A reverse discount model for dynamic demands

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ABSTRACT

In large industries like aerospace, automotive with high set up costs for manufacturing, the manufacturer often plays a dominant role in determining the batch sizes. The obvious choice for the manufacturer is to produce everything in one set up, to minimize his overall costs. Under this scenario, it may be worthwhile for the buyer to offer the manufacturer an increase in the wholesale price to entice the manufacturer to reduce the batch sizes. Such a mechanism is in effect a "reverse discounting" procedure. This paper examines the economics of reverse discounting in the context of deterministic but dynamic demand. The formulation involves the solution of Mixed Integer Linear Programming Problem (MILP).

KEYWORDS: Reverse Discount, Supply Chain Coordination, Mixed Integer Linear Programming

Main Area: OR in Industry (IND)

1. INTRODUCTION

A supply chain is composed of independent entities that are linked with each other through material, information and financial flows (Stadler, 2005). Effective functioning of the Supply Chain requires that the chain work as a single entity. Conflicting interests of the different entities or partners often come in the way of efficient and effective functioning of the chain (Cachon, 2003). Several mechanisms for Supply Chain Coordination have evolved over time to ensure proper functioning of the chain. One such mechanism that has been applied and studied by many involves offering quantity discounts. In the traditional Economic Ordering Quantity (EOQ) model as is developed by Harris (1915) that essentially involves finding the optimal ordering quantity when the ordering involves a tradeoff between the ordering cost and inventory carrying cost, the buyer would like to order in batch sizes equal to his EOQ. The same may not be economical for the supplier, as given a choice, the supplier perhaps would like to supply in relatively much higher batch sizes (Monahan, 1984). When the buyer is the dominant player, onus lies on the supplier to offer quantity discounts to motivate the buyer to increase the batch size. This mechanism of the supplier offering discounts, in effect, acts as a compensation to ensure higher batch sizes from the buyer, and helps in mitigating the conflict, and achieving the desired coordination in the supply chain. There are instances, however, in many large industries, where the manufacturer as the supplier plays a dominant role in determining the batch sizes (Esmaeilli et al., 2009). The obvious choice for the manufacturer, saddled with a high setup cost, is to produce everything in one set up, to minimize his overall costs. Under this scenario, it may be worthwhile for the buyer to offer the supplier an increase in the wholesale price to entice the supplier to reduce the batch sizes. Such a mechanism is in effect a “reverse discounting” procedure. Most of the work in “quantity discounts” would fall under the former category where buyer is the dominant player in the chain, and the supplier offers the discount. “Reverse discount” with supplier as the dominant player and buyer offering discounts, has been reported only in one study. Traditional EOQ assumptions have been common in all the studies. Two types of discounting procedure is common in the literature, “all units” and “incremental” type. In this paper, any reference to discount would mean “all units” discount.

The objective of this paper is to work out the economics of reverse discounting in the context of deterministic but dynamic demand. After a brief review of the existing literature in section 2, we develop the model in section 3 and present a computational exercise with several runs of the deterministic Mixed Integer Linear Programming (MILP) model. Insights from the results of the model are summarized in section 4. The concluding remarks and limitations of the study are presented in the final section.

2. BRIEF LITERATURE SURVEY

For an excellent and detailed review of the “quantity discount” models, the reader is referred to the article by Benton and Park (1996). In their taxonomy, they have classified discounting models into models with “non time-phased” (static) and “time –phased” (dynamic) demand. Within each of these categories, they have considered two separate discounting scheme, “all units” and “incremental”. For each type of scheme in turn, they have separated models with “buyer’s” perspective from those focusing on “Buyer-supplier” perspective. Initial models (Hadley and Whitin 1963, Buffa and Miller 1979, to name a few) were typically non-time phased, focusing only on the “buyer’s” perspective. Under the assumption of static, deterministic demand, the models addressed the problem of determining the EOQ of the buyer, given a discount schedule by the supplier. Supplier’s economics was not considered. Crowther (1964) is reported to be the first to consider the economics of the buyer as well as the supplier. Monahan (1984) developed it further and formulated the problem to determine the discount to be offered and the resulting batch size that maximizes the profit of the supplier, without making the buyer worse off. Banerjee (1986) extended the model to account for situations where the supplier has a finite production rate instead of infinite production rate as assumed by Monahan. Further, Lee and Rosenblatt (1986) examined the economics by relaxing the lot-for-lot assumption. Latest models on discounting as a mechanism for supply chain coordination under static but price dependent demand have been presented by Li and Liu (2006) and Shin and Benton (2007).

Traditionally inventory models with time-phased or dynamic demands have been formulated as Mixed Integer Linear Program (MILP) and solved optimally by Wagner-Whitin's algorithm (1958). Application of quantity discounts on the same has been addressed by authors like Mather (1970), Callarman and Whybark (1977), Benton and Whybark (1982) and others. In all these models, the focus has been on the buyer's perspective. However, as noted in Benton and Park (1996), models with "Buyer-Supplier" perspective in the context of "time-phased" demands remains an open area. The literature till 2012 also confirms the same in that we could not find any work in this domain.

The general assumption in all the above models has been that the buyer has the power to dictate his terms in deciding the lot sizes. In both the "buyer" and the "buyer-supplier" perspective discussed above, the buyer remains the dominant player. As pointed out in the introduction, in many large industries like aerospace, automobiles and heavy equipment, the supplier takes the dominant position and decides on the delivered lot sizes (Esmaelli, 2009). In general, for a given planning horizon, the supplier would like to set up only once and would want the buyer to carry the entire inventory. In this context, it may be worthwhile for the buyer, to propose a price increase to the supplier, expecting the supplier to decrease the batch size. This proposal of price increase constitutes "reverse discount". This economics of "reverse discount" for static demand scenario has been examined by Chakraborty and Chatterjee (2012).

In the view of the above discussion, this paper attempts to address the research gap by examining and analyzing "reverse discounts" in the context of "time-phased" or dynamic demands.

3. MATHEMATICAL MODEL

Consider a supply chain with one buyer and one supplier. The buyer faces dynamic deterministic demand. The supplier is the dominant player, and he decides on the batch size. Nature of costs suggests that the resulting batch size is relatively higher than what the buyer would like. Conflict in such cases is resolved with the buyer offering a price increase to compensate the supplier for any loss that he may incur in case he decides to decrease the batch size. In this section, we develop the mathematical model of this scenario. The assumptions and notations used in the model are first presented below.

Assumptions

1. Demands are known and deterministic in nature
2. Supplier will follow a lot for lot policy and hence does not keep any inventory
3. No backlogging is allowed
4. Infinite production/ procurement rate for the supplier
5. Capacity at the supplier is infinite

Notations

Data

w	The wholesale price charged initially by the supplier (i.e. unit cost of purchase for the buyer)
c	The unit variable cost incurred by the supplier to produce his goods
H	The holding cost in \$ per unit \$ per unit time
A_R	The ordering cost of the buyer
A_S	Set up cost of the supplier
S_1	Supplier's initial profit
S_2	Supplier's final profit
B_1	Buyer's initial cost
B_2	Buyer's final cost
d_i	Demand in the i^{th} period

Decision Variables

Δw	The value of reverse discount offered
I_i	Inventory carried by the buyer in i^{th} period
x_i	The buyer's ordered quantities in the i^{th} period
y_i	A binary variable denoting whether production has taken place in the i^{th} period

The supplier's optimal strategy is to set up once in the entire planning horizon and not to keep any inventory. Buyer gets all that is ordered in one order and keeps inventory. It is possible to develop Joint Economic Lot Sizing (JELS) model taking into account the buyer and supplier economics together. Though such models provide overall optimal solutions, implementation becomes a problem as these models result in differential benefits to the two partners. This may be unacceptable to one of the partners (Lu, 1995 and Sucky, 2005, 2006). Our concern here is to see that the proposal of price increase from the buyer becomes acceptable to the supplier. Thus, we formulate the problem as that of maximizing buyer's cost savings without making the supplier worse off from his initial position.

Formulation

The various stages of the formulation are:
Stage 1: The buyer observes the final demand and orders the supplier. The supplier's optimal strategy would be to produce only once in the entire planning horizon and not to keep any inventory as well, given that the supplier has an infinite production capacity to satisfy any demand (the infinite capacity can be relaxed and subsequently the resulting supply pattern for the supplier can be formulated). Based on the supplier's proposal, the buyer calculates his costs and the supplier, his profits.

Stage 2: The buyer then offers a proposal of wholesale price increase to the supplier, keeping in mind that the resultant proposal should not decrease the profits of the supplier and in turn maximize his resulting cost savings. The resulting mathematical model can be formulated as follows:
Let $d_1, d_2, \ldots, d_n$ be the demands known for the 'n' periods.

Let the Buyer's Initial cost (when the supplier supplies all that is being demanded in one set up)

$$=B_1= w.D+ A_R+ w.H.\,(D_1+D_2+\ldots + D_{n-1}) \quad (1)$$

where, $D= d_1+d_2+\ldots\ldots+d_n$ and $D_1=D-d_1$, $D_i=D_{i-1}-d_i$

The first term in (1) corresponds to the total price of the items paid to the supplier; the second term indicates the ordering costs incurred by the buyer for the single order that he places with the supplier and the last term gives the total inventory carrying costs for the buyer.

Similarly the Supplier's Initial Profit= $S_1 = (w-c)*(d_1 + d_2 + \ldots + d_n) - A_S$ (2)

The first term indicates the profit the supplier earns by selling the items while the second term is the set up cost incurred by him.

On proposing a wholesale price increment of 'Δw' to the supplier, the buyer also proposes in parallel, his ordering pattern to the supplier. The resulting cost of the buyer thus becomes

$B_2 = (w+\Delta w).D + (y_1+y_2+\ldots+y_n).A_R + (w+\Delta w).H\,(I_1+I_2+\ldots+I_{n-1})$

And the corresponding supplier's profit becomes

$S_2 = (w+\Delta w-c)*(d_1+d_2+\ldots+d_n) - A_S\,(y_1+y_2+\ldots+y_{n-1})$

Change in buyer's cost is given as: $\Delta B = B_1 - B_2 =$

$\Delta w.D + (y_1+y_2+\ldots+y_n).A_R + (w+\Delta w).H\,(I_1+I_2+\ldots+I_{n-1}) - w.H\,(D_1+D_2+\ldots+D_{n-1}) - A_R$ (3)

Change in Supplier's profit is given as: $\Delta S = S_2 - S_1 = \Delta w.D - A_S\,(y_1+y_2+\ldots+y_n) + A_S$ (4)

Thus, the buyer's decision problem is to choose, the offered price increase 'Δw' and the ordered quantities $x_1, x_2, \ldots, x_n$ that will maximize his cost savings ΔB subject to the condition that the supplier should not be made worse off. The mathematical programming model for the buyer corresponding to the above is given as:

Max $B_1 - B_2 = \Delta B$
Subject to $\Delta S \geq 0$
$x_t + I_{t-1} = I_t + d_t$ (5)
$x_t \leq B y_t$ (6)
$x_t, I_t \geq 0$ (7)
$y_t = \{0,1\}$
B is a very large positive number
Equation (5) is the inventory balance equation; equation (6) is related to set up cost while (7) indicates feasibility condition for production and inventory quantities. It is further to be noted that $I_t \geq 0$ specifies that demand cannot be backlogged.

From the above discussion, we can write the resulting MILP (Mixed Integer Linear Programming) problem as follows:

We will illustrate the above model through an example:

Example 1: Consider the example with the given 4 period demand data d_1= 235, d_2= 178, d_3=367 and d_4=431;
Buyer's ordering cost= 50, Initial wholesale price= 25; Inventory Holding Cost=0.05, Supplier's set up cost= 500
Buyer's Initial cost=33081.25,

Supplier's Initial Profit= 29775.00,
Reverse Discount offered= 0.8258
Buyer's Final Cost after offering the reverse discount= 31654.85,
Supplier's Final Profit= 29775.00,
Net Savings for the buyer= 1426.401

4. NUMERICAL RESULTS

The model in (5) is solved for some demand series (increasing and decreasing both) for various combinations of other parameters like A_R (50, 250, 1000), H (0.05, 0.25, 0.4), w (25, 200, 1000) and A_S (500, 2500, 10000) for a twelve period planning horizon. The objective is to gain insights to the reverse discounting procedure. Specifically, we would like to investigate some conditions under which the buyer is more likely to offer reverse discount. The results are presented below in the form of observations.

Observation 1: Average savings decreases with an increase in C_O/H ratio.

Ar/H	125	200	625	1000	2500	4000	5000	20000
%Savings	70.078%	59.371%	70.064%	46.653%	70.010%	59.283%	22.293%	0.000%

The above table shows there is a statistically significant (at 5% level of significance) decrease in the percentage of additional gains with the increase in C_O/H.

Observation 2: Average savings decreases with an increase in the coefficient of variation of demand

	Coefficients	P-value
Intercept	0.487731322	0
Coefficient of variation	-0.071304468	3.47E-14

The above table shows there is a statistically significant (at 5% level of significance) decrease in the percentage of additional gains with the increase in coefficient of variation of demand.

Observation 3: Average savings increases with an increase in the mean demand

	Coefficients	P-value
Intercept	0.421078	0
Mean demand	3.97E-05	4.33E-55

The above table shows there is a statistically significant (at 5% level of significance) increase in the percentage of additional gains with the increase in mean demand.

Observation 4: Percentage of reverse discount decreases with an increase in the mean demand

	Coefficients	P-value
Intercept	0.083798	0
Mean demand	-3.1E-05	4.1E-192

The above table shows there is a statistically significant (at 5% level of significance) decrease in the percentage of reverse discount offered with the increase in mean demand.

5. CONCLUDING REMARKS

In this paper, we have considered the buyer offering a reverse discount to the supplier to entice him to order in lower batch sizes and more frequently. In the context of deterministic and dynamic demand, we have formulated the problem as that of maximizing the benefit to the buyer without allowing the supplier to be worse off. Several runs of the model lead us to some insights to the performance of reverse discount mechanism. For example, we find that a decrease in the ratio of buyer's ordering cost to inventory holding cost is likely to increase the savings of the buyer. This in turn may suggest the buyer tries to bring down the ordering costs. Similar observations may be inferred from the results obtained. Finally it should be noted that the current study has a number of limitations. No theoretical results could be presented for the MILP model formulated. As the model formulated is of finite horizon, the results for different length of planning horizon would be naturally different. In future studies attempt may be made to infer on some "optimum planning horizon". Further, the assumption of infinite capacity at the supplier is also a limitation and can be relaxed in further studies.

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