CHANCE-CONSTRAINED AND STOCHASTIC VIABLE MANAGEMENT OF AN HYDROELECTRIC DAM

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ABSTRACT.
We study the management of a dam hydroelectric production where economic and tourist stakes compete about the reservoir water use. We first consider the expected gain stemming from the production as the criterion to maximize. In addition, the tourist issue is modelled so as to ensure a reference storage level during the tourist season, at a given probability level. This leads to a chance-constrained optimal control problem, that we solve by the Uzawa algorithm. Albeit the optimal strategy meets the a priori specified requirements, the deviation of the gain from its expected value is significative. Therefore, handling the dam management this way may not be satisfactory when the dispersion of the reached gains is of interest. Thus, we propose a viability approach to the problem: doing our best to respect the reference storage level during the tourist season while guarantying an acceptable gain. To this end, we symmetrize the two stakes by maximizing the probability for this joint thresholds event to occur.
KEYWORDS. stochastic optimal control, chance constraints, stochastic viability, hydroelectric dam management

Hydroelectricity is the main renewable energy in many countries. It provides a clean (no greenhouse gases emissions) and fast-usable energy that is cheap and substitutable for the thermal one. It is all the more important to ensure its proper use that it comes from a shared limited ressource: the reservoir water. This is the dam hydroelectric production management purpose. This management is subject here to the following tourist constraint: *to ensure a reference storage level* s_{ref} *during the tourist season* $\mathcal{T}$ *at a probability level* p_{ref}.

We first proceed by solving a chance-constrained stochastic optimal control problem. We first proceed by modelling the optimization process as a chance-constrained stochastic optimal problem which belongs to the finite-horizon Markov decision processes framework (see Puterman (2005)). A part of these optimization processes have been studied by authors who solved it by different algorithms (see for example Dupacova et al. (1991), Prékopa (1995), Dentcheva et al. (2001), Nemirovski and Shapiro (2006a), Nemirovski and Shapiro (2006b), Andrieu et al. (2010), Dentcheva and Martinez (2011) and Nemirovski (2012)). We solve it by the Uzawa and the stochastic dynamic programming algorithms.

Then, we propose a stochastic viability approach (see Doyen and De Lara (2010)) that symmetrizes the economic and the tourist stakes. These two modellings offer us complementary views on the link between the gain which stems from a dam hydroelectric production and the risk for a storage level trajectory not to complying with the tourist constraint.

1. Dam modelling

We present the dynamics of the dam, and the production model.

1.1. Dynamics of the dam. Let time t vary in $\{0, ..., T\}$. The following positive real valuated random variables are defined on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$:

- S_t, the *storage level* at the beginning of period $[t, t+1[$, (*state*)
- U_t and Z_t, the *hydroturbine* and the *regulative* outflows during $[t, t+1[$, (*control*)
- A_t and C_t, the *inflows* and the *production earnings* during $[t, t+1[$. (*noise*)

We assume that the noise random variables are mutually and step by step independent. They are uniformly distributed on a discrete set. The dynamics of the reservoir storage level reads:

$$S_{t+1} := S_t + A_t - (U_t + Z_t) \text{ with } S_0 := s_0.$$

The bound constraints are:

$$\underline{s}_{t+1} \leq S_{t+1} \leq \overline{s}_{t+1} \quad \text{and} \quad \underline{q}_t \leq U_t \leq \overline{q}_t, \quad \forall t \in \{0, \dots, T-1\}. \tag{1}$$

The hydroturbine and the regulative outflows U_t and Z_t are distinct control processes, a priori. However, the latter is not associated to any earnings and its only way to get valuated is to ensure the storage level upper bounds. So, we can consider that:

$$Z_t = \max\{S_t + A_t - U_t - \overline{s}_t, 0\}.$$

Then, the control reduces to one random variable and the reservoir dynamics becomes:

$$S_{t+1} = f_t^s(S_t, U_t, A_t) := \min\{S_t + A_t - U_t, \overline{s}_{t+1}\} \quad \forall t \in \{0, \dots, T-1\} \quad \text{and} \quad S_0 := s_0. \tag{2}$$

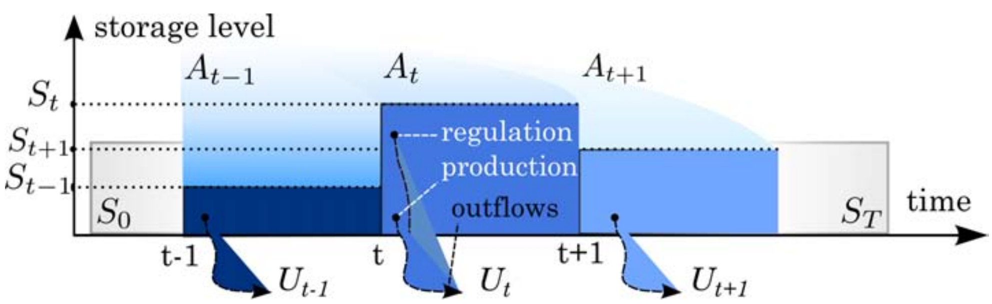


FIGURE 1. Reservoir dynamics of the dam

1.2. **Dam production.** The hydroelectric production obeys the following valorization mechanism:

- the production at time t, $\eta_t : \mathbb{R}_+^3 \to \mathbb{R}_+$ is linearly retributed,

$$c_t(S_t, U_t, A_t, C_t) := C_t \times \eta_t(S_t, U_t, A_t);$$

- a final storage level appreciation function $v_f(S_T)$ prevents the reservoir from being empty at the end of the study period, T.

The total gain is

$$\sigma_T = \sum_{t=0}^{T-1} C_t \, \eta_t(S_t, U_t, A_t) + v_f(S_T) \, . \tag{3}$$

2. Chance-constrained stochastic optimal management of a dam

We address the problem of the optimization of the expected gain σ_T under a tourist chance constraint.

2.1. **Problem statement.** The expected value of the total gain σ_T is the problem criterion:

$$\mathbb{E}\left[\sigma_T\right] \quad \text{where} \quad \sigma_T = \sum_{t=0}^{T-1} C_t \, \eta_t(S_t, U_t, A_t) + v_f(S_T) \, . \tag{4}$$

The tourist chance constraint is

$$\mathbb{P}\left(\{S_\tau \geq s_{\text{ref}}, \ \forall \tau \in \mathcal{T}\}\right) \geq p_{\text{ref}} \text{ where } \mathcal{T} \subset \{1, \ldots, T-1\} \, . \tag{5}$$

The problem is to find a non-anticipative stochastic process $U := (U_0, \ldots, U_{T-1})$ that maximizes (4) subject to the bounds and to the chance constraints (1) and (5). To be more specific, we adopt the Hazard-Decision and classical information pattern framework: U_t is measurable with respect to all of the past and current realizations of the noise at time t. We denote this property by the symbol $\preceq$ as follows:

$$U_t \preceq \sigma(S_0, W_0, \ldots, W_t) \quad \forall t \in \{0, \ldots, T-1\} \text{ where } W_t = (A_t, C_t). \tag{6}$$

Chance constraint goes back to the work of A. Charnes and W.W. Cooper ((Charnes and Cooper, 1959)), and using it raises theoritical and numerical difficulties (see (Grassmann, 1999), (Prékopa, 1995), (Henrion, 2002), (Ruszczynski and Shapiro, 2003)). We opt for writing the chance constraint as an expectation over indicator functions:

$$\mathbb{P}\left(\{S_\tau \geqslant s_{\text{ref}}, \ \forall \tau \in \mathcal{T}\}\right) = \mathbb{E}\left[\mathbb{1}_{\{S_\tau \geqslant s_{\text{ref}}, \ \forall \tau \in \mathcal{T}\}}\right] = \mathbb{E}\left[\prod\nolimits_{\tau \in \mathcal{T}} \mathbb{1}_{\{S_\tau \geqslant s_{\text{ref}}\}}\right] .$$

We then introduce the random process $\pi = (\pi_t)_{t \in \{0, \ldots, T\}}$, defined by the dynamics

$$\pi_{t+1} = f_t^\pi(S_t, U_t, A_t, \pi_t) := (\mathbb{1}_{\{S_{t+1} \geq s_{\text{ref}}\}} \times \mathbb{1}_{\{t \in \mathcal{T}\}} + \mathbb{1}_{\{t \notin \mathcal{T}\}}) \, \pi_t, \forall t \in \{1, \ldots T-1\} \text{ and } \pi_0 = 1 \, , \tag{7}$$

to propagate the information about the reference storage level respect during the tourist season all over the study period. We have that

$$\mathbb{P}\left(\{S_\tau \geqslant s_{\text{ref}}, \ \forall \tau \in \mathcal{T}\}\right) = \mathbb{E}\left[\pi_T\right] \, . \tag{8}$$

We handle the bounds and the measurability constraints (1) and (6) by means of the control admissible set $\mathfrak{U}$:

$$\mathfrak{U} := \{U = (U_0, \ldots, U_{T-1}) \,|\, (1) \text{ and } (6) \text{ hold}\} \, . \tag{9}$$

The chance-constrained stochastic optimal management now reads:

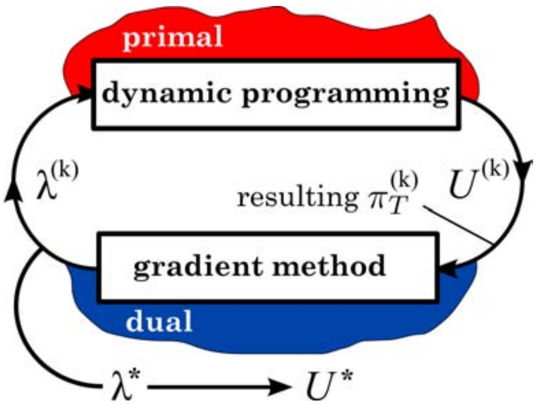
$$\max_{U \in \mathfrak{U}} \quad \mathbb{E}\left[\sum_{t=0}^{T-1} C_t \eta_t(S_t, U_t, A_t) + v_f(S_T)\right] \quad \text{s.c.} \quad \begin{cases} S_{t+1} = f_t^s(S_t, U_t, A_t) \\ \pi_{t+1} = f_t^{\pi}(S_t, U_t, A_t, \pi_t) \\ S_0 = s_0, \pi_0 = 1 \quad \mathbb{P} - p.s. \\ \mathbb{E}[\pi_T] \geq p_{\text{ref}} \end{cases} \tag{10}$$
$$\forall t \in \{0, \dots, T-1\}.$$

2.2. **Solving the chance constrained problem by the Uzawa algorithm.** We dualize the chance constraint (5) by a multiplier λ which is a scalar. We assume that a saddle point exists and we solve the Lagrangian by an iterative update of the λ value:

$$\min_{\lambda \in \mathbb{R}_+} \max_{U \in \mathfrak{U}} \mathbb{E}\left[\sum_{t=0}^{T-1} C_t \eta_t(S_t, U_t, A_t) + v_f(S_T) + \lambda(\pi_T - p_{\text{ref}})\right]. \tag{11}$$

The Uzawa algorithm features are:

- a *primal maximization* (reduced to an unconstrained problem) by dynamic programming;
- a *dual minimization* by a gradient step update of λ.



2.2.1. *Primal maximization by stochastic dynamic programming.* The Bellman optimality principle applies since, in (10), the criterion is time separable and the noise processes are step by step independent. So, we can write a dynamic programming backward induction with the state (S_t, π_t) at every time step t: *for every $(t, \bar{s}, \bar{\pi})$ such that $\underline{s}_{t+1} \leq \bar{s} \leq \bar{s}_{t+1}$,*

$$\begin{cases} V_T(\bar{s}, \bar{\pi}) = \lambda^{(k)}\,(\bar{\pi} - p_{\text{ref}}) + v_f(\bar{s}) \\ V_t(\bar{s}, \bar{\pi}) = \mathbb{E}\left[\max_{u \in \mathfrak{U}_t} C_t \eta_t(\bar{s}, u, A_t) + V_{t+1}\left(f_t^s(\bar{s}, u, A_t), f_t^{\pi}(\bar{s}, u, \bar{\pi}, W_t)\right)\right] \\ \qquad \forall t \in \{0, \dots, T-1\}\ . \end{cases} \tag{12}$$

2.2.2. *Dual minimization by a gradient step algorithm.* The (k)-th optimal solution $U^{\sharp(k+1)}$, we get by solving (12), gives us $S^{\sharp(k+1)}$ and $\pi^{\sharp(k+1)}$ by (10), so the reached tourist constraint probability level respect is $\mathbb{E}\left[\pi^{\sharp(k+1)}\right]$. Then, we can update the multiplier to a value $\lambda^{(k+1)}$ thanks to the following ρ gradient step algorithm:

$$\lambda^{k+1} = \max\left\{\lambda^{(k)} + \rho\left(p_{\text{ref}} - \mathbb{E}\left[\pi^{\sharp(k+1)}\right]\right), 0\right\}. \tag{13}$$

2.3. **Numerical experiment.**

2.3.1. *Numerical instance.* We test the algorithm (12) and (13) with a real case based simple instance of a dam. The bounds in (6) are constant over time. The dam characteristics are:

- $\{\min, \max\}$ bounds on S = $\{0, 76\}$ hm^3;
- time steps number T = 1092 (3 steps a day over a year);
- tourist reference storage level = 57.5 hm^3 from time 543 to time 732 (tourist season $\mathcal{T}$);
- $\{\min, \max\}$ bounds on U = $\{0, 61.7\}$ m^3s^{-1}.

The unities are given in m^3s^{-1} for the inflows and in € MWh^{-1} for the earnings. The stochastic universe is finite. The noise processes are white and uniformly distributed. The required probability level of the tourist constraint is $\text{p}_{\text{ref}} = 0.9$.

2.3.2. *Numerical results and dispersion of the gain.*

- expected total gain = 2.1×10^6 €;
- probability level of the tourist constraint respect = 0.9;
- iterations number, run time = 20, 10 mn on Intel i7 2720QM;
- optimal value of lambda = 5.8×10^6 in (11).

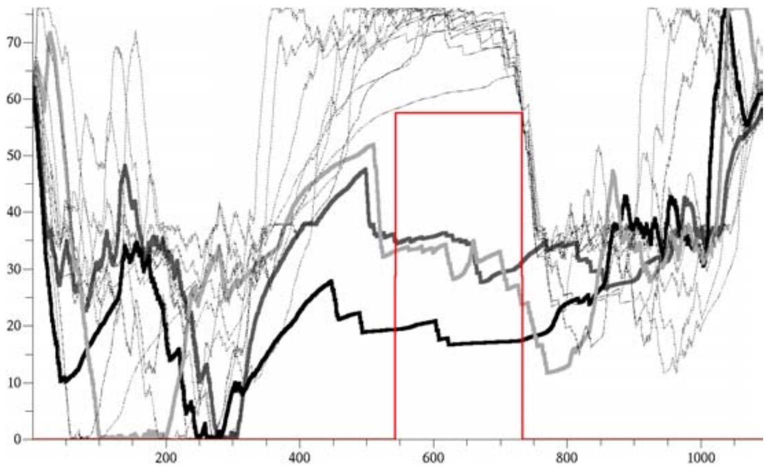


FIGURE 2. Storage level trajectories, those which fail to the tourist constraint respect are bolded. The tourist constraint is represented with the red square.

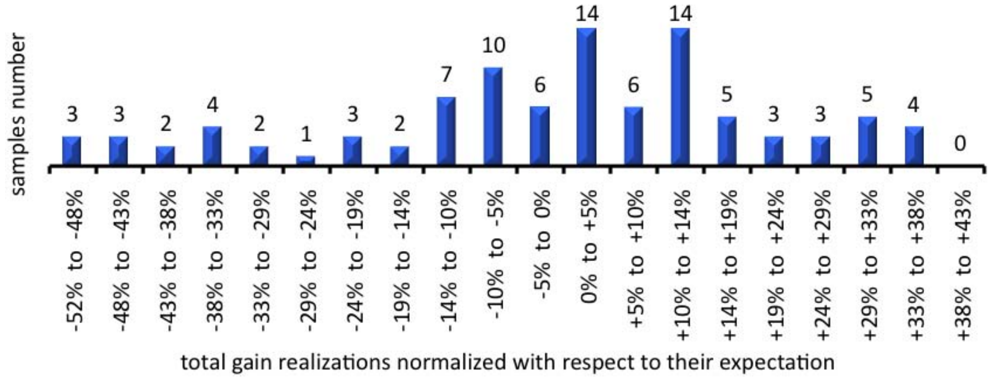


FIGURE 3. Distribution of 100 simulated total gains

We observe that the deviation of the random variable $\sigma_{\mathcal{T}}$ from its expected value is significative: the standard deviation is more than 25% of the expectation. Thus, handling the dam management this way may not be satisfactory if the dispersion of the total gain is of interest. Let us have a look at some representative noise scenarios (upon the intakes in Figure 4 and the earnings in Figure 5).

We note that the realizations of the noise random variables may vary significantly. This has all

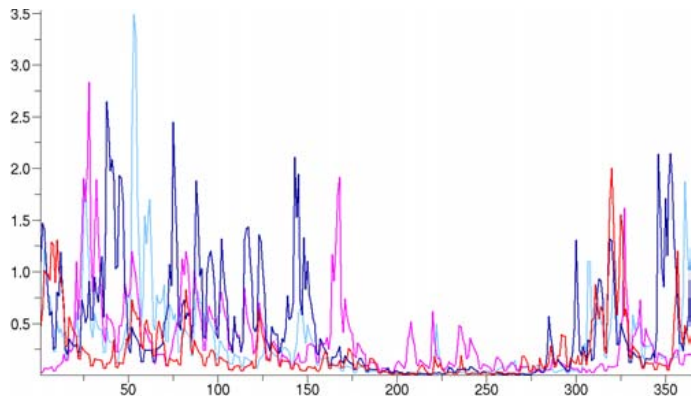


FIGURE 4. Four intakes scenarios sample (in hm3, a step a day over a year)

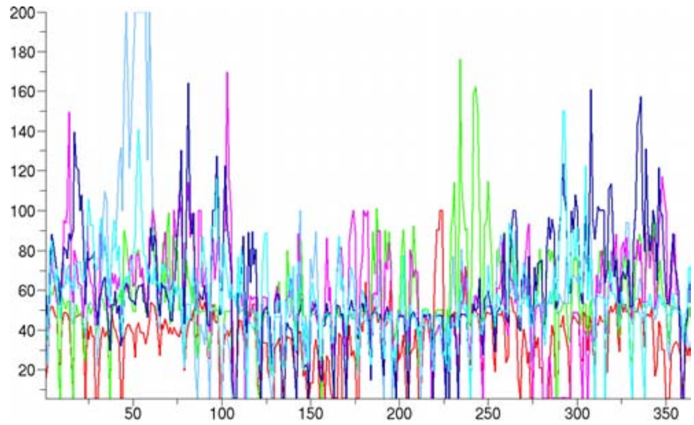


FIGURE 5. Six instant earnings scenarios sample (in euros, a step a day over a year)

the more a non-negligible effect on the simulated accumulated gain values that they combine. They explain the dispersion of the total gain random variable.

The expectation of a random variable is a way to characterize it, but it is neither the only one nor captures and reflects all that characterizes it. We turn to a different approach in the next section.

3. Stochastic viability approach to the dam management

Here, we want to ensure both a guaranteed total gain brought by the hydroelectric production and a guaranteed minimal level in the dam for the tourist issue, with a certain probability.

We call *thresholds guaranty event* and we note $\mathcal{G}$ the event that the reached $\sigma_{\mathcal{T}}$ has a given minimum value g_{ref} and the reference storage level s_{ref} is ensured during the tourist season:

$$\mathcal{G} = \{\sigma_{\mathcal{T}} \geq g_{\mathrm{ref}} \text{ and } S_\tau \geq s_{\mathrm{ref}},\ \forall \tau \in \mathcal{T}\}.$$

To maximize the probability for this event to happen is a mean to address the management of the dam production by symmetrizing the economic and the tourist stakes whereas there was asymmetry in (6). This is a stochastic viability approach ((De Lara and Doyen, 2008) and (Doyen and De Lara, 2010)) that we study in the next section.

3.1. **Description of the approach.** To meet the tourist constraint (to ensure the reference level s_{ref} throughout the tourist period $\mathcal{T}$) while ensuring a satisfactory total gain (a total gain greater than or equal to g_{ref}) in the stochastic framework, corresponds to ensuring the almost sure achievement of the event $\mathcal{G}$:

$$\sigma_T(s_0, U, \omega) \geq g_{\text{ref}} \text{ and } S_\tau(s_0, U, \omega) \geq s_{\text{ref}},\ \forall \tau \in \mathcal{T} \quad \forall \omega \in \Omega' \,/\, \mathbb{P}(\Omega') = 1$$
$$\Leftrightarrow \quad \mathbb{P}(\{\sigma_T \geq g_{\text{ref}} \text{ and } S_\tau \geq s_{\text{ref}},\ \forall \tau \in \mathcal{T}\}) = 1.$$

However, such a probability-one respect may be unachievable or too costly. Based on this observation, the *stochastic viability approach* is:

- to define as *viable* every trajectory such that $\mathcal{G}$ occurs;
- to replace the $\mathbb{P}(\mathcal{G})$ condition by a confidence probability level $\mathbb{P}(\mathcal{G}) \geq \alpha$, with $\alpha \leq 1$;
- to maximize the confidence probability level α such that $\mathbb{P}(\mathcal{G}) \geq \alpha$ is possible.

We represent the gain process dynamics by:

$$\begin{cases} \sigma_T = f_T^\sigma(S_T) := v_f(S_T) + \sigma_{T-1} \\ \sigma_{t+1} = f_t^\sigma(S_t, \sigma_t, U_t, W_t) := C_t \times \eta_t(S_t, U_t, W_t) + \sigma_t, \quad \forall t \in \{0, \dots T-2\} \\ \qquad \sigma_0 = 0\,. \end{cases}$$

Now, we consider the problem:

$$\begin{aligned} &\max_{U \in \mathfrak{U}} \quad \mathbb{P}(\{\sigma_T \geq g_{\text{ref}} \text{ and } S_\tau \geq s_{\text{ref}},\ \forall \tau \in \mathcal{T}\}) \\ &\text{s.c.} \quad \begin{cases} S_{t+1} = f_t^S(S_t, U_t, A_t),\ \forall t \in \{0, \dots, T\} \\ \sigma_{t+1} = f_t^\sigma(S_t, \sigma_t, U_t, W_t),\ \forall t \in \{0, \dots T-1\} \\ S_0 = s_0 \text{ and } \sigma_0 = 0 \end{cases} \\ &\qquad \forall t \in \{0, \dots, T\}. \end{aligned} \tag{14}$$

3.2. **Solving the stochastic viability problem by dynamic programming.** Assuming again the step by step independence of the noise random variables, the following proposition allows us to solve the problem by a multiplicative dynamic programming equation (see (Doyen and De Lara, 2010) for the proof).

Proposition 1. *(Hazard-Decision solving)*
Let us consider the following backward induction:

$$\begin{cases} V_T(s, \sigma) = \mathbb{1}_{\{\sigma \geq g_{ref}\}} \\ \forall t \in \mathcal{T}: \quad V_t(s, \sigma) = \mathbb{E}\left[\max_{u \in \mathfrak{U}_t} \quad \mathbb{1}_{\{s \geq s_{ref}\}} \times V_{t+1}\left(f_t^S(s, u, A_t), f_t^\sigma(\sigma, u, W_t)\right)\right] \\ \forall t \notin \mathcal{T} \cup T: \quad V_t(s, \sigma) = \mathbb{E}\left[\max_{u \in \mathfrak{U}_t} \quad V_{t+1}\left(f_t^S(s, u, A_t), f_t^\sigma(\sigma, u, W_t)\right)\right]. \end{cases} \tag{15}$$

If the noise random variables are step by step independent, then

$$\forall (s_0, \sigma_0) \in \mathbb{R}^2_+ \quad V_0(s_0, \sigma_0) = \max_{U \in \mathfrak{U}} \mathbb{P}\left(\left\{\sigma_T \geq g_{ref} \text{ and } S_\tau \geq s_{ref},\ \forall \tau \in \mathcal{T}\right\}\right).$$

Thus, the equation (15) *determines the solution of the stochastic viability problem* (14).

We shall now test numerically the resulting algorithm.

ALGORITHM: for a given starting state (s_0, σ_0)

for every gain value g_{ref} to guarantee **do**
  for every storage level s_{ref} to guarantee **do**
    solve:

$$\begin{cases} V_T(s,\sigma) = \mathbb{1}_{\{\sigma \geq g_{ref}\}} \\ \forall t \in \mathcal{T}: \quad V_t(s,\sigma) = \mathbb{E}\left[\max_{u \in \mathfrak{U}_t} \quad \mathbb{1}_{\{s \geq s_{ref}\}} \times V_{t+1}\left(f_t^S(s,u,A_t), f_t^\sigma(\sigma,u,W_t)\right)\right]. \\ \forall t \notin \mathcal{T} \cup T: \quad V_t(s,\sigma) = \mathbb{E}\left[\max_{u \in \mathfrak{U}_t} \quad V_{t+1}\left(f_t^S(s,u,A_t), f_t^\sigma(\sigma,u,W_t)\right)\right]. \end{cases}$$

    save: $\alpha^*(s_{ref}, g_{ref}) = V_0(s_0, \sigma_0)$
  end for
end for

3.3. **Numerical experiment.** We go back to the numerical instance we have studied in the first section. We make s_{ref} vary from 20 to $70\,hm^3$ and we make g_{ref} vary from $0,5$ to $2,5$ millions euros. The Figure 6 shows the maximal viability probability and the Figure 7 shows its isograph, as function of the guaranteed gain g_{ref} in Meuros and guaranteed stock s_{ref} in hm3. In Section 2, we maximized the expected gain subject to the tourist 0.9-level chance constraint (of having at least 57.5 hm3 in the tourist season), and obtained an optimal expected gain of about $2,1$ millions euros. In Figure 7, we see that guaranteeing jointly such a gain value and the tourist constraint is feasible with about 55% probability. In fact, if we keep s_{ref} to 57.5 hm3 and want a viability probability of 0.9, we cannot guarantee a gain higher than 1.5 millions euros. By symmetrizing the economic and the tourist stakes, the stochastic viability approach offers a complementary view on the problem, and more specifically on the influence of the tourist-associated risk parameterization on the gain random variable.

4. Conclusion

The traditional approach, when going from deterministic to stochastic control, consists in taking the expected value of the original criterion (risk-neutral approch in economics). The constraints may be taken in various senses, such as robust, in probability one, or with a given probability level. This is the way we handled the dam management issue in the first part. However, the realized optimal random gain displayed a significative dispersion. This is why, in the second part, we proposed a stochastic viability approach that symmetrizes the economic and the tourist stakes, and guarantees minimal thresholds. Thus, the stochastic viability approach offers a complementary view to stochastic optimal control under chance constraint.

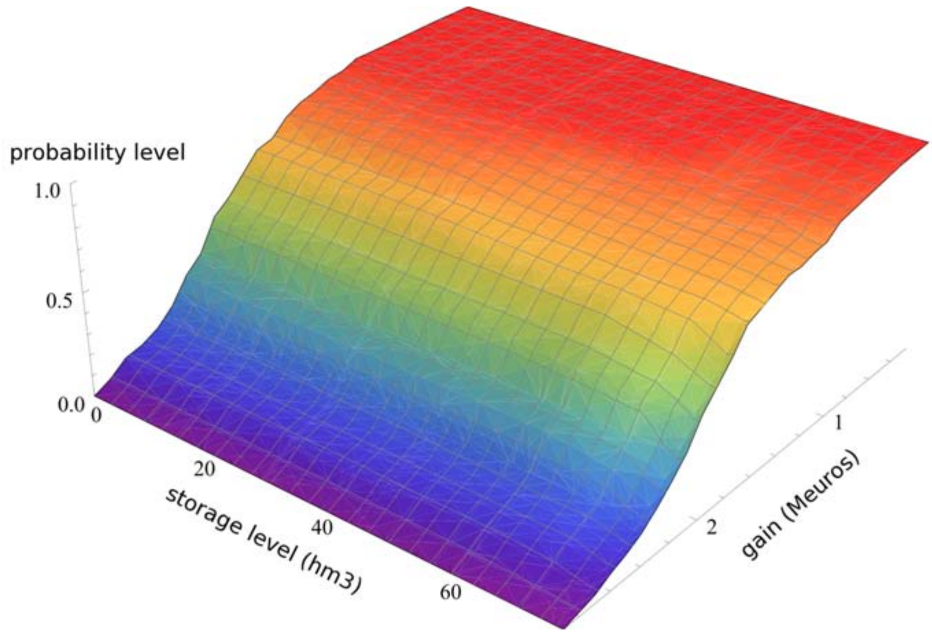


FIGURE 6. Maximal viability probability as function of the guaranteed gain g_{ref} in Meuros and guaranteed stock s_{ref} in hm3

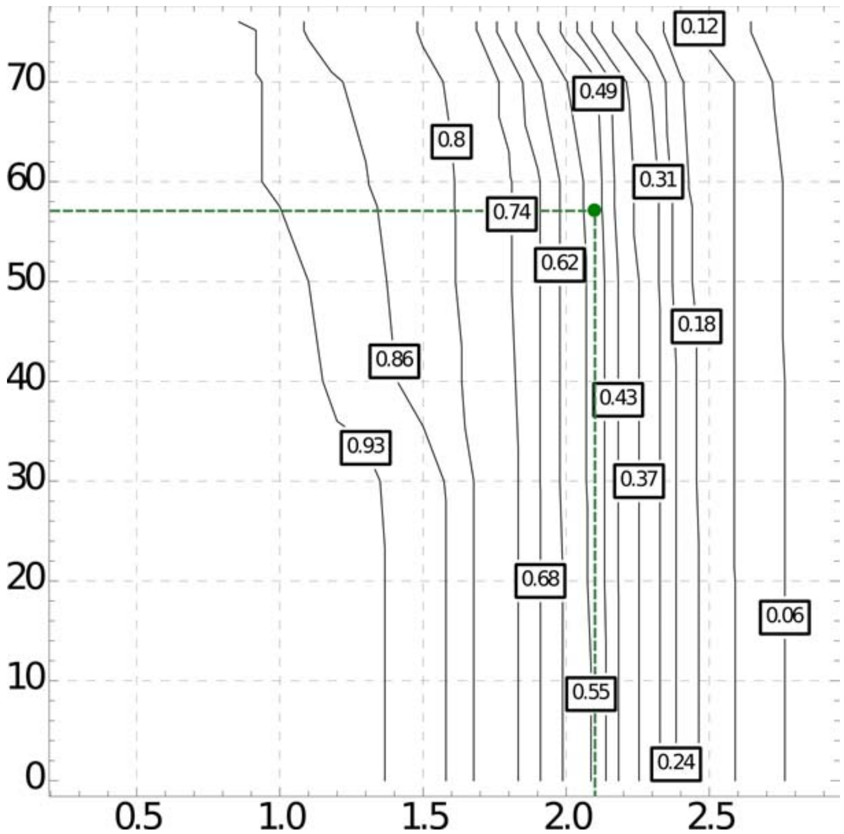


FIGURE 7. Guaranteed gain g_{ref} in Meuros as abscissa and guaranteed stock s_{ref} in hm3 as ordinate

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