NETWORK RESILIENCY AND REINFORCEMENT DECISIONS IN DISASTER-PRONE AREAS

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ABSTRACT

In this work we deal with a network subject to random edge failures and study the problem of optimally determining the set of links to be reinforced so as to minimize the sum of investment and expected flow costs. The problem arises, for example, within the context of humanitarian logistics in disaster-prone areas where mitigating measures must be undertaken to increase network resiliency and guarantee the provision of help to the population. The problem is formulated as a mixed integer non-linear program and belongs to the class of stochastic programming problems with endogenous uncertainty – *i.e.*, those in which the probability distribution of the random parameters is decision-dependent. The proposed approach includes a convexification technique for polynomials of binary variables, an efficient cut-generation algorithm and the incorporation of importance sampling concepts into the stochastic programming framework so as to allow the solution of large instances of the problem.

KEYWORDS. Stochastic programming. Endogenous uncertainty. Network reinforcement.

1. Introduction

The impact of natural or man-made disasters can be very significant both in terms of losses of human lives and damages to affected regions, as vastly documented during recent catastrophic events including earthquakes, hurricanes and floods. Besides the immediate death toll and destruction of infra-structure, the effects of these calamities usually last long after the initial strike. When an earthquake strikes a city, for example, utility services such as water, electricity and gas supply may have to be interrupted for weeks before necessary repairs are carried out. On top of that, several roads and bridges are usually affected, rendering the transportation network severely impaired. As pointed out by Tomasini and Van Wassenhove (2009), more casualties actually happen due to the isolation to which many residents are forcefully put to rather than by the event itself.

In face of that, regions that are prone to the occurrence of natural disasters must take preventive measures in order to mitigate potential damages and, most importantly, devise emergency plans so that they are able to provide care for those affected by such events. It is thus imperative to assess the vulnerability of the existing transportation network and to take steps aimed at guaranteeing that it will be possible to either evacuate people to safe locations or to provide them with basic resources in post-disaster days.

The objective of the network reinforcement problem in the context of humanitarian logistics is to determine the optimal set of investments on retrofitting the links of a transportation network so as to minimize the sum of (deterministic) investment costs and expected (probabilistic) costs incurred when transporting people and/or resources after a catastrophic event – investments in bridges and tunnels, for example, may increase their resilience so that an earthquake is less likely to render them unusable. Such investments usually involve very large sums of money and a limited budget must be optimally allocated.

The remaining of this article is organized as follows: Section 2 provides the mathematical formulation of the problem along with the identification of the difficulties that prevent it from being solved exactly by existing approaches; Section 3 describes the approach to deal with the difficulties outlined in the preceding section and proposes an algorithm for obtaining a global optimal solution to the problem. Section 4 presents promising computational results and Section 5 concludes with final remarks and directions of future research.

2. Background and problem description

Though there has been an extensive amount of work dealing with network reinforcement problems in general, the literature on the problem within the humanitarian logistics context is very limited. Viswanath, Peeta and Salman (2004) – which subsequently resulted in Peeta et al. (2010) – were the first to state the problem, motivated by the risks of an earthquake hitting Istambul, Turkey. They limit the scope of their model to the case where one is interested in maintaining connectivity between origin (O) and destination (D) pairs and their approach relies on the enumeration of the O-D paths (which, for practical purposes and due to computational difficulties, is limited to listing a pre-defined number of paths by using a k-shortest path algorithm). Next, they propose an approximation of the objective function based on the first order terms of its Taylor series expansion. As they recognize in their article, the disadvantage of this approach is that by ignoring higher order terms they neglect the potential synergies of simultaneously investing in more than one link.

Liu, Fan and Ordonez (2007) and Fan and Liu (2003) also study the stochastic network protection problem. In the former, the problem follows the same outline as that described above and they propose an extension of the L-Shaped method of Van Slyke and Wets by using generalized Benders decomposition. In the latter, the second-stage problem involves the determination of a Nash equilibrium by solving an MPEC (mathematical program with equilibrium constraints) which results from the consideration that users may choose their best-perceived routes along the network. Their solution method relies on the application of the Progressive Hedging algorithm of Rockafellar and Wets. Both papers, however, make the explicit assumption that the decision to invest on the reinforcement of a link eliminates the probability that it might become unavailable after the disaster. They argue that it would be preferable and more realistic to maintain a probabilistic view on link failures but doing so would lead the problem to fall under the class of stochastic programming problems with decision-dependent uncertainties (also referred to as endogenous uncertainties) for which "mathematical analysis (…) is very sparse, and is only limited to convex problems of special structures" thus relying "heavily on heuristic methods to solve problems with realistic sizes due to computational difficulties".

Although not dealing with the same problem, there are some related works on the investment in links of a stochastic network, including Wollmer (1980), Wallace (1987) and Wollmer (1991). Additionally, there is also a significant body of work on the development of plans for disaster preparedness and response which adopt a different perspective from that of mathematical programming. Instead, these works usually take a somewhat heuristic view to determine critical links of a network based on a set of pre-defined criteria – e.g., Sohn et al. (2003), Basoz and Kiremidjian (1995) and Bana Costa, Oliveira and Vieira (2008).

Mathematically, the problem is formulated by assuming we are given an undirected graph $G = (N, E)$ with vertex set N and edge set E. Vertices represent locations where survivors and/or resources may be located, and edges represent the roads, bridges and tunnels which comprise the transportation network. For ease of presentation, a deterministic supply or demand h_i is associated with each vertex i. Edges have non-negative transportation costs c_e, capacity u_e and are assumed to be available after the occurrence of the disastrous event with probabilities p_e^C. As also stated in related works, it is assumed that each edge fails independently of the others – although this is not a necessary assumption for the methods proposed in this work. The survival probability of an edge may be increased to p_e^I if an amount r_e is invested in it. We associate the availability status of an edge (*i.e.*, whether the edge remains operational or not) to the value of a random variable ξ_e, which is equal to 1 if the edge e is operational and 0 otherwise. A network configuration is given by the availability status of each network link.

Assuming that we are able to enumerate all the possible scenarios S of network configuration, the problem may be formulated as follows:

$$(\boldsymbol{P}) \quad Min \quad \sum_{e\in E} r_e x_e + \sum_{s\in S} p_s \left(\sum_{e\in E} c_e y_{es} + \sum_{i\in N} d_i z_{is} \right) \qquad (2.1)$$

$$subject\ to: \quad Ax \le b \qquad (2.2)$$

$$W_s y_s + z_s = h_s \qquad \forall s \in S \qquad (2.3)$$

$$p_s = \prod_{e\in E} \left(\mathrm{p}_{es}^C + \left(\mathrm{p}_{es}^I - \mathrm{p}_{es}^C \right) \cdot x_e \right) \qquad \forall s \in S \qquad (2.4)$$

$$y_{es} \le u_e \xi_{es} \qquad \forall s \in S, \forall e \in E \qquad (2.5)$$

$$x \in \{0,1\}^{|E|};\ y, z \in \mathbb{R}^+ \qquad (2.6)$$

where:

ξ_{es}	realization of random variable ξ_e in scenario s;
p_{es}^C	probability of the availability status of edge e in scenario s, given that no investment is made on it (i.e., $P(\xi_e = \xi_{es} \vert x_e = 0)$ or, alternatively, $q_e^C \cdot \xi_{es} + (1 - q_e^C) \cdot (1 - \xi_{es})$;
p_{es}^I	probability of the availability status of edge e in scenario s, given that a

reinforcement investment is made on it (i.e., $P(\xi_e = \xi_{es} | x_e = 1)$ or, alternatively, $q_e^I \cdot \xi_{es} + (1 - q_e^I) \cdot (1 - \xi_{es})$;

d_i penalty cost for the non-fulfillment of demand of vertex i;

p_s continuous variable equal to the probability of scenario s ;

x_e binary variable which is equal to 1 if an investment is to be made on edge e, 0 otherwise;

y_s vector of continuous flow variables of scenario s;

z_s vector of continuous slack variables for the demand and supply of each vertex in scenario s.

The objective function (2.1) to be minimized provides the sum of deterministic costs incurred in the first stage due to decisions of reinforcement investments and expected second-stage costs of routing commodities through the network and demand curtailment. Expressions (2.2) and (2.3) represent, respectively, the sets of first-stage constraints (such as budget limitations, minimum investment in each region, etc.) and second-stage constraints (such as mass-balance equations on the realized network configuration of each scenario). Expression (2.4) defines variables p_s as a function of investment decision variables x_e and constraint (2.5) determines the upper bound of the flow in edge e, according to the realization of the random variable ξ_e in scenario s.

Problem (2.1) – (2.6) is a mixed-integer nonlinear program for which solution methods are usually not guaranteed to find a global optimal solution. In particular, there are three main difficulties associated with its formulation that prevent existing algorithms to obtain global optimal solutions. These obstacles are briefly described below:

- **Non-linearity due to product of first and second stage variables.** In standard stochastic programming problems the probability of a scenario is known and it thus usually becomes a coefficient of the objective function. In the case of the class of problems being studied in this work, the expression for the expected value of second stage costs – $\sum_{s \in S} p_s (\sum_{e \in E} c_e y_{es} + \sum_{i \in N} d_i z_{is})$ – involves the product of first stage variables p_s – since, as described earlier, first stage decisions affect the probability of occurrence of each possible outcome – and second stage variables y_{es} and z_{is}.
- **Non-linearity due to the expression for the scenarios' probabilities.** A second source of non-linearity arises from the expression that defines variables p_s themselves, which represent the probabity of occurrence of each possible network configuration after taking into account first stage investment decisions. In this case, the expression involves non-linear terms of order up to $|E|$ due to products of binary variables x_e: $p_s = \prod_{e \in E} \left(\mathrm{p}_{es}^C + \left(\mathrm{p}_{es}^I - \mathrm{p}_{es}^C \right) \cdot x_e \right)$. These non-linear terms arise from the product of the probability of occurrence of the outcome of each random variable that composes a scenario.
- **Scenario generation.** The vast majority of stochastic programming models deal with random variables whose probability distribution is independent of the decision variables. This *a priori* knowledge of the joint probability distribution allows one to obtain scenarios for the realization of the random variables and their respective probabilities of occurrence – either by sampling from it in a Monte Carlo fashion or by constructing them based on a given criteria (*e.g.*, moment matching such as in Kaut and Wallace 0 or minimization of distances between probability measures – Romisch 0 or Hochreiter and Pflug 0) – which may then be used to numerically compute the expectation of second stage costs. Since the probability distribution of the random variables is not known beforehand in this particular problem (*i.e.*, it can only be computed after first stage decisions are determined), one cannot rely on existing scenario generation methods.

3. Proposed methodology

In this Section, a reformulation scheme which overcomes the difficulties associated with the existence of non-linear terms in the problem formulation will be presented, along with a way to circumvent the impossibility of applying usual scenario generation methods.

3.1 Reformulation scheme and cut generation algorithm

The product between variables $\mathrm{p_s}$ and $\mathrm{y_{es}}$ in the objective function may be removed by observing that the feasible regions of the second-stage problems – sets of constraints (2.3) and (2.5) – are decoupled from first-stage variables. The second-stage problem of each scenario may then be solved independently of the others:

$$\forall \mathrm{s} \in \mathrm{S}, \mathrm{g_s} = \quad \text{Min} \quad \sum_{e\in E} c_e y_{es} + \sum_{i\in N} d_i z_{is} \tag{3.1}$$

$$\text{subject } to\text{:} \quad W_s y_s + z_s = h_s \tag{3.2}$$

$$y_{es} \leq u_e \xi_{es} \qquad \forall e \in E \tag{3.3}$$

$$y, z \in \mathbb{R}^+ \tag{3.4}$$

As shown above, we denote by $\mathrm{g_s}$ the value of the optimal solution of problem (3.1) – (3.4) for a given scenario s, which then allows us to re-write problem (2.1) – (2.6) as follows:

$$(\boldsymbol{P_1}) \qquad \text{Min} \quad \sum_{e\in E} r_e x_e + \sum_{s\in S} p_s g_s \tag{3.5}$$

$$\text{subject } to\text{:} \quad Ax \leq b \tag{3.6}$$

$$p_s = \prod_{e\in E} (\mathrm{p}_{es}^C + (\mathrm{p}_{es}^I - \mathrm{p}_{es}^C) \cdot x_e) \qquad \forall s \in S \tag{3.7}$$

$$x \in \{0,1\}^{|E|} \tag{3.8}$$

In the following, without loss of generality, we assume that $\mathrm{g_s} \geq 0, \forall \mathrm{s} \in \mathrm{S}$. A remaining difficulty in solving problem (3.5) – (3.8) lies on the product of binary variables $\mathrm{x_e}$ in the definition of variables $\mathrm{p_s}$ – each equation defined in the set of constraints (3.7) is a polynomial of order |E|. The particular structure of the polynomials defined in the set of constraints (3.7) – specifically, the fact that they may be written as the product of linear terms in the form $\mathrm{a} \cdot \mathrm{x} + \mathrm{b}$, where $\mathrm{a} > 0$ and $\mathrm{a} + \mathrm{b} > 0$ – allows for the application of the proposed convexification technique described below. By relying on the fact that $\mathrm{a} = \mathrm{b} \cdot \mathrm{c} \rightarrow \mathrm{a} = \exp(\ln \mathrm{b} + \ln \mathrm{c})$, each equation in (3.7) may be re-written as:

$$\mathrm{p}_s = \exp\left(\sum_{e\in E} \ln(\mathrm{p}_{es}^C + (\mathrm{p}_{es}^I - \mathrm{p}_{es}^C) \cdot x_e)\right) \tag{3.9}$$

Since x is a vector of binary variables, the expression within the summation operator may also be re-written in such a way that variables $\mathrm{x_e}$ are not part of the logarithmic expression. This is accomplished by observing that the argument of each logarithm is $\mathrm{p_{es}^C}$ if $\mathrm{x_e}$ is equal to 0 and $\mathrm{p_{es}^I}$ otherwise, leading to:

$$\mathrm{p}_s = \exp\left(\sum_{e\in E}\{\ln(\mathrm{p}_{es}^C) + [\ln(\mathrm{p}_{es}^I) - \ln(\mathrm{p}_{es}^C)]\cdot x_e\}\right) \qquad (3.10)$$

A continuous variable may then be defined as the logarithm of the probability of each scenario, thus being an affine function of variables $\mathrm{x_e}$ (this auxiliary variable is introduced for ease of presentation but it is not strictly necessary):

$$w_s = \ln(\mathrm{p}_s) = \sum_{e\in E}\{\ln(\mathrm{p}_{es}^C) + [\ln(\mathrm{p}_{es}^I) - \ln(\mathrm{p}_{es}^C)]\cdot x_e\} \qquad (3.11)$$

Having the value of the natural logarithm of the probability of a scenario given by expression (3.11), the actual value of its probability (i.e., the value of $\mathrm{p_s}$) may be obtained by a piecewise linear approximation of the exponential function. Since we are dealing with minimization problem and the exponential function is convex, this approximation may be represented by a set of linear constraints which can be incorporated into the problem:

$$(\boldsymbol{P_2}) \quad Min \quad \sum_{e\in E} r_e x_e + \sum_{s\in S} g_s \hat{p}_s \qquad (3.12)$$

$$\text{subject } to: \quad Ax \le b \qquad (3.13)$$

$$w_s = \sum_{e\in E}\{\ln(\mathrm{p}_{es}^C) + [\ln(\mathrm{p}_{es}^I) - \ln(\mathrm{p}_{es}^C)]\cdot x_e\} \qquad \forall s \in S \qquad (3.14)$$

$$\hat{p}_s \ge \alpha_k + \beta_k \cdot w_s \qquad \forall s \in S, \forall k \in K \qquad (3.15)$$

$$\hat{p} \in \mathbb{R}^+, w \in \mathbb{R} \qquad (3.16)$$

$$x \in \{0,1\}^{|E|} \qquad (3.17)$$

where:

K	set of linear constraints that approximate the exponential function
α_k, β_k	coefficients of the k-th segment used to approximate the exponential function
w_s	continuous variable equal to the natural logarithm of the probability of scenario s
$\hat{p}_s$	continuous variable equal to the approximation of the probability of scenario s

Given an approximation to the exponential function (i.e., given a set of cuts in the form $\mathrm{y} \ge \exp(\mathrm{w_0}) + \exp(\mathrm{w_0})\cdot(\mathrm{w} - \mathrm{w_0})$ that provide a piecewise linear approximation to the exponential function) and assuming it is computationally feasible to enumerate and solve the second stage problems for all possible network configurations, one should be able to solve problem (3.19) – (3.24) using commercially available solvers. However, the number of cuts necessary for a reasonable approximation of the exponential function may grow to be very large, leading to computational difficulties. The observation that only a small fraction of these cuts will be active at the optimal solution of the problem – only |S| cuts represented in the set of constraints (3.15) will be actually binding – naturally points towards the design of an algorithm that dynamically generates the cuts to construct the piecewise linear approximation to the exponential function. The following algorithm (ALG1) may be used in order to obtain a solution to the problem for which the gap of the global optimum solution is less or equal to ε [1]:

```
Initialize the set of cuts K = ∅, the lower bound LB = −inf, upper bound UB = +inf
and define the maximum percentage error ε
While |(UB − LB)/UB| > ε
    Solve problem P_2 with the currently defined set of cuts K
    Set LB = v(P_2)
    Set UB = ∑_{e∈E} r_e x_e^* + ∑_{s∈S} g_s · exp(w_s^*)
    For each scenario s ∈ S
        Add the cut α_k = exp(w_s^*) · (1 − w_s^*) and β_k = exp(w_s^*) to the cut set K
    End For
End While
```

The algorithm works by gradually constructing a better approximation of the second stage cost function through the addition of cuts around the optimal values of variables w_s^* found at each iteration.

3.2 Scenario generation

While the number of possible network realizations is computationally tractable, the algorithm presented in the previous Section may be used in order to obtain a solution which is within a tolerance level ε from the global optimum of the original problem. However, if one wants to be able to solve large-scale problems, it becomes imperative to have an estimate of the expected value of the second stage cost function which is not based on the complete enumeration of all possible network configurations.

As previously mentioned, the decision-dependent nature of the uncertainties involved in the problem makes it impossible to utilize traditional scenario generation methods such as Monte Carlo sampling, moment matching or minimization of distances between probability measures. We propose to overcome this obstacle by merging the concepts from importance sampling into a stochastic programming framework, as discussed next.

In statistics, importance sampling is a technique used to estimate the properties of a certain distribution while only having samples drawn from a different one. In the context of simulation studies, importance sampling is usually employed as a variance reduction technique used in conjunction with the Monte Carlo method.

As detailed in Rubinstein (1981) the method relies on a simple observation to compute the expected value of a random variable $X \sim F_1(x)$ based on samples from another distribution $F_2(x)$:

$$\mathbb{E}_{f_1}\{x\} = \int_x x f_1(x)dx = \int_x x\frac{f_1(x)}{f_2(x)} f_2(x)dx = \mathbb{E}_{f_2}\left\{x\frac{f_1(x)}{f_2(x)}\right\} \tag{3.18}$$

For a given set of samples x_i $(i = 1, \dots, N)$ drawn according to a probability density function $f_2(X)$, the importance sampling estimator of the mean of distribution $f_1(X)$ is then defined as:

[1] $v(P_2)$ denotes the value of the optimal solution of problem P_2 and γ^* indicates the value of variable γ at the optimal solution

$$\hat{\mu}_X^{IS} = \frac{1}{N}\sum_{i=1}^{N} x_i \cdot \frac{f_1(x_i)}{f_2(x_i)} \tag{3.19}$$

Following expression (3.19), each sample is weighted differently based on the likelihood ratio, i.e. the ratio between the probability of occurrence of that sample under the distribution of interest and the one from which the samples were drawn. This estimator is proved to be consistent – it converges to μ_X with probability 1 as the sample size grows to infinity – and unbiased – its expected value is μ_X, whatever the sample size.

Although the final (post-investment) probability distribution of the availability of the edges is not known a priori, the initial distribution (i.e., the one which does not consider any reinforcement investments) may be used to generate scenarios of network configuration, for which the probability of occurrence may be easily calculated. Additionally, since the convexification technique previously discussed makes it possible to compute the probability of occurrence of any scenario given the first-stage investment decisions (or, at least, an approximation to its value), we may join these pieces of information in order to compute the importance sampling estimator of the expected value of the second stage cost function. Problem (3.12) – (3.17) is thus reformulated in a way which does not require the full enumeration of all possible network configurations but relies on a smaller subset of randomly generated scenarios, as shown below:

$$(\boldsymbol{P_3}) \quad \text{Min} \quad \sum_{e\in E} r_e x_e + \frac{1}{|S|}\sum_{s\in S} g_s\left(\frac{\hat{p}_s}{p_s^{INI}}\right) \tag{3.20}$$

$$subject\ to: \quad Ax \leq b \tag{3.21}$$

$$w_s = \sum_{e\in E}\{\ln(\mathrm{p}_{es}^C) + [\ln(\mathrm{p}_{es}^I) - \ln(\mathrm{p}_{es}^C)] \cdot x_e\} \qquad \forall s \in S \tag{3.22}$$

$$\hat{p}_s \geq \alpha_k + \beta_k \cdot w_s \qquad \forall s \in S, \forall k \in K \tag{3.23}$$

$$\hat{p} \in \mathbb{R}^+, w \in \mathbb{R} \tag{3.24}$$

$$x \in \{0,1\}^{|E|} \tag{3.25}$$

where:

p_s^{INI}	probability of sampled scenario s, calculated based on the initial probability distribution of the availability of each edge, i.e. $p_s^{INI} = \prod_{e\in E} p_{es}^C$

Based on a set of scenarios of network realizations, sampled according to the initial probability distribution of the edges' availabilities, a solution to problem (3.20) – (3.25) may be found using the algorithm described above, appropriately modified to solve problem P_3.

4. Computational results

Computational tests were perfomed to analyze the performance of the proposed reformulation scheme and algorithm. All tests were conducted on a Pentium 4 3.00 GHz computer with 2 GB of RAM. Models and algorithms were implemented using the modeling language MOSEL and solved by XPRESS 19.00.04.

The first results are those obtained for the set of instances described in Viswanath et al.(2004). These are all small-size problems which served as a "proof of correctness" for the proposed methodology. Since no other work in the literature deals with the problem in its original form (Fan, Liu (2010) and Liu, Fan, Ordonez (2006) dismiss the probabilistic nature of the problem by assuming that investment on an edge completely eliminates the probability of that edge failing afterwards), several other instances were created in order to assess the performance of the methodology in solving medium and large-size instances.

All the instances solved in Viswanath et al. (2004) refer to a graph which contains 4 vertices and 5 edges, as depicted in Figure 1. All 28 instances were solved to optimality in less than 1.0 second and average solution time was 0.313 second. Additionally, a total of 30 medium-sized randomly-generated instances (consisting of networks with up to 12 edges) were solved in under 1000 seconds by the proposed algorithm with full scenario enumeration and gap tolerance level set to no more than 1%.

We observed that the total time required to solve the instances increased very rapidly with respect to the number of edges in the network – just as an illustration of this fact, the average time needed to solve the instances with 11 edges was 40.2 seconds, while the average time consumed by the algorithm in solving the instances with 12 edges was 368.1 seconds. A critical example is provided by an instance of the problem with 10 vertices and 15 edges (and, consequently, 32,768 possible scenarios of network configuration) which was solved by full scenario enumeration. Figure 2 presents the performance of the algorithm – data points represent the upper and lower bounds obtained at each iteration:

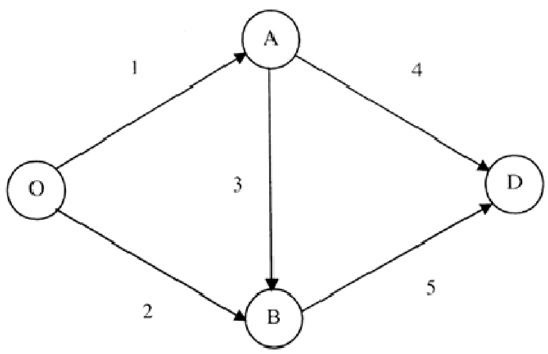

Figure 1 – Graph corresponding to the instances solved in Viswanath et al. (2004)

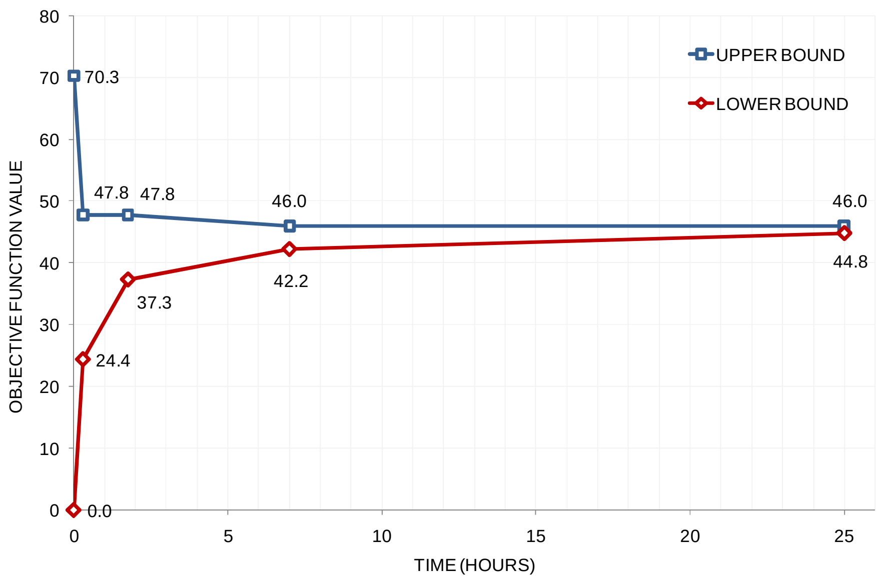

Figure 2 – Algorithm perfomance on an 15-edge instance with full scenario enumeration

In the case of the 15-edge instance, it took a total of 25 hours for the algorithm to narrow the gap down to 2.57%, which clearly leads to the conclusion that full scenario enumeration is currently not a viable option when one tries to solve large scale problems and a sample-based version of it becomes a necessity. Another 18 larger-sized instances, with the number of edges ranging from 15 to 40, were randomly generated and solved by the proposed approach – Table 1 reports obtained results2 .

The instances with 15 edges (v10e15_1, v10e15_2 and v10e15_3) all refer to the same graph of the example for which the convergence of the algorithm was shown in Figure 2. Each one of them was solved using a different set of 500 scenarios (out of the 32,768 possible network configurations), sampled according to the initial probability distribution of the edges' availabilities. It is interesting to observe that even though the number of scenarios used in these instances is significantly smaller than the total number of possible scenarios, the solutions found for these problems in under 60 seconds have an objective function value which is close to that found after 25 hours in the case of full scenario enumeration.

Table 1 – Results for the large-size instances

Id	*# Scen*	*# TotScen*	*UB*	*LB*	*% Gap*	*# Iter*	*MainTime*
v10e15_1	500	3.28E+4	44.57	44.40	0.37%	6	41.0
v10e15_2	500	3.28E+4	44.84	44.84	0.00%	6	27.5
v10e15_3	500	3.28E+4	47.04	46.92	0.25%	6	27.5
v10e20_1	500	1.05E+6	81.83	81.72	0.13%	8	1,169.7
v10e20_2	500	1.05E+6	81.47	80.76	0.87%	9	2,725.2
v10e20_3	500	1.05E+6	81.69	81.33	0.45%	10	1,713.1
v10e20_4	500	1.05E+6	78.47	78.16	0.39%	10	3,164.3
v10e20_5	500	1.05E+6	77.33	77.33	0.00%	10	4,028.0
v12e25_A	300	3.36E+7	75.53	74.96	0.75%	8	2,528.2
v12e25_B	300	3.36E+7	52.44	52.10	0.65%	12	3,133.6
v12e25_C	300	3.36E+7	70.32	70.19	0.18%	11	1,882.2
v12e25_D	300	3.36E+7	43.10	42.93	0.41%	8	810.2
v13e30_1	200	1.07E+9	32.44	32.32	0.36%	9	516.4
v13e30_2	200	1.07E+9	38.86	38.68	0.47%	11	6,332.0
v13e30_3	200	1.07E+9	32.41	32.41	0.01%	7	1,086.4
v13e30_4	200	1.07E+9	33.17	32.90	0.84%	9	1,095.7
v13e30_5	200	1.07E+9	34.46	34.17	0.84%	9	3,457.3
v16e40_1	200	1.10E+12	19.56	19.47	0.48%	7	4367.5

5. Conclusions

This works aims at contributing to the solution of the network reinforcement problem within the area of humanitarian logistics, which is formulated as a MINLP and characterized as a stochastic programming problem with endogenous uncertainties. The proposed re-formulation scheme overcomes the nonlinearities that arise in the original

[2] *#TotScen* indicates the total number of network configuration scenarios, *#Scen* indicates the number of scenarios actually used when solving the problem, *UB* reports the value of the best solution found while *LB* indicates the value of the solution to the last approximated problem, *% Gap* presents the percentage gap between the upper and lower bounds; *# Iter* indicates the number of iterations of the algorithm needed to reach the final solution, and *MainTime* report the total time for the convergence of the algorithm.

formulation presented in the literature and the incorporation of the importance sampling concepts allows us to solve large otherwise-untractable instances of the problem by using sample scenarios even though the final probability distribution of the random variables is not known a priori. The proposed approach was able to solve the instances available in the literature in very short time. Additionally, larger instances of the problem were created in order to assess the performance of the developed algorithms solutions within 1% of the global optimal have been found in reasonable time.

Regarding improvements on the specific problem discussed in this work, there are some issues that can be dealt with more efficiently such as the solution of very similar problems in each iteration of the algorithm or the adoption of a branch-and-cut framework.

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