage (1955): “The imposition of additional restrictions upon the freedom of action of any agency can obviously never increase the value of the best opportunity available to that agency” (p. 233).

2. Computation of the optimal portfolio of large projects with multi-period budget constraints.
3. Robustness analysis of the optimal portfolio of large projects found in step 2. Since these projects represent 75% of the budget, it is important to know to what extent the uncertainty of the PVs of these projects could affect the optimal portfolio.
4. Computation of the optimal portfolio of small-scale projects considering the annual budget constraints and zonal constraints.
5. Final comparison of the integration of combination of the two portfolios computed in steps 2 and 4 with the overall portfolio formed by EDPD.

The following sections describe in more detail each of these five steps.

3.1 Step one: Computation of the efficient frontier for the large-scale projects

The decision support system PROBE allows the computation and display of the efficient frontier for any set of projects, given a range of variation for the budgetary constraint. Looking firstly at Figure 1, which, similarly to the prioritization approach, is restricted to the convex efficient portfolios. The dots on the frontier line correspond to successively enlarged portfolios from the left to the right (respectively, a portfolio with zero projects, a portfolio with one project, P10, the first project in Table 2, and so on, until including all the projects ranked by decreasing order of the *NPV* / *C* ratio).

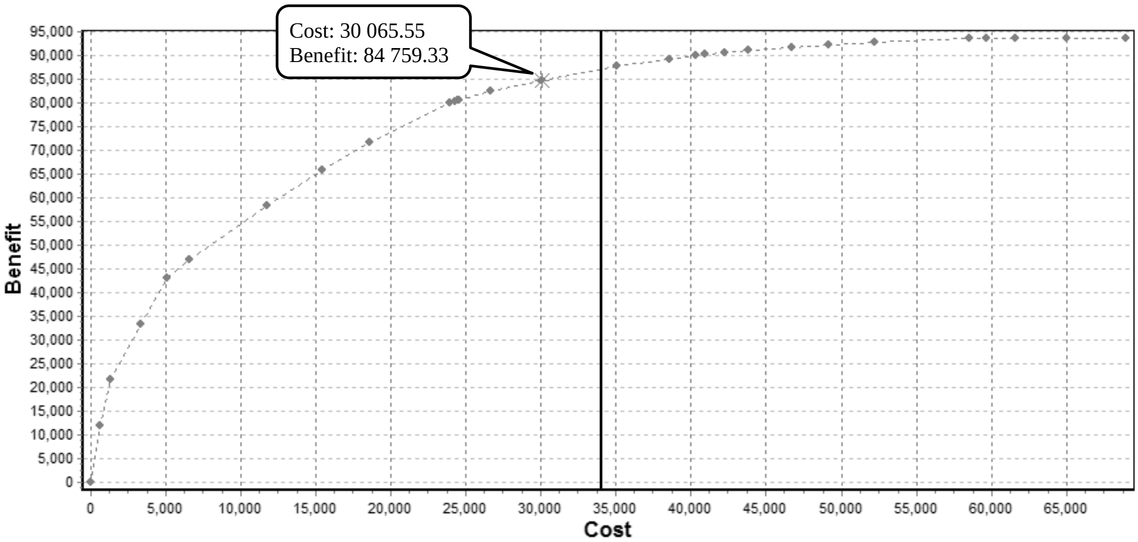

Figure 1. PROBE display of the convex efficient frontier (units in 10^3 Euros).

As it is evident from the analysis of the convex efficient frontier in Figure 2, the best affordable portfolio in these circumstances corresponds to the portfolio marked with a star dot, which costs 30 065.55×10^3 Euros, has an *NPV* of 84 759.33×10^3 Euros and is formed by the first 14 projects in Table 2 (which are also those selected by the prioritization approach). However, this portfolio does not exhaust the available budget reported in the last column of Table 1 (34 315×10^3 Euros). The search for affordable portfolios with higher NPVs, given the budgetary constraint of 34 315×10^3 Euros, implies to analyze all other non-convex efficient portfolios.

The graph in Figure 3 is a display of PROBE that presents all the efficient solutions given by the optimization program (1), by varying the budget (*B*) between zero and the sum of the costs of all large-scale projects (as detailed in Lourenço et al. *forthcoming*). It shows that there are several (non-convex) efficient portfolios between the best affordable convex efficient portfolio and the budget limit.

$$\begin{aligned} &\text{maximize} && \sum_{j=1}^{28} v_j x_j \\ &\text{subject to:} && \sum_{j=1}^{28} c_j x_j \le B \qquad (1) \\ & && x_j \in \{0,1\}, \quad j = 1,\ldots,28\,, \end{aligned}$$

where v_j is the *NPV* of the large-scale project *j*, c_j is the investment cost of project *j*, $x_j = 1$ if project *j* is included in the optimal portfolio and $x_j = 0$ otherwise.

Table 2. Large-scale projects ranked by decreasing benefit-to-cost ratios.

Priority order	Project	*NPV* (10^3 Euros)	Cost (10^3 Euros)	*NPV*/Cost
1	P10	12 016.67	602.38	19.95
2	P16	9 741.32	700.00	13.92
3	P15	11 447.06	2 023.48	5.66
4	P17	9 741.32	1 773.83	5.49
5	P23	3 896.55	1 436.11	2.71
6	P05	11 411.82	5 221.40	2.19
7	P03	7 511.86	3 683.78	2.04
8	P18	5 971.11	3 144.54	1.90
9	P19	8 181.24	5 375.40	1.52
10	P14	411.00	306.82	1.34
11	P11	220.43	174.57	1.26
12	P02	148.97	126.00	1.18
13	P13	1 735.00	2 112.67	0.82
14	P06	2 324.98	3 384.57	0.69
15	P04	2 993.71	4 964.12	0.60
16	P07	1 467.35	3 528.12	0.42
17	P09	732.40	1 788.49	0.41
18	P01	232.61	587.13	0.40
19	P12	487.14	1 318.71	0.37
20	P28	508.86	1 550.00	0.33
21	P08	549.83	2 902.53	0.19
22	P21	466.13	2 482.62	0.19
23	P22	535.50	3 084.23	0.17
24	P27	898.95	6 229.88	0.14
25	P25	1.24	1 199.68	0.00
26	P26	1.28	1 914.76	0.00
27	P24	1.00	3 452.94	0.00
28	P20	0.46	3 905.33	0.00

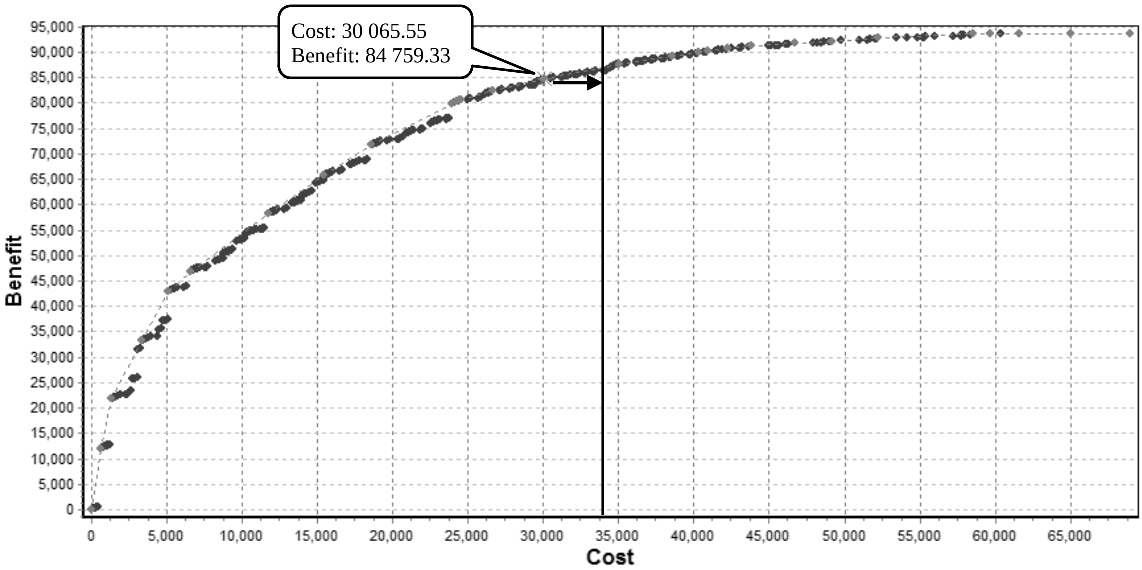

Figure 2. PROBE display of the efficient frontier—the convex efficient portfolios are linked with a dotted line (units in 10^3 Euros).

The magnified graph in Figure 3 allows the visualization of the 22 efficient portfolios that exhibit, without exceeding the budget, higher NPVs than the best affordable convex efficient portfolio previously found. Among them it is possible to identify the optimal solution (B in Figure 3), which includes the following 15 projects, respectively presented by decreasing order of benefit-to-cost ratio: P10, P16, P15, P17, P23, P05, P03, P18, P19, P14, P11, P02, P06, P04 and P12. This portfolio has a *NPV* of 86 505.18×10^3 Euros and a cost of 34 235.71×10^3 Euros, thus it provides an increase of 1 745.85×10^3 Euros in *NPV* and of 4 170.16×10^3 Euros in investment cost in comparison with the best affordable convex efficient portfolio previously found. It is also interesting to report the discussion undertaken with the EDPD officers regarding Figure 3. The head of the Department of Network Planning explained that the graph could help him negotiate with the board of the company an increase in the budgetary limit, since he could argue that with a small amount of money it would be possible to change from the portfolio signaled as B (in Figure 3) to portfolio C (the next convex efficient portfolio), while a change from portfolio A to portfolio C (the two contiguous convex efficient portfolios) would require a much larger amount of money. This discussion is related to the usual separation between hard and soft budgetary constraints (hard and soft capital rationing) which one can find frequently in financial literature (see, e.g., Brealey and Myers 2003, pp. 108-109). Budgetary constraints are considered to be soft if it is possible for a company or a department to expand its budgetary limits in the presence of profitable investment projects, financing their activities through the capital markets or the banking system. However, after the 2008 financial crisis the limits of credit are in several cases very narrow.

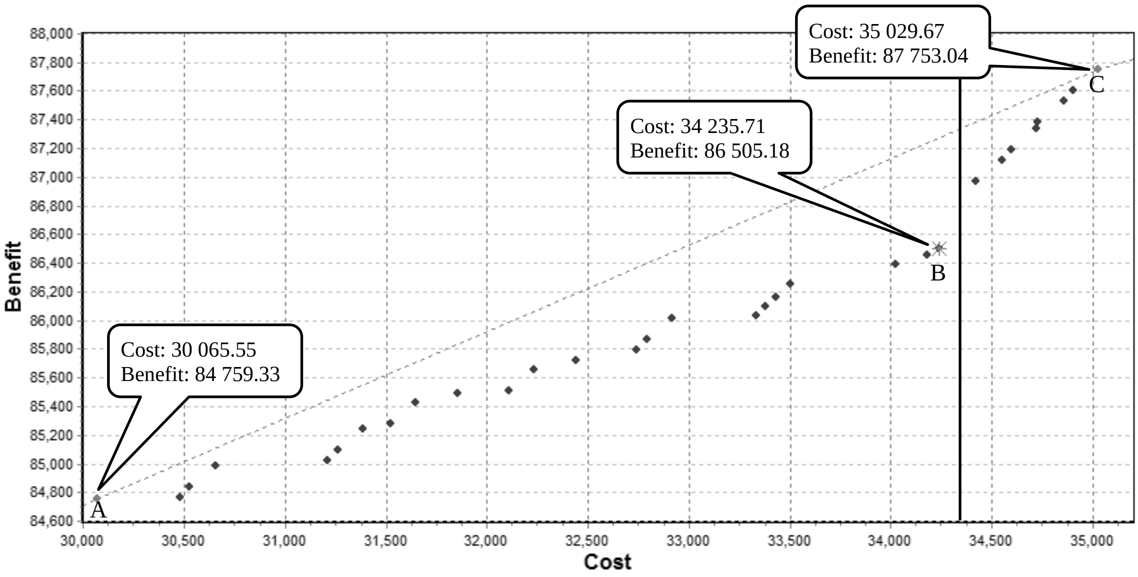

Figure 3. PROBE zoomed graph showing the efficient portfolios between two convex efficient portfolios—the optimal portfolio is signaled with a star dot (units in 10^3 Euros).

3.2 Step two: Computation of the optimal portfolio of large-scale projects with multi-period budget constraints

The optimal portfolio found in the previous step would imply spending more money than was available for each of the first two years of investment—6 985.77×10^3 Euros in the first year and 1 621.7×10^3 Euros in the second year. Therefore, the optimal portfolio found is not a feasible solution for the EDPD problem. The optimization program had to be updated to model (2):

$$\begin{aligned} &\text{maximize} && \sum_{j=1}^{28} v_j x_j \\ &\text{subject to:} && \sum_{j=1}^{28} c_{j,1} x_j \le 8766,\ \sum_{j=1}^{28} c_{j,2} x_j \le 10549,\ \sum_{j=1}^{28} c_{j,3} x_j \le 15000 \qquad (2) \\ &&& x_j \in \{0,1\},\quad j = 1,\dots,28, \end{aligned}$$

where v_j is the *NPV* of the large-scale project *j*, $c_{j,y}$ is the cost of project *j* on year *y* ($y = 1,\ldots, 3$), and $x_j = 1$ if project *j* is included in the optimal portfolio and $x_j = 0$ otherwise.

The optimal feasible portfolio found by PROBE includes the following 11 projects: P10, P16, P15, P17, P23, P05, P03, P18, P19, P01 and P28. Only the first nine of the 15 projects included in the previous portfolio remained (P10, P16, P15, P17, P23, P05, P03, P18, P19), the last six projects were excluded (P14, P11, P02, P06, P04 and P12), and two new projects (P01 and P28) entered the portfolio. The consequence of splitting the overall budget into three annual amounts is that the optimal feasible portfolio presents a 6.8% decrease in *NPV* (5 844.76×10^3 Euros). As understood by the EDPD officers this is the “cost” of introducing rigid multi-period budgetary constraints.

3.3 Step three: Robustness analysis of the optimal portfolio of large-scale projects

The changes in the composition of the optimal portfolio that occurred in the previous step led to a further discussion on how robust would be the chosen portfolio, in face of the uncertainty surrounding the expected cash flows of the projects, namely, the uncertainty of their PVs. In this case, where only one benefit criterion is considered (*NPV*), we may define robustness as follows. Let *p* denote an optimal feasible portfolio of projects, with a benefit $v^p = \sum_{j\in p} v_j$ and an investment cost $c^p = \sum_{j\in p} c_j$. Let *d* denote a portfolio of projects with a benefit $v^d = \sum_{j\in d} v_j$ and an investment cost $c^d = \sum_{j\in d} c_j$. Assume, for instance, that the benefits of *p* and *d* can vary within an uncertainty domain $\Re_v$ respectively given by $\underline{v}^p \le v^p \le \overline{v}^p$ and $\underline{v}^d \le v^d \le \overline{v}^d$, where $\underline{v}^p = \sum_{j\in p} \underline{v}_j$, $\overline{v}^p = \sum_{j\in p} \overline{v}_j$, $\underline{v}^d = \sum_{j\in d} \underline{v}_j$, $\overline{v}^d = \sum_{j\in d} \overline{v}_j$ and $\underline{v}_j \le v_j \le \overline{v}_j$ for all *j*. We say that portfolio *d* is a competitor of portfolio *p* if and only if (i) $\sum_{j\in d\setminus p} \underline{c}_j \le \sum_{j\in p\setminus d} \overline{c}_j$ and (ii) there is a combination of feasible benefit value scores *v* such that $\sum_{j\in p\setminus d} v_{ij} - \sum_{j\in d\setminus p} v_{ij} < 0$. We say that the choice of portfolio *p* is undoubtedly robust when *p* has no competitors. (Portfolio robustness evaluation in problems with multiple benefit criteria require a different definition; see Lourenço et al. *forthcoming* .)

The decision support system PROBE was used to find out the conditions that could lead other portfolios to beat the optimal feasible portfolio and to measure the potential regret caused by keeping this choice under uncertainty. The robustness of the optimal portfolio was tested against simultaneous variations (±α) of the projects’ PVs, for the range [0%, 20%]. It can be seen in Figure 4 that for $\alpha \le 5\%$ no portfolio beats the optimal, with $\alpha = 6\%$ there is only one competitor portfolio, and this number grows to nine competitors when $\alpha = 20\%$. The differences between the composition of the optimal and the competitor portfolios are shown in Table 3.

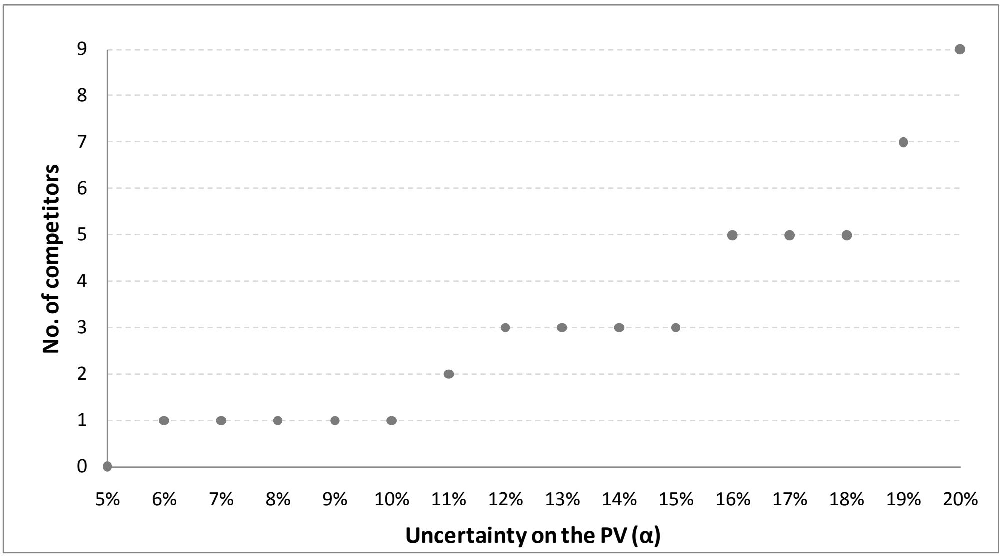

Figure 4. Uncertainty on the *PV* vs. number of competitors.

Table 3. Differences in composition between the optimal and the competitor portfolios.

Detected for the first time when...	Projects exclusive to the optimal portfolio	Projects exclusive to the competitor portfolio
α = 6%	P28	P02, P11
α = 11%	P01, P28	P02, P11
α = 12%	P28	P11
α = 16%	P01, P28	P11
α = 16%	P28	P02
α = 19%	P01, P28	P02
α = 19%	P01, P19, P28	P02, P04, P09
α = 20%	P01, P19, P28	P02, P04, P11, P14
α = 20%	P01, P19, P28	P04, P09

Table 4 shows that most of the projects that are included in the optimal affordable portfolio are not replaced in the competitor portfolios. It can be seen e.g., that 91% of the projects are kept with a variation on the PVs of 10%, and 73% still remain in the portfolio when α = 20%. These figures show a significant stability of the optimal portfolio.

Table 4. Stability analysis of the optimal portfolio.

Uncertainty on the *PV*	No. of stable projects	Percentage of stable projects
$0\% \leq \alpha \leq 5\%$	11	100%
$6\% \leq \alpha \leq 10\%$	10	91%
$11\% \leq \alpha \leq 18\%$	9	82%
$19\% \leq \alpha \leq 20\%$	8	73%

Another relevant aspect of robustness analysis is the regret evaluation. For an optimal portfolio *p* and a competitor portfolio *d*, the maximal regret (i.e. loss) in *NPV* caused by selecting *p* instead of *d* corresponds to the absolute value of $\sum_{j\in p\setminus d} \underline{v}_{ij} - \sum_{j\in d\setminus p} \overline{v}_{ij}$. For *k* competitors (with *k* > 1) the maximal regret of selecting the optimal portfolio *p* corresponds to the maximum of the *k* maximal (pairwise) regrets. Figures 5 and 6 show, for different levels of uncertainty and in absolute and relative terms, the maximal regret in the overall *NPV* of the optimal portfolio. We can see in Figure 6 that for α = 6% the maximal regret in *NPV* represents only 0.03% of the minimum *NPV* of the optimal portfolio, under this range of variation; for α = 20% the maximal regret in *NPV* is 406.3×10^3 Euros, which is only 0.69% of the minimum *NPV* of the optimal portfolio. These small potential losses were considered irrelevant by the decision-makers at EDPD, and confirmed the robustness of the optimal portfolio found in the previous step.

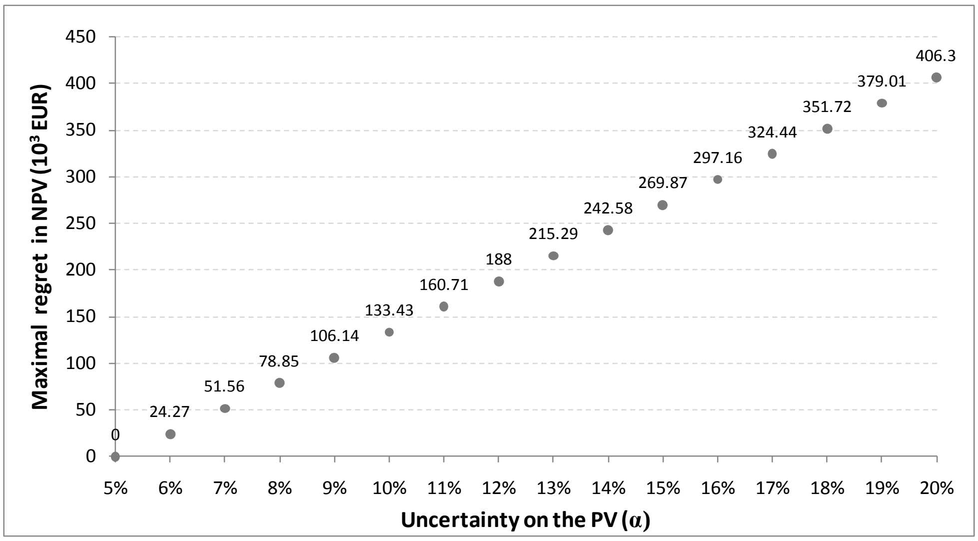

Figure 5. Uncertainty on the *PV* vs. maximal regret in *NPV*.

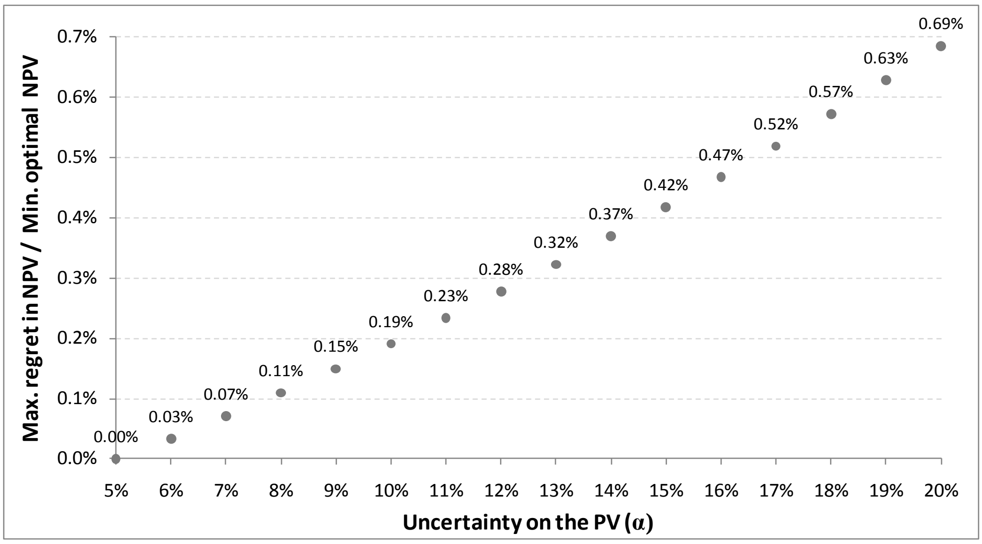

Figure 6. Uncertainty on the *PV* vs. max. regret in *NPV* / min. *NPV* of the optimal portfolio.

3.4 Step four: Computation of the optimal portfolio of small-scale projects

PROBE was used again, this time to find out the optimal portfolio of small-scale projects. Nine constraints were added to the software—three annual budget constraints and six zonal constraints concerning the first year of investment, as respectively shown in program (3). From the 344 projects that were competing for the budget, PROBE selected 265 projects. The optimal affordable portfolio has a *NPV* of 23 554.7×10^3 Euros and requires an investment of 4 637.09×10^3 Euros.

$$\begin{array}{ll}
\text{maximize} & \sum_{j=1}^{344} v_j x_j \\
\text{subject to:} & \sum_{j=1}^{344} c_{j,1} x_j \le 4323,\ \sum_{j=1}^{344} c_{j,2} x_j \le 4914,\ \sum_{j=1}^{344} c_{j,3} x_j \le 5000, \\
& \sum_{j\in Z1} c_{j,1} x_j \ge 255,\ \sum_{j\in Z2} c_{j,1} x_j \ge 296,\ \sum_{j\in Z3} c_{j,1} x_j \ge 387, \\
& \sum_{j\in Z4} c_{j,1} x_j \ge 70,\ \sum_{j\in Z5} c_{j,1} x_j \ge 281,\ \sum_{j\in Z6} c_{j,1} x_j \ge 534 \\
& x_j \in \{0,1\}, \quad j = 1,\ldots,344,
\end{array} \tag{3}$$

where v_j is the *NPV* of the small-scale project *j*, $c_{j,y}$ is the investment cost of project *j* on year *y* (*y* = 1,…, 3), *Zn* denotes the set of small-scale projects that belong to geographic zone *n* (*n* = 1,…, 6), $x_j = 1$ if the small-scale project *j* is included in the optimal portfolio and $x_j = 0$ otherwise.

3.5 Step five: Final comparison

We can now compare the results of the portfolios obtained with both approaches (Table 5). Starting by the large-scale projects we can see comparing rows (2) and (3) to row (1) that the investment costs of the PROBE solution are closer to the budget than the costs of the EDPD solution. As a consequence, it was possible to reach a higher *NPV*, by 372.07×10^3 Euros. The same reasoning applies to the small-scale projects. Here, the increase in *NPV* was 49.46×10^3 Euros. Overall, as it can be seen in row (12) the PROBE (optimization) approach leads to a higher benefit than the EDPD procedure.

Table 5. Benefits and costs of the solutions found (in 10^3 Euros)

Type of project / Approach	*NPV*	Costs			
		Year 1	Year 2	Year 3	Total
Large-scale projects					
(1) Budget for new large projects		8 766.00	10 549.00	15 000.00	34 315.00
(2) PROBE portfolio	80 660.42	8 741.63	9 828.18	7 528.24	26 098.05
(3) EDPD portfolio	80 288.35	8 684.56	9 273.69	6 303.24	24 261.49
(4) = (2) – (3)	372.07	57.07	554.49	1225	1 836.56
Small-scale projects					
(5) Budget for new small projects		4 323.00	4 914.00	5 000.00	14 237.00
(6) PROBE portfolio	23 554.70	4 322.76	314.33	0	4 637.09
(7) EDPD portfolio	23 505.24	4 320.58	237.98	0	4 558.56
(8) = (6) – (7)	49.46	2.18	76.35	0	78.53
Total					
(9) Budget for new projects		13 089.00	15 463.00	20 000.00	48 552.00
(10) PROBE portfolio	104 215.10	13 064.39	10 142.51	7 528.24	30 735.10
(11) EDPD portfolio	103 793.60	13 005.14	9 511.67	6 303.24	28 820.10
(12) = (10) – (11)	421.53	59.25	630.84	1 225.00	1 915.09

4. Effect of adding an environmental impact criterion

PROBE allows including multiple benefit criteria. This feature is used in this section to analyze the extent to which the introduction of an environmental impact (*EI*) criterion would affect the stability of the optimal feasible portfolio of large-scale projects found in Section 3.2. The second column of Table 6 shows the partial value scores of the projects assuming a linear value function on the *NPV* criterion and the third column shows partial value scores of the projects on the *EI* criterion, for which a four-level impact scale was defined. Using a linear additive value model to aggregate the two partial value scores of each project, it would be enough to assign a weight of 5% to the new criterion in order to alter the composition of the selected portfolio, with project P28 being replaced by projects P02 and P11, as shown in the fourth and fifth columns of Table 6. Increasing that weight to 25% would not produce further changes. It is worthwhile noting that a weight of 5% assigned to the *EI* criterion would require EDPD to be willing a tradeoff of about 59×10^3 Euros to mitigate a significant noise impact.

Table 6. Effects of adding a new criterion.

Project	Value scores		Best portfolio	
	NPV	*EI*	Considering *NPV* only	Considering *EI* also, with a weight of 5%
P01	7.8	37.5	P01	P01
P02	4.97	100		P02
P03	250.4	37.5	P03	P03
P04	99.79	100		
P05	380.39	100	P05	P05
P06	77.5	100		
P07	48.91	100		
P08	18.33	37.5		
P09	24.41	37.5		

Table 6. (continued)

Project	Value scores		Best portfolio	
	NPV	*EI*	Considering *NPV* only	Considering *EI* also, with a weight of 5%
P10	400.56	62.5	P10	P10
P11	7.35	100		P11
P12	16.24	100		
P13	57.83	100		
P14	13.7	62.5		
P15	381.57	100	P15	P15
P16	324.71	62.5	P16	P16
P17	324.71	62.5	P17	P17
P18	199.04	100	P18	P18
P19	272.71	62.5	P19	P19
P20	0.02	100		
P21	15.54	62.5		
P22	17.85	62.5		
P23	129.89	100	P23	P23
P24	0.03	62.5		
P25	0.04	62.5		
P26	0.04	0		
P27	29.97	62.5		
P28	16.96	100	P28	

5. Final remarks

This paper describes a case study in the domain of capital budgeting involving different types of projects and budget constraints in multiple time periods. Two approaches for project portfolio selection were discussed: first the pragmatic prioritization procedure that has been followed by the company (EDPD) and the alternative PROBE approach based on 0–1 linear programming. The uncertainty involving the expected cash flows of the projects was also analyzed in terms of its potential consequences for the selection of the portfolio that, respecting all the constraints, could offer the highest cumulative *NPV* to the company. With the help of PROBE, several measures and scenarios of uncertainty were built in order to reassure the decision-makers about the robustness of the final investment decision. Finally, it is worth mentioning that besides being concerned with the *NPV*, EDPD is also considering whether to take into account other aspects when evaluating their projects. This implies a multicriteria extension of the initial NPV model, e.g. with the addition of a criterion related to the environmental impacts of the large-scale projects. In this case, a new multicriteria portfolio decision analysis revealed that a small weight assigned to environment impacts could provoke changes in the composition of the best portfolio.

Acknowledgment

The authors want to thank the collaboration of Head and technical staff of the Department of Network Planning of EDP Distribuição – Energia, S.A.

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