A MULTI-PRODUCT LOT-SIZING IN A BAKERY MANUFACTURING PROCESS

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ABSTRACT

The purpose of this paper is to present an Integer Linear Programming model for a lot-sizing problem. The problem considers demands, inventory policies, backorder costs and the search of an efficient use of resources (machines and workers). The real-world case used to illustrate the model is from a Colombian company, which produces raw material for the bakery industry. The short term planning for the company under study is critical, because there is a multi-product environment where resources are shared between different references and processes. The computational experiments show the effectiveness of the proposed model.

KEYWORDS. Production planning, Inventory control, Multi-product lot-sizing.

Main area: IND – IO en Industria

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1. Introduction

The operations planning in the short term field of manufacturing is a widely studied area. There are many cases of research projects by considering the specific characteristics of a particular manufacturing system. These studies try to represent a real-world case offering support for future decision-making processes. The definition of lot-sizing problem (LSP) considers features and decision variables from the production and inventory control area, and their impact on the service level. Usually, the objective function for the LSP is to minimize the sums of the total holding cost, of the stock-out cost, and of the other costs, which affect the operation of each system.

In particular, our problem considers several characteristics which increase the complexity of the traditional LSP. Karimi et al. (2003) propose a general classification about the most frequent issues to consider in a lot-sizing problem. In this work, we have considered planning horizon, number of levels, number of products, several patterns of demands, holding inventory, back-orders, and setups structures. Different models by including one item for uncapacitated and capacitated versions of the LSP have been proposed in Brahimi et al. (2006). In particular, these models consider one product, few production stages, and known demands.

Aksen et al. (2003) introduce the concept of immediate lost sales by considering inventory policies for a single product; while Absi and Kedad-Sidhoum (2008) consider inventory policies for a multi-item capacitated model by adding setup times. Robison et al. (2009) and Jans and Degraeve (2007), introduce three important concepts about inventory management: stockout, back-orders and holding cost. All of these approaches consider a dynamic demand environment.

Capacity constraints and relaxed demand constraints with a penalty are considered in Aksen (2007). This relaxation can include variables to represent back-orders and inventory levels among different periods. Other mathematical approaches can be found in Abad (2000), Toso et al. (2009) and Kovacs et al. (2009) propose set of constraints and set of variables to represent the multiple stage condition for the LSP. In these works, a new concept is introduced: scheduling with sequence-dependent setups. All conditions previously presented increase computational complexity increasing the computing time. Heuristic approaches for the LSP are proposed by Xie and Dong (2002) and Minner (2009). Some special characteristics to represent and solve stochastic models are described by Paternina-Arboleda and Das (2005). In this work, relevant references for a stochastic lot scheduling problem are considered.

A production program in a multi-product environment must consider three fundamental aspects: i) the type of products to schedule, ii) the amount of products to produce, iii) the time to make the products. The first aspect generally is solved by giving priority to an item for which is close to stockout or reorder point. The other two aspects are closely related, if small batches are scheduled. The way of scheduling lots, their sizes, and its frequency, are the main causes of the final performance for the objective function. Set larger lots imply a greater efficiency, but also imply more scheduling time for a single product at a given time. This decision could affect the responsiveness of the system.

In this paper, the considered real-world case has a batch system for the production of bakery raw materials industry. Currently, different dry products (dough improvers, premixes, and sugar among others), and a wide range of liquid essences are produced. In general, these products represent about seventy different references with approximately two thousand tons annually. In addition, a two-stage multi-product environment with deterministic demands allowing back-orders and holding inventory is considered. Due to similarity inside each family of products, setups are considered constant with an independent sequence.

2. Mathematical model

Let us introduce the following notation for our proposed formulation:

Sets:
1. set of products indexed by i, where i=1,..., I
2. set of shifts indexed by j, where j=1,..., J
3. set of days indexed by k, where k=1,..., K
4. set of machines indexed by m, where m=1,..., M

Subsets
1. subset of production machines indexed by m, where $m \in \alpha \subset M$
2. subset of packing machines indexed by m, where $m \in \beta \subset M$
3. subset of products with a production stage indexed by i, where $i \in \mu \subset I$
4. subset of products with production and packing stage indexed by i, where $i \in \theta \subset I$

Parameters
H_{im}: time required to produce one kilogram of each product i at machine m (hr/kg)
P_m: hard-work force required per shift at machine m (number of persons)
MO: total persons available
RP_i: maximum requirement of product i (kg)
IS_i: initial stock of product i (kg)
RD_{ik}: daily requirement of product i at day k (kg/day)
CI_i: holding cost of product i ($/kg-day)
CB_i: backorder cost of product i ($/kg-day)
CH_m: overtime cost at machine m ($/hr)
HM: maximum overtime (hr)
HT: time available per shift (hr/shift)
A: setup time (hours/setup)
TML: minimum lot-size (kg)

Decision Variables
x_{ijkm}: amount of product i scheduled in shift j of day k at machine m (kg)
y_{jkm}: 1 if shift j is scheduled in day k at machine m, 0 otherwise
he_{jkm}: overtime in shift j in day k at machine m (hr)
f_{ik}: backorder of product i in day k (kg)
s_{ik}: inventory of product i in day k (kg)
wip_{ik}: work in process of product i in day k (kg)
w_{ijkm}: 1 if product i is scheduled in shift j in day k at machine m, 0 otherwise

Objective Function

The performance of the system is defined by three different sets of costs: holding cost, backorder cost, and overtime cost. The first set (holding cost) is related to the operations required to control and manage items, and to the financial value of investment in products, work in process and raw materials. The second set (backorder cost) is associated to the decreasing value defined by the final customer. Finally, the third set (overtime cost) is considered as a linear ratio which represents additional time for each machine.

$$\text{Min } Z=\sum_i \sum_k CI_i s_{ik} + \sum_i \sum_k CB_i f_{ik} + \sum_j \sum_k \sum_m CH_m he_{jkm} \qquad (1)$$

Constraints

Constraints (2) ensure that the sum of persons in each shift for each machine must not exceed the total available hard-work force.

$$\sum_{m}\sum_{j} P_m\, y_{jkm} \leq MO, \ \forall\, k \tag{2}$$

Constraints (3) limit the production capacity. The maximum capacity is expressed as the sums of the time available for each scheduled HT, and of the overtime, minus the sum of the setup time employed by each assigned product. These constraints are considered as production efficiency if the size of the programmed batch is larger, and the number of setups is smaller.

$$\sum_{i} H_{im}\, x_{ijkm} \leq HTy_{jkm} + he_{jkm} - \sum_{i} Aw_{ijkm}, \ \forall\, j,k,m \tag{3}$$

Constraints (4) add overtime according to the maximum number of possible hours.

$$he_{jkm} \leq HMy_{jkm}, \ \forall\, j,k,m \tag{4}$$

Constraints (5), (6) and (7) make logical relations between shift programming and scheduled products during the available shifts by considering maximum and minimum lot-size, and setup time for each product.

$$w_{ijkm} \leq y_{jkm}, \ \forall\, i,j,k,m \tag{5}$$

$$x_{ijkm} \geq TMLw_{ijkm}, \ \forall\, i,j,k,m \tag{6}$$

$$x_{ijkm} \leq RP_i w_{ijkm}, \ \forall\, i,j,k,m \tag{7}$$

Constraints (8) include back-orders and inventory control to keep inventory balance among days. The basic expression considered is *initial stock + production-required product = final inventory*. The final inventory is shown as a linear combination of f_{ik} and s_{ik}. Indeed, only one of these variables will be activated each time, because the penalty cost of each of them in the objective function.

$$IS_i + \sum_{j}\sum_{m\in\alpha} x_{ij1m} - RD_{i1} = s_{i1} - f_{i1}, \ \forall i\in\mu \tag{8}$$

Finally, the multi-period issue is restricted in (9). The final inventory for a given period k-1 is calculated.

$$\sum_{j}\sum_{m\in\alpha} x_{ijkm} + s_{i\,k-1} - f_{i\,k-1} - RD_{ik} = s_{ik} - f_{ik}, \ \forall i\in\mu, k>1 \tag{9}$$

Equations (10) and (11), represent the condition expressed in (9) for products belonging to the subset θ.

$$IS_i + \sum_{j}\sum_{m\in\beta} x_{ij1m} - RD_{i1} = s_{i1} - f_{i1}, \ \forall i\in\theta \tag{10}$$

$$\sum_{j}\sum_{m\in\beta} x_{ijkm} + s_{i\,k-1} - f_{i\,k-1} - RD_{ik} = s_{ik} - f_{ik}\,,\quad \forall i\in\theta,\ k>1 \tag{11}$$

Due to products θ have two stage processes, which must be connected to ensure process flow and work in process. Constraints (12) described the first period, and constraints (13) described the multi-period stage.

$$\sum_{j}\sum_{m\in\alpha} x_{ij1m} - \sum_{j}\sum_{m\in\beta} x_{ij1m} = wip_{i1},\quad \forall i\in\theta \tag{12}$$

$$\sum_{j}\sum_{m\in\alpha} x_{ijkm} + wip_{i\,k-1} - \sum_{j}\sum_{m\in\beta} x_{ijkm} = wip_{ik}\,,\quad \forall\ i\in\theta,\ k>1 \tag{13}$$

Constraints (14) are the non-negativity constraints.

$$x_{ijkm}, he_{jkm}, f_{ik}, s_{ik}, wip_{ik} \geq 0,\quad \forall\ i,j,k,m \tag{14}$$

3. Results

Scenarios

Three different instances were selected to represent the behavior of the system and the relations between variables, constraints and parameters in the proposed model. These scenarios have different demands, initial stocks and production requirements. Table 1 shows the information of the initial stock; the overall demand, and the production requirement used by instances.

Table1.Instances features (kg)

Instance	Initial Stock	Overall Demand	Production Requirement
Month A	139944	374400	234456
Month B	159888	366600	206712
Month C	113029	352200	239171

Each scenario has different performance, because the different production requirements. Consequently, the computing time and the function objective value are different.

Computational Results

The tests were carried out by using CPLEX 12.3 on an Intel Core i5 2.3 GHz processor with 4 GB of memory. In the used real-world case, the computing time of the proposed model is relevant, in order to be considered as support decisions tool for a short period term. Indeed, computing time must be efficient according to the planning horizon. Figure 1 shows the results of the used computing time to find the optimal solution. The behavior for months A and C are similar respect to the computing time. It can be inferred by the fact of the relations with the total number of kilograms used by the company. In the case of month B, which amount is smaller almost by thirty tons, the computing time is reduced.

The proposed instances consider seventy products, five work stations, three shifts, and a planning horizon of twenty four days.

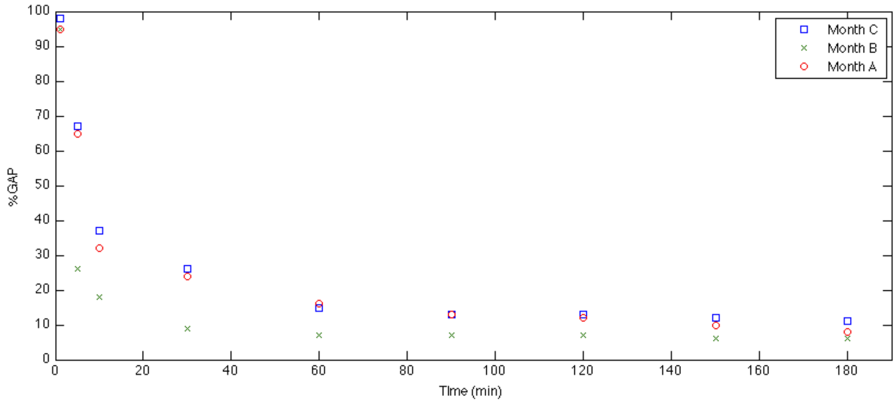

Figure 1: Evolution to reach optimal solution

Performance of the objective function

Table 2 details the costs obtained for each instance. We have compared the final results with the traditional planning method used by the company. It is worth to note that the proposed model outperforms the currently used method respect to the overall cost. Indeed, the main reductions of the costs occur in backorder cost and inventory cost.

Table 2.Comparison of the performance for each component of Z($)

Traditional planning method				
Instance	**Inventory**	**Backorder**	**Overtime**	**Overall Cost**
Month A	2616	3713	284	6612
Month B	1904	4833	33	6770
Month C	2587	4496	209	7292
Proposed Integer Lineal model				
Instance	**Inventory**	**Backorder**	**Overtime**	**Overall Cost**
Month A	1788	22	156	1966
Month B	1658	19	4	1681
Month C	1347	18	21	1385

Lot size

The lot size is a significant issue related to the efficiency and the cost of the production program. In a multi-product environment, a big lot size of a given product can delay the production for the other products, because the shared resources (machines and hard-work force). In the previously traditional planning method, the considered lots had an average of two thousand kilograms. The proposed ILP model is able to find a more equitable replenishment of the different products. Smaller lots are found as described Table 3. In this table, the following notation is used: Average is the average size of the lots, STD is the standard deviation for scheduled lots, Q1 is the first quartile (lowest 25% of lots), Q2 is the second quartile (data set in half), and Q3 is the third quartile, lowest 75%.

Table3. Lot size (kg)

Instance	Average	STD	Q1	Q2	Q3
Month A	1311	982	630	996	1714
Month B	1091	776	600	768	1222
Month C	1178	792	600	810	1500

The obtained lot sizes consider less than 1700 kg. Indeed, this lot size permits to produce many products frequently keeping healthy stocks levels, and avoiding stock-outs.

It is important to note that the final inventory of each product at the end of each month is close to the value of the safety stock. Since the frequency of production for each reference is related to its lot size and the evolution of inventory, it is no necessary to define a reorder point as a general rule.

Efficiency analysis

We have added some new variables and constraints which have no impact on the outcome, but that estimate other performance measures. The variables *at* represent the scheduled time and variables *st* represent the total time spent during setups.

$$st=\sum_{m}\sum_{j}\sum_{k} HT\, y_{jkm} \qquad (15)$$

$$at=\sum_{m}\sum_{j}\sum_{k}\sum_{i} A\, w_{ijkm} \qquad (16)$$

$$Efficiency = 1- \frac{at}{st} \qquad (17)$$

Table 4 presents the results obtained by the efficiency of the ILP model for the three months. Indeed, Table 4 shows scheduled time, setup time, and efficiency.

Table 4.Comparison of efficiency performance

Instance	Scheduled time (hr)	Setup time (hr)	Efficiency (%)
Traditional planning method			
Month A	980	84	91
Month B	1000	82	91
Month C	960	92	90
Proposed Integer Lineal Model			
Instance	**Scheduled time (hr)**	**Setup time (hr)**	**Efficiency (%)**
Month A	976	68.5	93
Month B	904	70.3	92
Month C	1016	77.3	92

The results of the proposed ILP model are slightly better than those proposed by the traditional planning method. In the traditional planning method bigger lots are chosen, so the efficiency is increased. Nevertheless, matching the demands of many products for a given month the production program must be changed decreasing the efficiency of the system.

4. Conclusions

In a multi-product environment, the size of each batch affects the level of inventory for a given product, and the references which use the same resources (persons and machines). In the particular real-world case, a common is to schedule small batches (values range from a minimum batch operation and 1800 kilograms). The different SKU's can be provided in a more effective way such as: It is possible to avoid an excessive inventory of some items and to avoid the risk of stockout for others items.

With the exception of products whose total requirements were low and could be covered by a minimum lot, all the rest were supplied by the production various amounts of similar size to each other, and programmed equally throughout the month.

The proposed objective function considers holding costs, backorder costs, and overtime cost. This approximation allows establishing requirements without additional costs. The minimum lot size and the simultaneity of the requirements for different products are the main conditions to stock a certain amount of inventory.

The total computing time of the proposed ILP model is considerably high because the number of variables and constraints. Moreover, since the Multi-product LSP is a monthly planning problem, the model is not solved frequently. Therefore, the computing time remains in an acceptable range for a tactical problem like Multi-product LSP. Additionally, results may be obtained with a 10% gap respect to the optimal solution, within a short computing time. These results allow using the result of the proposed model as an approximation method for production planning. The use of formal tools allows obtaining better results to understand complex systems. Nevertheless, the next step will be to test the performance of the model with similar problems on benchmark instances by considering a generalization of its principal characteristics.

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