# FRACTIONAL REGRESSION MODELS FOR ASSESSING THE SIGNIFICANCE OF CONTEXTUAL VARIABLES IN OUTPUT ORIENTED DEA MODELS

## Geraldo da Silva e Souza Eliane Gonçalves Gomes

Brazilian Agricultural Research Corporation (Embrapa) – SGE Parque Estação Biológica, Av. W3 Norte final, Asa Norte, 70770-901, Brasília, DF, Brazil {geraldo.souza, eliane.gomes}@embrapa.br

#### **ABSTRACT**

We extend the notion of fractional regression for output oriented DEA models using probability choice models combined with expected mean specifications related to the gamma and the truncated normal families of distributions. The gamma family is also an alternative to second stage regressions using bootstrap methods. We apply these methods to assess the significance of technical effects – type of unit, processes improvement and technology impact – affecting DEA efficiency scores computed to agricultural research production in Brazil, measured through Embrapa. We favor the fractional regression approach since it takes into account the whole sample and applies to DEA performance measurements in general. We conclude that type of unit and processes improvement are significant effects, with the latter effect negatively associated the efficiency of the units classified into the type of unit considered more efficient.

KEYWORDS. Covariates. Fractional regression models. Second stage DEA. Bootstrap.

#### **RESUMO**

Neste artigo a noção de regressão proporcional é expandida para os modelos DEA orientados a outputs, utilizando modelos de escolha probabilísticos combinados com especificações de médias esperadas relacionadas às famílias de distribuição gama e normal truncada. A família gama é também uma alternativa para regressões em dois estágios que se utilizam de métodos de reamostragem. Aplicam-se estes métodos na avaliação da importância de efeitos técnicos – tipo de unidade, melhorias de processos e impacto das tecnologias – que afetam a medida DEA de eficiência de produção da pesquisa agropecuária aplicada no Brasil medida através da Embrapa. Favorece-se a regressão proporcional, visto que esta abordagem leva em conta toda a amostra e se aplica a medidas de desempenho gerais com base em DEA. Conclui-se que tipo de unidade e melhoria de processos são efeitos significantes, sendo o último efeito negativamente associado à eficiência das unidades classificadas no tipo de unidade considerado mais eficiente.

PALAVRAS-CHAVE. Covariáveis. Regressão proporcional. Modelos DEA em dois estágios. Reamostragem.

#### 1. Introduction

In many applications of DEA one is faced with a setting where the main objective is to assess the significance of contextual variables in the efficiency measures. Typical approaches are based on two stage regressions. Some criticism to this order of ideas can be seen in the literature in the works of Simar and Wilson (2007, 2011), who establish the conditions under which it is valid. Basically it is assumed that the contextual variables are exogenous to the production process. Correlation between DEA measurements of different units and endogeneity of contextual variables may invalidate the statistical analysis. Banker and Natarajan (2008, 2011) also discuss the problem.

Our objective in this article is the assessment of the influence of the contextual variables type of unit (product, thematic or ecoregional oriented), processes improvement and technology impact on the DEA efficiency measurements, computed as performance measures for the Brazilian Agricultural Research Corporation (Embrapa) research centers. This is a DEA VRS model with a single pooled output measure and three inputs – personnel expenses, operational costs, and capital depreciation. Two types of models have been recently proposed in the literature that may be employed to study the statistical significance of contextual variables in the Embrapa's case study: the bootstrap approach of Simar and Wilson (2007, 2011) and the fractional regression.

Ramalho et al. (2010, 2011) proposed the fractional regression to assess the effect of covariates in a context where DEA scores are treated as descriptive measures of the relative technical efficiency of the sampled DMUs. They propose the use of flexible families of probability distributions to describe the response behavior of performance measures with values in the interval (0,1] and apply their specifications to agricultural data. They consider one and two part models that can be used with quasi maximum likelihood, nonlinear least squares and maximum likelihood estimation. In the two part model, firstly a binary choice model is fit by maximum likelihood to all units. The contextual variables affect the expected response (choice of being efficient) through a distribution function evaluated on a linear construct. The second part fits a nonlinear mean with values in (0,1) for the inefficient units. The dependence on the contextual variables in this instance is obtained via a monotonic function of another linear construct. Here we extend the two part model to DEA responses with values greater than or equal to one, which are typical of output oriented models. We combine the same choice model idea with the gamma and the truncated normal distributions. In this case efficiencies are fitted envisaging the interval  $(1,+\infty)$ .

The two stage approach of Simar and Wilson (2007) assumes that DEA scores measure efficiency relative to an estimated frontier, the true value of which is unobserved. This implies that estimates of efficiency from DEA models are subject to uncertainty because of sampling variation (Ramalho et al., 2010). From an empirical point of view, the statistical analysis of the two part fractional regression and the two stage approach of Simar and Wilson (2007) are similar. They assume a truncated normal distribution for the inefficient units and propose bootstrap estimation of the model parameters. The same idea can be used in the fractional regression. The difference is that the two stage regression does not take into account efficient DMUs. In this context, unity efficiency is viewed more as a natural consequence of the way DEA scores are defined than as informative as why units become efficient.

Our discussion proceeds as follows. Firstly, we summarize the Embrapa production model. This is the case study in which our fractional regression model and the two stage approach via bootstrap will be applied. Secondly, we describe the family of probability distributions we use in our fractional and bootstrap regressions. Thirdly, we apply and discuss the proposed models regarding our case study. Finally, we summarize our conclusions.

### 2. Embrapa's Production Model

Embrapa's research system comprises 42 research centers (DMUs) spread all over the country. Input and output variables have been defined from a set of performance indicators known to the company since 1991.

The set of production variables monitored by Embrapa, as considered here, comprises one output and a three dimensional input vector. The analysis is performed on a yearly basis. Here we restrict attention to 2009. Dynamic specifications are studied in Souza et al. (2010).

The input side of Embrapa's production process is composed of three factors: personnel, operational costs (consumption materials, travel and services less income from production projects), and capital measured by depreciation.

The output indicator is a pooled index of four categories: Scientific Production; Production of Technical Publications; Development of Technologies, Products, and Processes; Diffusion of Technologies and Image.

Inputs and output are indexes of complex computations that can be appreciated elsewhere. See Souza et al. (2007, 2010) and Souza and Gomes (2011) for more details.

Embrapa's production system is being monitored since 1996 for 37 research centers. Measures of efficiency and productivity are calculated and used for several managerial objectives. One of the most important is the negotiation of production goals with the individual research units. A proper management of the production system as a whole requires the identification of good practices and the implementation of actions with a view to improve overall performance and reduce variability in efficiency among research units.

Parallel to this endeavor is the identification of non-production variables that may affect positively or negatively the system. It is of managerial interest to detect controllable attributes causing the observed best practices.

Several attempts are in course in Embrapa to evaluate the effects of contextual variables in production efficiency. It is worth to mention Souza (2006), Souza et al. (1999, 2007, 2010) and Souza and Gomes (2011). Here we analyze the effect of three exogenous covariates: process improvement (PRO), impact of technologies (IMP), and type of a research center. PRO and IMP are considered continuous scores. Type is a categorical variable. The construct *IMP* is a score computed by Embrapa's administration reflecting perceptions regarding the quality of the reports on impact of the technologies developed by the research centers; it's about form and contents of the reports, and not about the importance of the technologies under concern. On the other hand, PRO is a value intended to measure the successful implementation of changes on some administrative processes. These processes are selected by local Embrapa's administration. Type is an exogenous classification based on the research focus of each unit. There are three types: units or research centers that focus their research on agricultural products (PRODUCT), research centers focusing on agricultural specific themes (THEMATIC), and research centers focused on agricultural research pertaining to issues related to environment and ecological aspects (ECOLOGICAL). We assume that all contextual variables satisfy the separability assumption of Simar and Wilson (2007).

The data on production (inputs and output), the DEA output oriented efficiencies under variable returns to scale, and contextual variables are shown in Table 1. The year of analysis is 2009.

#### 3. DEA, Contextual Variables and Statistical Models

Consider a production process with n production units, the Decision Making Units (DMUs). Each DMU uses variable quantities of s inputs to produce a single output y. Denote by  $Y = (y_1, ..., y_n)$  the  $1 \times n$  output vector, and by  $X = (x_1, ..., x_n)$  the  $s \times n$  input matrix. Notice that the element  $y_j > 0$  is the output of DMU r and  $x_j \ge 0$ , with at least one component strictly positive, is the  $s \times 1$  vector of inputs used by DMU j to produce  $y_r$ .

For each DMU j the DEA measure of efficiency  $\phi_r^*$  is the solution of the LP problem  $\max_{\phi,\lambda}\phi$  subject to  $\sum_r\lambda_r y_r \geq \phi\ y_j$  and  $\sum_r\lambda_r x_r \leq x_j$ ,  $\sum_r\lambda_r = 1$ ,  $\lambda = (\lambda_1,...,\lambda_n) \geq 0$ .

**Table 1.** Production Data, Efficiency Measurements and Contextual Variables. Inputs are X1, X2, X3. Output is Y. Efficiencies are EFF. Contextual variables are Processes Improvement (PRO), Impact of the Technologies (IMP), and Type of research center. Year = 2009.

	X1	X2	X3	Y	EFF	PRO	IMP	Туре
DMU1	1.9491	2.31	2.7117	1.5779	0.8529	71.38	1.42	THEMATIC
DMU2	0.9475	0.7801	0.6516	0.8873	0.4796	45.88	4.27	PRODUCT
DMU3	0.6054	0.6833	0.7612	1.5432	0.8341	88.38	3.53	THEMATIC
DMU4	1.3058	1.1456	1.1190	0.5541	0.2995	72.79	4.20	PRODUCT
DMU5	1.0482	1.1079	1.1601	1.3029	0.7042	88.88	2.86	THEMATIC
DMU6	0.6746	0.8532	0.6409	0.7294	0.3942	58.50	3.86	PRODUCT
DMU7	0.4377	0.5439	1.0545	1.8501	1.0000	58.42	2.22	THEMATIC
DMU8	1.021	0.7785	0.7123	1.0453	0.5650	80.68	4.61	PRODUCT
DMU9	0.9175	0.9185	1.8102	0.7664	0.4142	80.92	3.94	PRODUCT
DMU10	1.3485	0.9039	1.5332	0.7837	0.4236	95.13	4.10	PRODUCT
DMU11	0.9720	1.0944	1.0455	0.7466	0.4036	85.88	3.75	PRODUCT
DMU12	1.0433	0.7983	1.0437	1.0598	0.5728	57.75	4.10	THEMATIC
DMU13	1.0481	1.0375	0.7269	1.2256	0.6625	70.04	4.91	PRODUCT
DMU14	1.4299	1.4462	1.4492	1.0583	0.5720	81.88	4.07	PRODUCT
DMU15	0.9104	0.7062	0.7744	1.0922	0.5904	73.63	3.32	THEMATIC
DMU16	0.8805	0.838	0.9973	0.6600	0.3568	79.48	4.54	PRODUCT
DMU17	1.3737	1.7809	1.5852	1.1443	0.6185	47.43	4.72	PRODUCT
DMU18	1.0264	0.9054	0.9540	0.9172	0.4958	76.50	4.47	PRODUCT
DMU19	0.5765	0.5647	0.6141	1.8501	1.0000	92.25	4.96	THEMATIC
DMU20	0.6892	0.9250	1.0699	0.7055	0.3813	76.38	4.02	PRODUCT
DMU21	1.2903	1.1155	0.8306	0.5272	0.2850	73.38	3.91	<i>ECOLOGICAL</i>
DMU22	1.7702	1.7286	1.5338	0.5682	0.3071	85.38	4.22	<i>ECOLOGICAL</i>
DMU23	1.6006	1.715	1.8198	1.1389	0.6156	84.08	3.16	<i>ECOLOGICAL</i>
DMU24	0.7749	1.194	0.673	0.6848	0.3702	85.40	3.84	<i>ECOLOGICAL</i>
DMU25	0.5078	0.4727	0.2901	0.4944	1.0000	73.00	1.41	ECOLOGICAL
DMU26	0.7037	0.5547	0.4159	1.1163	1.0000	0.00	3.10	ECOLOGICAL
DMU27	0.6122	0.5341	0.6379	1.4728	0.8365	50.17	3.73	<i>ECOLOGICAL</i>
DMU28	1.1706	1.0919	0.8334	0.5575	0.3013	2.50	1.77	ECOLOGICAL
DMU29	0.6368	0.7740	0.5731	0.6497	0.3775	73.88	4.51	ECOLOGICAL
DMU30	0.7758	0.5738	0.6142	0.8509	0.4599	86.54	4.85	ECOLOGICAL
DMU31	1.0206	0.9173	0.6094	1.2273	0.6687	95.50	2.71	ECOLOGICAL
DMU32	1.3446	1.3243	1.1444	0.6782	0.3666	65.14	4.32	ECOLOGICAL
DMU33	2.3904	2.1439	1.5218	0.8324	0.4499	83.13	4.32	ECOLOGICAL
DMU34	0.6753	0.6457	0.7747	0.9863	0.5331	83.80	4.35	PRODUCT
DMU35	0.4118	0.4548	0.5033	1.5013	1.0000	18.88	4.67	PRODUCT
DMU36	0.7590	0.8277	1.0633	1.8501	1.0000	90.25	3.04	THEMATIC
DMU37	0.350	0.8103	0.7465	0.7627	1.0000	0.00	4.26	THEMATIC

In order to evaluate the effect of contextual variables in efficiency measures, we consider here an extension of the two part fractional regression model as defined in Ramalho et al. (2010) adapted for output oriented DEA models, and the Simar and Wilson (2007) bootstrap. The first part of the fractional regression model comprises a standard binary choice model that governs the probability of observing an efficient DMU. Ramalho et al (2010) suggest the use of the whole sample to estimate the model  $\operatorname{Prob}(\phi_j^*=1\mid z_j)=F(z_j'\beta)$ , where z is a vector of contextual variables,  $\beta$  is an unknown parameter vector, and F is a known probability distribution. Typical choices for F are the logistic and the standard normal distributions. Other possibilities may be seen in Ramalho et al. (2010). For the second part of the model we assume the specification  $E(\phi_j^*\mid z_j)=G(z_j'\theta)$ , for the DEA scores in the interval  $(1,+\infty)$ . This is an extension of a similar condition proposed in Ramalho et al. (2010).

Motivated by the gamma and the truncated normal distributions, we propose two specifications in this context. Firstly, the mean of a random variable of the form 1+H, where H has the gamma distribution with location parameter p and scale parameter  $\lambda_j^{-1} = \exp(z_j'\theta)$ . Here  $z_j$  is the observation of the vector of contextual variables for the inefficient DMU j. The parameters  $\theta$  are unknown. Another flexible family much used in stochastic frontier analysis is given by the truncated normal distribution. The corresponding mean in this case is given by  $z_j'\theta + \sigma\phi\left((1-z_j'\theta)/\sigma\right)/(1-\Phi\left((1-z_j'\theta)/\sigma\right))$ , where  $\phi(.)$  and  $\Phi(.)$  are the density and the distribution functions of the standard normal. This expression is the mean of the random variable  $z_j'\theta + u_j$ , where  $u_j$  is the  $N(0,\sigma^2)$  truncated at  $1-z_j'\theta$ .

Contextual variables and corresponding parameters may differ in the choice and inefficient models. In both cases the mean response for the efficiency measure is a monotonic function of the linear construct  $z'\theta$ .

Two nonlinear regressions for the whole sample are implied by these assumptions:

(1) Gamma assumption
$$E(\phi_j^* \mid z_j) = \left[1 + p \exp(z_j'\theta)\right] \left[1 - F(z_j'\beta)\right] + F(z_j'\beta)$$

(2) Truncated Normal

$$E(\phi_{j}^{*} \mid z_{j}) = \left[z_{j}'\theta + \sigma \frac{\phi((1 - z_{j}'\theta)/\sigma)}{1 - \Phi((1 - z_{j}'\theta)/\sigma)}\right] \left[1 - F(z_{j}'\beta)\right] + F(z_{j}'\beta)$$

In applications under exogeneity of the contextual variables these models may be estimated by quasi maximum likelihood or nonlinear least squares. Our choice is the nonlinear least squares, with standard errors and confidence intervals computed using nonparametric bootstrap based on centered residuals. This approach is different from the proposal of Ramalho et al. (2010) and takes into account the whole sample jointly. We believe that additional information on the contextual variables is gained in this context. In our application the data does not support different parameterizations for efficient and inefficient units jointly, as we have a small sample. Therefore, we assume  $\theta = \beta$ . In other words, our assumption is that contextual variables affect the response equally on efficient and on inefficient DMUs. A difficult that arises with the representations (1) e (2) is that the mean, in general, is no longer a monotone function of the linear construct  $z'\theta$ .

Restricting attention only to inefficient units, using maximum likelihood and assuming the truncated normal or the gamma distributions, the results will be equivalent to the two stage analysis proposed by Simar and Wilson (2007).

#### 4. Statistical Results

the data of Table 1 we assume that the linear construct  $\mu = \beta_0 + \beta_1 PRO + \beta_2 IMP + \beta_3 PRODUCT + \beta_4 THEMATIC$ affects expected efficiency according to one of the models discussed above. PRODUCT and THEMATIC are dummy variables.

Jointly, only model (1) seems to fit the data. The separate mean specifications do not produce stable nonlinear least squares results. Maximum likelihood for the inefficient units under the gamma and the truncated normal are stable and leads to similar statistical results.

We begin our discussion with the separate analysis, as proposed by Ramalho et al. (2010), using maximum likelihood methods and the bootstrap Algorithm #1 of Simar and Wilson (2007) to compute standard errors and confidence intervals for the inefficient units, based on 5,000 replications. Table 2 shows results of the choice model with the probit assumption that seems to produce the best fit.

**Table 2.** Results of the choice model with the probit specification.

Model Fit Statistics					
Criterion	Intercept Only	Intercept and Covariates			
AIC	37.893	37.814			
SC	39.504	45.868			
-2 Log L	35.893	27.814			

Testing Global Null Hypothesis: BETA=0						
Test	Chi-Square	DF	Pr > ChiSq			
Likelihood			_			
Ratio	8.0798	4	0.0887			
Score	7.4800	4	0.1126			
Wald	6.4687	4	0.1668			

Analysis of Maximum Likelihood Estimates

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	Parameter	DF	Estimate	Standard Error	Wald Chi-Square	Pr > ChiSq
	Intercept	1	1.4009	1.6314	0.7373	0.3905
	PRO	1	-0.0282	0.0179	2.4646	0.1164
	IMP	1	-0.1006	0.2944	0.1168	0.7326
	PRODUCT	1	-0.8279	0.8624	0.9214	0.3371
	THEMATIC	1	0.9818	0.6213	2.4972	0.1140

We see that only marginally the set of contextual variables is significant. *PRO* acts reducing the probability of being efficient and *THEMATIC* has an increasing effect. The analysis of Simar and Wilson (2007) is shown in Table 3 assuming the truncated normal distribution (results with the gamma distribution are basically the same). Only type of unit is significant at the 5% level.

**Table 3.** Results of the truncated normal specification for the inefficient units.

Parameter Estimate		Bootstrap Standard Error	DF	t Value	95% Bootstra Confidence	
Intercept	2.4618	1.1369	30	2,1654	0.0308	4.5015
PRO	0.0008	0.0101	30	0,0816	-0.0181	0.0225
IMP	-0.0278	0.2076	30	-0,1340	-0.4411	0.4126
PRODUCT	-0.2265	0.2917	30	-0,7765	-0.7991	0.3632
THEMATIC	-1.6402	0.6780	30	-2,4192	-3.3985	-0.6399
sigma	0.6423	0.1067	30	6,0197	0.3968	0.8267

Table 4 shows the statistical results for the joint estimation of the specification (1). Correlation between predicted and observed values is 0.692 ( $R^2$ =0.479). Nonlinear least squares did not converge for (2). The bootstrap (nonparametric) results are based on 5,000 replications. Figure 1 illustrates the bootstrap distributions.

**Table 4.** Nonlinear least squares for  $E(\phi_j^* \mid z_j) = [1 + p \exp(z_j'\theta)] [1 - F(z_j'\beta)] + F(z_j'\beta)$ .

Parameter	Estimate	Bootstrap Standard Error	95% Bootstrap Confidence	
Intercept	-1.0188	0.7098	-5.2688	1.9396
PRO	0.0528	0.0176	0.0137	0.0852
IMP	-0.6738	0.3174	-1.1485	0.1667
PRODUCT	0.0770	0.6443	-1.4742	1.2034
THEMATIC	-4.1007	0.7654	-4.6618	-1.4163
$v (p=\exp(v))$	1.0673	0.0968	0.9292	1.3139

One can see in Table 4 that Type and PRO are significant effects. The model is more informative regarding the effect of the contextual variables on efficiency. Figure 2 shows the derivative of the expected mean response as a function of the linear construct  $\mu$ . We see that the mean response increases or decreases depending on the level of the contextual variables. The same behavior is observed with the marginal effect of PRO. The negative value of THEMATIC is in the direction of more efficiency. For PRO is more difficult to disentangle the marginal effect. For all thematic centers, which are the more efficient, PRO has a negative effect. For the other types the response will vary with the level of  $\mu$ .

We see that processes improvement has not been adequate for benchmark units and do not lead to overall improvement for the less efficient units. The impact of technologies on efficiency, as actually measured, is not statistically important. Appropriate presentation of reports will not lead to more efficiency in production. Managers should look more carefully into production and costs profiles and into processes actually carried out within benchmarks units that could indeed lead to the increase of overall performance.

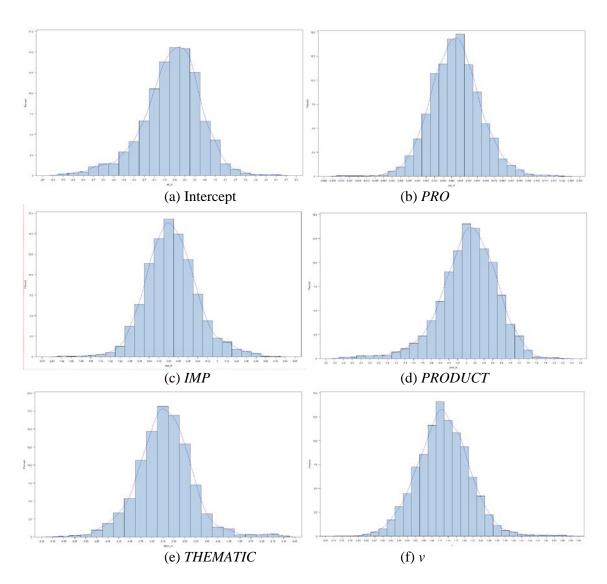
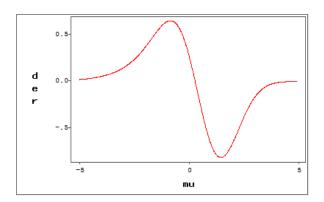


Figure 1. Bootstrap distributions.



**Figure 2.** Derivative of the expected mean response as a function of the linear construct  $\mu$ .

#### 5. Summary and Conclusions

We propose and fit a family of probability distributions for DEA output oriented measures of efficiency as an extension of fractional regression models for responses outside the (0,1) interval. The objective of the analysis was to study the effect of contextual variables in the DEA performance measure computed for Embrapa's research centers.

The models considered allow for the dependence on a vector of contextual variables via a linear construct. As in fractional regression, the models have two parts. These are a choice model explaining the expectation of being efficient, and a flexible family describing the expected mean behavior of the inefficient firms. These are motivated by the gamma and the truncated normal distributions.

The separate analysis for efficient and inefficient units seem to provide support for the two stage regression of Simar and Wilson (2007), since we do not detect significant effects in the choice model related to the efficient units. For the inefficient units both approaches lead to the same results when we fit the data using maximum likelihood under the assumptions of the gamma or the truncated normal.

Combination of the two part models estimated via nonlinear least squares and bootstrap leads to different conclusions regarding the marginal effects of the contextual variables. The impression is that the inclusion of efficient units adds relevant information to the statistical analysis. This point is particularly important for the instrumentalist approach, where we look at efficiency measurements more as measures of performance than as realizations associated with a true unknown production process.

We conclude that the fractional regression approach is more informative. The set of contextual variables studied in Embrapa's application is defined by PRO – Process improvement, IMP - impact of technologies and Type (Product, Thematic and Ecoregional). Type and PRO are statistically significant. The type category Thematic includes the more efficient units. The response effect to PRO varies with the expected mean efficiency level and is negatively associated with performance for the Thematic research centers.

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