VEHICLE ROUTING IN A STAR-CASE NETWORK

Lic. Luis Alfonso Infante Rivera
PISIS-UANL
luisinfanterivera@gmail.com

Dr. Igor Litvinchev
PISIS-UANL
igorlitvinchev@gmail.com

ABSTRACT

We address the problem of routing a fleet of different vehicles from a central depot to customers with known demands and time windows. It is assumed that a vehicle has to return back to the central depot after visiting a customer, such that a transportation network has a star-case structure. The mathematical model is presented together with a numerical study.

KEYWORDS. Vehicle routing, Star-case transportation network, Lagrangian relaxation.

Main area : L & T - Logistics and Transport.

1. Introduction

In many organizations, the management of distribution activities constitutes a major decision-making problem. The efficient utilization and routing of the fleet of vehicles lies at the heart of almost all distribution-routing problems. In particular, a natural question facing a distribution manager is: How many and what size vehicles are needed in order to accommodate demand at minimal cost? This question gives rise to the problem we formulate and address in this paper.

Variants of the vehicle routing problem generally share the following characteristics. A set of routes must be designed for the vehicles, originating from and terminating at a central depot. The routing costs associated with the vehicles form one component of total distribution costs. The other essential component consists of fleet acquisition and maintenance costs.

We consider vehicle routing problem with multiple vehicle types as a problem of simultaneously determining the composition and routing of heterogeneous fleet of vehicles in order to service a prescribed set of customers (clients) with known delivery demands and time windows from a central depot or plant. It is assumed that each type of vehicle is available with infinite supply. The objective is to minimize the sum of the vehicle acquisition costs and routing costs.

The literature on vehicle routing problem (VRP) is quite extensive, see e.g. Golden (2008), Pop (2012), Toth (2002) and the references therein. Frequently it is assumed that there exists a direct route (not passing through the central depot) between any pair of clients, such that a transportation network is represented by a complete graph. The routing problem in specialized and/or incomplete networks is paid much less attention (see, e.g., Basnet (1999), Beaulieu (2002), Labbe (1991),). The central assumption of our model is its star-case transportation network. In a few words only direct routes from the central depot to the clients are allowed. The connection client-client either is not permitted, or is not used. There are at least three reasons to consider such a network:

a) The transportation network has a star-case configuration from the beginning. This situation occurs in some underground networks (mine, subway, etc.) where there are no physical routs between the clients and all connections have to pass through the center.

b) The transportation network is a complete graph, but only direct routes are allowed by the rules. This is the case, for example, in a gasoline supply by PEMEX (Mexican State Petrol Company), where the demand of a gasoline station has to be an integer multiple of a vehicle (pipe) capacity and by security reasons the pipe has to return back to the center immediately after visiting a gasoline station.

c) The transportation network is a complete graph, but client's demand is significantly greater than the capacity of the vehicle. In this case, even if there are physical connections client-client, they are (almost) not used. Since to satisfy the (large) client's demand, the vehicle has to return back to the center to reload. The later is typical for the high-level routing where the demand points are large distribution centers, but not the final customers.

Thus, in contrast to the classical vehicle routing problem, a client has to be visited multiple times (maybe by the same vehicle) to satisfy its demand. We also assume that a split delivery is allowed. Soft time windows are considered allowing the vehicle to start service at the client before or after its time window, respectively. As a result, the vehicle incurs additional costs. A corresponding mixed-integer programming model is formulated and results of a numerical study are discussed.

2. Model Formulation

The following parameters are used to characterize the model:

$K =$ number of vehicles,

$I =$ number of clients; index 0 denotes the depot,

$J_k =$ maximal number of trips (depot-client-depot) for vehicle k ,

$d_i =$ demand of client i,
$c_{ik} =$ cost of the round-trip travel from the depot to client i for vehicle k,
q_k = capacity for vehicle k,
t_{ik} = time for one way travel from the depot to client i for vehicle k,
f_k = fixed (acquisition) cost for vehicle k,
E_i = earliest time allowed for beginning delivery at client i,
L_i = latest time allowed for beginning delivery at client i,
e_{ik} = cost (per time unit) for violating the earliest time for delivery,
l_{ik} = cost (per time unit) for violating the latest time for delivery.

The decision variables are as follows:

x_{ijk} : = 1, if client i is visited by vehicle k at its trip j ; = 0 otherwise,
y_k : = 1, if vehicle k is used at least for one client $i \neq 0$; = 0 otherwise,
$s_{jk} \geq 0$ - the time to begin trip j by vehicle k,
$w^+_{ijk} \geq 0$ - time violation for the latest delivery at trip j,
$w^-_{ijk} \geq 0$ - time violation for the earliest delivery at trip j.

Note that we use index j as an internal discrete time counter for the vehicle k, such that at any trip j vehicle k either visits a client, or stays at the depot (visits client 0).
The model is mathematically stated as:

$$\min \sum_k f_k y_k + \sum_{i,j,k} \left(c_{ik} x_{ijk} + e_{ik} w^+_{ijk} + l_{ik} w^-_{ijk} \right) \tag{1}$$

$$\text{s.t.} \quad x_{0jk} + \sum_{i \neq 0} x_{ijk} = 1 \quad \text{for all } j, k, \tag{2}$$

$$\sum_{j,k} q_k x_{ijk} \geq d_i \quad \text{for all } i \neq 0, \tag{3}$$

$$J_k - \sum_{j \in J_k} x_{0jk} \leq y_k J_k \quad \text{for all } k, \tag{4}$$

$$s_{j+1,k} \geq s_{jk} + \sum_i 2t_{ik} x_{ijk} \quad \text{for all } j, k, \tag{5}$$

$$w^-_{ijk} \geq E_i x_{ijk} - (s_{jk} + t_i x_{ijk}) \quad \text{for all } i \neq 0, j, k, \tag{6}$$

$$w^+_{ijk} \geq (s_{jk} + t_i x_{ijk}) - L_i - M(1 - x_{ijk}) \quad \text{for all } i \neq 0, j, k, \tag{7}$$

$$x_{ijk}, y_k \in \{0,1\}, w^+_{ijk}, w^-_{ijk}, s_{jk} \geq 0. \tag{8}$$

The objective function (1) minimizes the total fixed cost, travel cost, and the cost for deviation from time windows. Constraints (2) ensure that at any trip a vehicle visits exactly one client or stays at the depot (visiting client 0). Constraints (3) state that the demand of every client is satisfied. Constraints (4) jointly with minimizing the objective ensure that vehicle is not in use if it stays at the depot for all trips. Constraints (5) state that the trip starts only after the vehicle returns back to the depot completing the previous trip. constraints (6) jointly with minimizing the objective ensure that if at a given trip the client is not visited ($x_{ijk} = 0$),then there is no cost for violating the earliest delivery time ($w^-_{ijk} = 0$). Otherwise, the violation time is calculated in a standard way. Similarly, constraints (7) state the same for violating the latest delivery time. Here

M is a large positive constant ($M \geq s_{jk} - L_i$). Note that by (6) and (7) $w^{-}_{ijk} > 0$ only for $s_{jk} + t_i < E_i$, while $w^{+}_{ijk} > 0$ only for $s_{jk} + t_i > L_i$. Hence for $E_i \leq L_i$ we always have $w^{+}_{ijk} w^{-}_{ijk} = 0$. A solution to (1) – (8) indicates the vehicles used (variables y_k), provides allocation of clients to vehicle together with the order of visits (variables x_{ijk}), and the vehicle schedule (variables s_{jk}).

3. Computational Experiment

Linear mixed integer model (1)-(8) was tested on graphical terminals Sun Ray connected with server SunFireV440 having 4 processors UltraSPARC IIIi –1.28GHz. The commercial solver ILOG CPLEX 12.0 was used together with modeling language AMPL20021031. Real problems data were kindly provided by SINTEC (http://www.sintec.org).

A set of 10 randomly generated problems with 30 clients was considered for E_i = 08:00am and $L_i =$ 05:00pm for all clients. The costs for violating the earliest and the latest delivery time were equal for all clients, and only three types of vehicles were considered. Parameters varying within the test problems were demands, travel costs, and travel times. For 3600 seconds CPU time all problems were solved within 40-60% of optimality (gap in branch-and-bound technique). Leaving more time (up to 6 hours) did not improve the gap significantly. A more detailed description of computational results one can find in Litvinchev (2012).

The case study was considered representing the distribution of goods in Valle de Mexico area. The distribution network has 13 clients, 1 production plant sending products to the clients, hard time windows ($E_i =$ 08:00am and L_i = 05:00pm, the costs for violation the time window e_{ik}, l_{ik} were set to 10^5), with 50 (30 single-size and 20 double-size) vehicles available. The results of modeling provide us a schedule for routing an optimal selection of vehicles to satisfy the demand and to meet the hard time windows.

Vehicle	Destination	Star time	Travel Time	Next Time available
1	Reyes	3.37	9.26	12.63
	Vallejo	12.63	7.26	19.89
2	Coacalco	2.875	10.25	13.125
3	Huixquilucan	3.87	8.26	12.13
	Reyes	12.13	9.26	21.39
4	Huixquilucan	3.87	8.26	12.13
	Chalco	12.13	9.26	21.39
5	Chalco	3.87	9.26	13.13
6	Chalco	3.37	9.26	12.63
7	Coacal	2.875	10.25	13.125
8	Reyes	3.37	9.26	12.63
	La Viga	12.63	7.26	19.89
9	Chalco	3.37	9.26	12.63
10	Chalco	3.37	9.26	12.63
11	Reyes	3.37	9.26	12.63
12	Coacal	2.875	10.25	13.125
13	Coacal	2.875	10.25	13.125
14	Coacal	2.875	10.25	13.125

Vehicle	Destination	Star time	Travel Time	Next Time available
15	Reyes	3.37	9.26	12.63
	Tlalpan	12.63	8.26	20.89
16	Tlalnepantla	4.37	7.26	11.63
	Mixcoac	11.63	9.26	20.89
17	Iztapa	3.96	8.08	12.04
	Chalco	12.04	9.26	21.3
18	Texcoco	2.37	11.26	13.63
	Cuatitlan	13.63	3.26	16.89
19	Texcoco	2.37	11.26	13.63
20	Mixcoac	3.37	9.26	12.63
	Huixquilucan	12.63	8.26	20.89
21	Huixquilucan	3.87	8.26	12.13
	Zaragoza	12.13	9.26	21.39
22	Tlalnepantla	4.37	7.26	11.63
	Coacalco	11.63	10.25	21.88
23	Coacalco	2.87	10.25	13.12
24	Texcoco	2.37	11.26	13.63
25	Texcoco	2.37	11.26	13.63

The table above is a sample of the schedule obtained by optimization of the model. For example, vehicle 1 goes to Reyes at 3.37 hours, returns back to the depot at 12.63 hours, and then is available to go to Vallejo. In this schedule the first 14 vehicles are single-size while the next 11 are double capacity vehicles. Thus we use 25 of 50 vehicles available and obtain savings in total transportation cost without loosing the level of service.

The solution presented in the Table was obtained by CPLEX 12.2 after running 3600 seconds CPU time and with gap 68% reported. To improve the estimation of the gap, Lagrangian dual bound (Guignard (2003)) was calculated relaxing constraints (3). The corresponding dual problem was solved by the subgradient technique. If after 5 consecutive iterations of the sub-gradient technique the dual bound was not improved, the half of the step size scaling parameter was used. The process stops if the step size scaling parameter is less than 0.0001, or if the maximum number (300) of iterations is reached. Relaxing constraints (3) a gap 32% was obtained for the same feasible solution after 429 seconds CPU time.

4. Conclusions

The principal difference of the model (1)-(8) from classical vehicle routing formulations (see, e.g., Toth (2002)) is in the assumption that the transportation network has a star-case topology where only direct routes from the central depot to the clients are allowed. Another difference from the classical case is that each client can be visited multiple times (maybe by the same vehicle) to satisfy its demand. Our computational experience shows that for mid-sized problems a good suboptimal solution can be found in a reasonable time by commercial software. Meanwhile, for larger problems, and especially, for sensitivity analysis we need faster techniques taking into account specifics of the problem. Note, that in the problem of distribution of goods the number of clients is relatively small, but the time windows are thin and hard. On the contrary, the gasoline company problem has a great number of clients, and large and smooth time windows. We believe that developing fast and/or approximate methods using the specifics of real problems is an interesting area for future research.

References

C. Basnet, L. R. Foulds and J. M. Wilson (1999), Heuristics for Vehicle Routing on Tree-Like Networks, *Journal of the Operational Research Society* 50(6), 627-635.

Beaulieu, M., Gamache, M. (2006), An enumeration algorithm for solving the fleet management problem in underground mines, *Computers & Operations Research*, 33, 1606–1624.

Golden, B.L., Raghavan, S. and E. A. Wasil, (2008) *The Vehicle Routing Problem: Latest Advances and New Challenges*. Springer,

M. Guignard (2003), Lagrangian relaxation, *TOP*, 11, 151-228.

M. Labbe, G. Laporte and H. Mercure (1991), Capacitated Vehicle Routing on Trees Operations Research 39(4), 616-622

Litvinchev, I., Infante, L. (2012), Routing in a star-case network, Technical Report PISIS-2012-01 Graduate Program in Systems Engineering, UANL, San Nicolas de los Garza, México.

Pop, P., Kara, I., Marc, A.H. (2012), New mathematical models of the generalized vehicle routing problem and extensions, *Applied Mathematical Modelling*, 36, 97-107.

Toth, P. and Vigo, D., (editors) *The Vehicle Routing Problem,* SIAM monographs on discrete mathematics and applications, Society for Industrial and Applied Mathematics, 2002.