INTEGRATING PRODUCTION AND DISTRIBUTION DECISIONS IN THE SPECIALITY OILS SUPPLY CHAIN

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ABSTRACT

We propose linear programming models for decoupled and integrated planning in a divergent supply chain for specialty oils. The optimization problem involves decisions on production, inventory, internal transportation, sales and distribution to customers. The integrated objective is to maximize contribution. In the decoupled approach, an internal price system pursues to align sellers with this objective. We solve numerical examples of the models to illustrate potential effects of integration and coordination and discuss the advantages and disadvantages of the integrated over the decoupled approach. While the total contribution is higher in the integrated approach, the sellers' contribution may be lower. We suggest contribution sharing rules to make both the company and sellers better off under the integrated planning.

KEYWORDS. Supply chain management. Integrated planning. Contribution sharing.

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1. Introduction

Integrating decisions about production with other functions in the supply chain, such as inventory and distribution, has proved to be of significant relevance in organizations. An important body of Operations Research literature has been devoted to this issue, as reviewed by Erengüç et al. (1999). The basic idea of an integrated model is to simultaneously optimize decision variables of different functions that have traditionally been optimized in a sequence where the output of one stage was used as the input to other stage (Sarmiento and Nagi, 1999). Aligning decisions under the same goal can be challenging when the objectives of the different functions are in conflict.

In this paper, we address a problem of tactical planning in a divergent supply chain. Our motivation comes from a project in which we are working with a company in the speciality oils industry. The logistics network is composed of refineries, hubs, depots and sales offices. Although owned by the company, the sales offices are managed independently and the decision on how to ship to customers is decentralized. According to the demand they observe, the sellers make decisions on type and amount of products to order, and from which storage location to order from. This decision is mainly driven by an internal price set by the company and the distribution cost calculated by the seller. This price is set for each product and each location where it is stored. After a sale is realized, the seller receives a percentage of the contribution margin (revenue minus the internal price and minus the cost of distribution to customers), and the rest of the revenue is received by the company itself.

We formulate linear programming models to represent this supply chain, considering decisions on production, inventory, internal transportation, sales and distribution to customers. In a first approach, we propose decoupled models to represent the situation where sales and distribution to customers are decided separate from the rest of the functions in the supply chain. Then, we integrate all the decisions in the same model and analyze its potential to improve the performance in comparison to the decoupled models.

Integrating planning has been one of the main topics studied by recent literature in the oil supply chain. Pinto et al. (2000) work on planning and scheduling applications for refinery operations. Neiro and Pinto (2004) propose a model for a petroleum supply chain in the context of the Brazilian company Petrobras, integrating sources, terminals, refineries, distribution centres and consumers. Bengtsson et al. (2011) integrate production and logistics decisions under uncertainty in ship arrivals. Guyonnet et al. (2009) explore the benefits of an integrated model involving unloading, oil processing, and distribution. In these works, one of the main challenges is given by the numerous non-linear constraints appearing from computing the properties of the products after being processed. A recent overview of refinery planning and scheduling by Bengtsson and Nonås (2010) have identified the handling of non-linearities as one of the main issues in the agenda for future work. A distinction of the problem we deal with is that fixed and unique recipes are used to mix each final product from semi finished products. This characteristic allows us to approach the problem by linear programming, in both the decoupled and the integrated approach. A second distinction of our problem is the sales mechanism involved in the supply chain. Normally, in the oil planning literature it has been assumed that the objectives of the sales units are aligned with the objectives of the whole company. In the decoupled version of the problem approached in this article, we give insights in the case when both parts are not aligned. This has been a research topic in other industrial contexts (e.g. Ouhimmou et al. 2008; Feng et al. 2008, 2010). As for the integrated planning approach, while it results in higher total contribution, the sellers' contribution may be lower. The agreement among the actors in the supply chain has been identified by Erengüç et al. (1999) as a particularly important issue on the integration of production and distribution, because these agreements will determine to a large extent whether each component of the chain will be motivated to achieve the cost reductions by integrating decisions across the chain. In the numerical examples of our problem, we discuss contribution sharing rules that make both the sellers and the company better off in the integrated case.

2. Specialty oils supply chain

The oil industry has been identified as a typical example of divergent supply chain (Viswanadham and Raghavan, 2000; Lasschuit and Thijssen, 2004). This is the case of the supply chain for speciality oils that we face in our problem, which is characterized by a divergent product structure as well as a divergent physical structure. A representation of the supply chain is presented in Figure 1. Next, we describe its main parts.

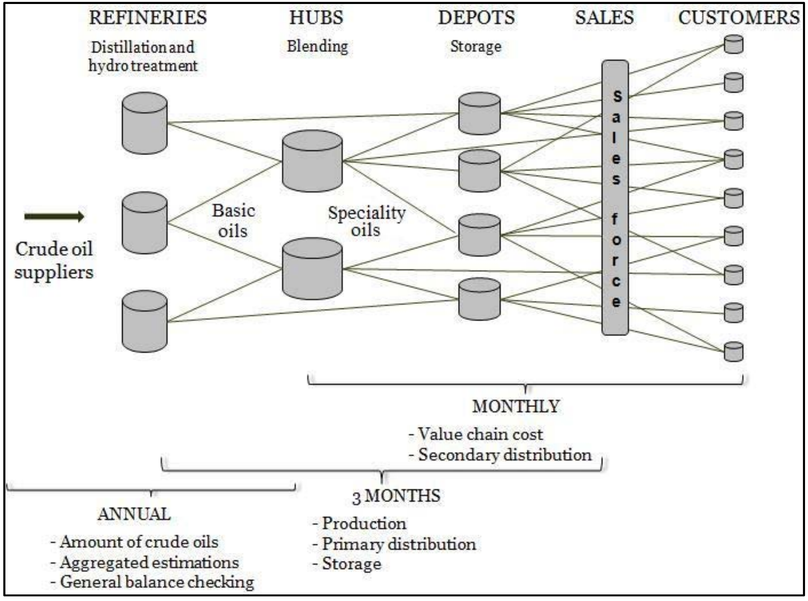

Figure 1: Supply chain for speciality oil products and planning levels.

- **Refineries and products**

The refineries are supplied with crude oil from external suppliers. There are different types of crude oil, some of them containing more percentage of one or another component. In the refineries, the crude oils are exposed to a series of processes, in order to generate saleable products. There are two product segments, that we call *basic oil products* and *speciality oil products* (or simply *basic oils* and *speciality oils*). The processes in the refineries and hubs differ somewhat for different products, but they can be simplified to three steps: distillation, hydrotreatment and blending.

During the distillation process, the crude oil is divided into several fractions. The characteristics of the fractions depend on which crude oil and *run-mode* are used. The run-mode defines the division between the fractions and the generation of different distillates. This determines the characteristics of the different fractions, for instance, in terms of the hydrocarbons that will be contained within them, viscosity and point of ignition. Given a run-mode and a type of crude oil, the proportions between the distillates obtained from the process are fixed. Hence, if it is desired to generate more of a certain distillate, then more of the other distillates obtained in this run-mode will also be generated.

During the hydrotreatment process, the distillates obtained from the distillation receive properties such as density, volatility flashpoint, pour point and colour. The products resulting from this stage correspond to the basic oil products. Some of them are already saleable products, but they can also be used for blending, in order to generate the more sophisticated speciality oil products.

The blending process does not take place at the refineries, as the distillation and hydrotreatment processes do, but in the hubs later in the supply chain. Here, the basic oils are mixed based on recipes in order to create desired properties for the speciality oil products.

- **Storage locations**

The saleable products are transported to depots that serve as storage locations. The hubs also act as storage locations of saleable products. The refineries serve as storage locations, but only for crude oils. From the refineries, some few products will be sent directly to the depots, while most will go through one of the hubs.

- **Sales**

Sellers perform the product transactions with the customers, in a number of local markets. The sellers play an important role in the supply chain, since they decide from which storage location to ship a product in order to satisfy a customer requirement.

- **Customers and demand**

The customers for basic and speciality oil products include a number of firms in construction, road building, pipe coating and automotive industries. Depending on the type of product, different patterns of sales demand are observed; some present high seasonality in demand, with peaks during the northern hemisphere summer, while the demand for other products is more stable. In practice, there is little flexibility to cope with the seasonal variations. High levels of inventory are the result of trying to counteract the seasonality.

- **Transportation**

We distinguish primary and secondary transportation or distribution. The primary distribution corresponds to the transportation of oils within the facilities (refineries, hubs and depots), while the secondary distribution corresponds to the transportation of the saleable products to the customers. The transportation of crude oil from supply sources to refineries is carried out by ships, the same as from refineries to hubs and depots. From hubs to depots and from these locations to the customers, the means of transport varies more, since the volumes are smaller and variable. For the transportation of products to the customers, tank trucks are used more often. On occasion, a combination of ship and truck is used and less often, train and tank trucks are also used.

- **Supply chain planning**

The current planning and management of the supply chain is performed in three main levels, as Figure 1 shows. The strategic planning considers decisions on how much of each crude oil will be used in a year and performs aggregated estimations in order to check that there will be a production balance between the different products. Our research focuses on the tactical level, which includes two stages. One stage is performed by the planners at the refineries and hubs. They perform a production plan, considering a horizon of three months. Decisions involved in the plan are the amount of each product to produce in each location and the primary distribution. A second stage involves the planning of the secondary distribution, from hubs and depots to the customers. This planning is based on a mechanism with internal pricing. For each depot, each product is given an internal price. The sum of this internal price plus the distribution cost from the storage location to the customer results in a figure that we call the *value chain cost*.

- **Value chain cost description**

An internal pricing mechanism considers the assignment of premiums to the sellers, depending on their sales results. One significant part of the premiums is the difference between the sales price and the value chain cost. In consequence, for each sale, a main goal of the sellers is to maximize this difference so as to maximize their own premiums. The value chain cost is calculated as follows:

Value chain cost = Cost of goods sold + Primary distribution cost + Secondary distribution cost.

The *Cost of goods sold* (COGS) includes raw material cost, cost for externally procured products, exchange rates and processing costs in refining and blending. The *Primary distribution cost* is related to the distribution to storage facilities, including depot freight and associated costs of running depots and hubs. The *Secondary distribution cost* includes the transport cost to the customer, a cost for filling the product in drums and other variable costs (such as import taxes). In practice, the company centralizes the calculation of COGS and the Primary distribution cost, resulting in what is called the *internal price*. Hence,

Value chain cost = Internal price + Secondary distribution cost.

For each sale opportunity the secondary distribution cost is calculated by the seller and added to the internal price, thus completing the total value chain cost. The sale price is based on a negotiation between the seller and the customer. When the sale is realized, the seller receives a premium, the main share of which is proportional to the gross result of the sale (revenue minus total value chain cost). The seller has a choice from which depot to supply the customer from (assuming availability). Both the internal price and the secondary distribution cost depend on which depot the product will be shipped from. Hence, it is not necessarily convenient for the seller to order the product from the closest depot (or the one with cheapest transportation cost), because the same product can have different internal prices in different depots. It is also not always best for the seller to order from the depot with the lowest internal price, because the transportation cost from the depot to the customer might be too high. The idea from the company is that this mechanism should be self regulatory and make the sellers act in such a way that, while acting in their own interests, they minimize the total long term cost of distribution for the company. In practice, however, this control mechanism is not exempt from imperfections.

3. Planning models

In this section we formulate linear programming models to represent the tactical planning including the refineries and echelons downstream. In Appendix A, we introduce the notation of sets and parameters that are used through the remainder of the article.

- **Fully decoupled model**

We first consider a fully decoupled case, where there is no coordination between sales and operations units. While the sales units focus on their sales premiums, the operations units focus on supplying at minimum cost. For this case, we develop a decoupled model that is composed of two sub-models: the sales sub-model and the operations sub-model. In the sales sub-model, the sales units do their planning separate from the other echelons of the supply chain, by considering only the sales prices and the value chain costs to maximize their premiums. In the operations sub-model, production and primary distribution are planned together and the decisions from the sales sub-model are considered as input.

The formulation of the sales sub-model is presented in Appendix B. The objective function (1) maximizes the total premium obtained by all the sellers, through the whole planning horizon. Constraint (2) states that each seller will order for each customer at most the amount that this customer demanded, considering that it is possible to serve the same customer from different depots. Constraint (3) corresponds to the non-negativity of the variables.

The formulation of the operations sub-model is presented In Appendix D. Objective function (6) minimizes the total cost through the whole planning horizon up to the depot level (i.e., excluding distribution cost to the customers). The first term is the cost of processing crude oils at the refineries; the second term is the cost of production at the hubs; the third term is the primary distribution transport costs; the next three terms are the total costs of the average inventory per period; the last term is the cost for unsatisfied demand. Constraint (7) sets the initial level of inventories of crude oils, basic oils and speciality oils. Constraint (8) sets the initial values of crude oils refined in each mode and refinery. Constraint (9) sets the initial values of basic oils utilized in each hub for producing each type of speciality oil. Constraint (10) represents the flow conservation of crude oils at the refineries. Constraints (11), (12) and (13) state the conservation of flow of basic oils at the refineries, hubs and depots, respectively. Constraints (14) and (15) give the conservation of flow of speciality oils at the hubs and depots, respectively. Constraint (16) states that the company supplies at most the amount requested by the sellers. Note that the quantities $w^p{}_{agkt}$ ordered by the sellers play the role of demand parameters in the operations sub-model (from the solution to the sales sub-model, the sellers have already decided on the location from which to order). Constraints (17) state non-negativity of the variables.

- **Decoupled model with coordination constraints**

In practice, the company attempts to set conditions in order to achieve certain balance between production and sales of different products from different depots. In Appendix C, we incorporate two coordination constraints into the sales sub-model. Constraint (4) sets an upper bound α on the proportion between two different products that the same seller can order from the same depot. Constraint (5) imposes a maximum quantity λ for each product that can be ordered in total from sellers in the same region.

- **Integrated planning model**

In Appendix E, we propose a model that integrates sales and operations decisions, under a same objective function (18) of maximizing the resulting contribution of sales minus variable costs over the planning horizon. The decision on how to fulfil demand is made centrally, as well as the decisions on production, inventory and primary and secondary distribution.

4. Numerical results

We provide numerical results for the implementation of the models in an instance whose dimension is given in Table 1. We consider a time horizon of three months split in 12-week periods and weekly demand forecasts as given. In order to keep the article within a reasonable extension, we do not provide details on the data. We remark that the decoupled and integrated versions use the same parameter values and, for fair comparison, we have considered a penalization on unfulfilled demand ψ high enough to satisfy all demand, thus the revenue result remains unaffected in all cases. Also, the prices considered are such that it is convenient for the sellers to accept all demand from their customers.

Table 1: Instance description.

Refineries: 2	Hubs: 2	Depots: 3	Crude oils: 2	Basic oils: 2
Specialty oils: 4	Regions: 3	Sellers: 3	Customers: 9	Periods: 12

We used AMPL to code the models and CPLEX 10.0 to solve them on an Intel Core2 Duo 2.27GHz processor with 2GB of RAM. It took less than a second to find the optimal solution. The results are shown in Table 2.

Table 2: Costs, revenue and contribution of the optimal solution to instance I1.

	DM	DCα	Δ%DM	DCλ	Δ%DM	DCαλ	Δ%DM	Integrated	Δ%DM
Down-to-depot costs	30,905	30,350	-1.80 %	30,725	-0.58 %	30,211	-2.25 %	30,811	-0.31 %
2ry distribution costs	4,079	3,930	-3.67 %	3,997	-2.02 %	3,856	-5.48 %	908	-77.74 %
Total costs	34,985	34,280	-2.01 %	34,722	-0.75 %	34,067	-2.62 %	31,719	-9.33 %
Revenue	68,352	68,352	0.00 %	68,352	0.00 %	68,352	0.00 %	68,352	0.00 %
Contribution	33,367	34,072	2.11 %	33,630	0.79 %	34,285	2.75 %	36,633	9.79 %

The second column of Table 2 shows the result obtained for the fully decoupled model (DM), expressed in monetary units. The next column corresponds to the solution of the decoupled model with the coordination constraint (4). The percentage figure corresponds to the difference between this solution and the DM solution. Note a reduction of 2.01% in total costs is achieved when introducing this coordination constraint. The column DCλ shows the results when the coordination constraint (5) is considered. In this case, a reduction of 0.75% in total costs is obtained compared to the DM case. The column DCαλ corresponds to the solution when both coordination constraints (4) and (5) are considered simultaneously, leading to a drop of 2.62% in total costs and an increase of 2.75% in contribution. The last column shows the results of the integrated model, which outperforms all previous ones. A reduction of 9.33% in total costs and an increase of 9.79% in contribution are achieved by the integrated compared to the DM case. Note the secondary distribution cost from the integrated solution is dramatically lower than in the DM case, with a 77.74% reduction. When compared to the DCαλ case, the integrated model leads to a reduction of 6.89% in total costs and an increment of 6.85% in contribution.

An observation concerns the premium amounts obtained by the sellers in each model. Table 3 shows and compares these amounts.

Table 3: Premium amounts obtained by the sellers in each model.

	DM	DCα	Δ%DM	DCλ	Δ%DM	DCαλ	Δ%DM	Integrated	Δ%DM
Premium seller 1	607	607	0.00 %	600	-1.09 %	595	-1.96 %	227	-62.62 %
Premium seller 2	647	626	-3.17 %	641	-0.92 %	626	-3.17 %	453	-29.98 %
Premium seller 3	596	596	0.00 %	596	0.00 %	596	0.00 %	105	-82.42 %
Total premium	1,850	1,829	-1.11 %	1,837	-0.68 %	1,817	-1.75 %	784	-57.59 %

The premiums obtained by the sellers in the integrated model exhibit high differences in comparison to all the other cases. In particular, when comparing with the fully decoupled case, in the integrated case the sellers receives between 29.98% and 82.42% lower premium. It is therefore arguable whether, under the premiums obtained in the integrated model, the sellers would still be encouraged to sell high volumes or not. However, given that the integrated model leads to higher total contribution, finding another mechanism to share the contribution among sellers and the company could keep the incentives for the sellers to achieve high sales volumes. Then, the question arises of how to find an allocation such that all stakeholders are motivated to use the integrated approach.

Table 4 shows the percentage of the contribution that the premiums of the sellers represent in each problem (i.e., the percentage that the results of Table 3 represent over the contribution results presented in the last row of Table 2). The share that the sellers get in the integrated solution is considerably lower than in the other cases.

Table 4: Premium of the sellers as percentage of the total contribution.

	DM	DCα	DCλ	DCαλ	Integrated
%P/C Seller 1	1.82 %	1.78 %	1.78 %	1.73 %	0.62 %
%P/C Seller 2	1.94 %	1.84 %	1.91 %	1.83 %	1.24 %
%P/C Seller 3	1.79 %	1.75 %	1.77 %	1.74 %	0.29 %
%P/C Total	5.54 %	5.37 %	5.46 %	5.30 %	2.14 %

Table 5 shows the *equivalent premium*, that we define as the premium amount obtained by the sellers considering the same percentage they received in the original case of the corresponding problem (DM, DCα, DCλ, DCαλ) but applied to the contribution from the integrated solution. Note that all the equivalent premiums so obtained are greater than the premiums received by the sellers in the original case that were presented in Table 3.

Table 5: Equivalent premium amounts.

	DM	DCα	DCλ	DCαλ
Equivalent premium seller 1	666	652	654	635
Equivalent premium seller 2	710	673	698	669
Equivalent premium seller 3	655	641	650	637
Total equivalent premium	2,031	1,967	2,001	1,942

Table 6 shows the contribution after total premium for each of the other problems according to different ways of calculating the premiums. The contribution after premiums in the integrated solution corresponds to 36,633 - 784 = 35,849.

Table 6: Contribution after premium allocations.

	DM		DCα		DCλ		DCαλ	
Original contribution after premium	31,518		32,243		31,793		32,468	
Integrated's contrib. after equivalent premium	34,602	9.79 %	34,666	7.52 %	34,632	8.93 %	34,691	6.85 %
Integrated's contrib. after identical premium	34,783	10.36 %	34,804	7.94 %	34,796	9.44 %	34,816	7.23 %

The first row of Table 6 shows the contribution after premium in the original case, derived from subtracting the total premium values of Table 3 from the contribution values of Table 2. The second row shows the result of the contribution from the integrated case (36,633) minus the total equivalent premiums of Table 5 and the percentage of improvement achieved by using the equivalent premium rule with respect to the original case. The resulting contribution after premium computed by this rule outperforms the original cases with an increase ranging from 6.85% to 9.79%. The last row of Table 6 shows the result of the contribution from the integrated case (36,633) minus the *identical premium*, that we define as the same absolute premium amount obtained by the sellers in the original case (the total values in Table 3). The resulting contribution after premium in this case is between 7.23% and 10.36% higher than in the corresponding original cases.

Reallocation rules based on equivalent premiums and identical premiums are two examples of simple ways of reallocating the additional contribution among the sellers and the company, such that all the sellers are better off than in the decoupled case while the company also obtains a better result.

More analysis and numerical results reinforcing the contents of this article can be found in Guajardo et al. (2012).

5. Concluding remarks

By using linear programming, we have formulated decoupled and integrated planning models for a divergent supply chain of speciality oil products. The advantages of the integrated approach over the decoupled planning is that the decisions on secondary distribution are made together with previous echelons in the supply chain, thus providing a better match with production and storage units. These advantages lead the integrated model to achieve important decreases in total costs and increases in contribution in comparison to the decoupled models, as we illustrated in the numerical results.

We also discussed the premiums obtained by the sellers. They may obtain lower premiums in the integrated solution, thus the practical situation may not allow an integrated model to be implemented. In order to motivate the sellers, the development of a revenue/contribution sharing principle might be required. This has successfully been developed in other industries (e.g. Frisk et al., 2010), but, to our knowledge, it has not been addressed in oil related literature. We explored two rules for reallocating contribution such that all the sellers and the company were better off in the integrated planning than in the decoupled planning. Developing pricing mechanisms with allocation of premiums is part of our future research agenda. Ideally, such internal prices should generate solutions where the decoupled and integrated models provide the same solutions. In the literature, there are several decomposition schemes that can be useful for this purpose, for example, based on Lagrangian relaxation (Lidestam and Rönnqvist, 2011; Pirkul and Jayaraman, 1998).

We are currently collaborating with a main company in the speciality oils industry. We believe the proposed integrated model has the potential to improve the current supply chain planning, for example, in how to achieve a better mix of products, the timing at which they are produced and how to distribute them. A further research issue is the possibility to delay mixing of some oils to the depots and incorporating the uncertainty of demand. Other possible extension is the integration of the procurement decision, which in our model would be possible to achieve by defining η (the amount of crude oil incoming to refineries) as a decision variable instead of a parameter, and adding the corresponding cost in the objective function. A further issue is the possibility of closing down or opening depots. This requires the model to include binary variables and to assign fixed cost for the use of depots.

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APPENDIX A: NOTATION

Indexes and sets

$a \in A$: set of sellers.
$j \in J$: set of geographic regions.
$a \in L_j$: set of sellers that belong to region j.
$k \in K$: set of customers.
$i \in I$: set of crude oils.
$b \in B$: set of basic oil products.
$s \in S$: set of speciality oil products.
$p \in P$: set of saleable products, the union of set B and set S (basic oils and speciality oils).
$r \in R$: set of refineries.
$h \in H$: set of hubs.
$d \in D$: set of depots.
$f \in F$: set of storage locations for basic oils, the union of sets R, H and D (refineries, hubs and depots).
$g \in G$: set of storage locations for saleable products, the union of set H and set D (hubs and depots).
$m \in M$: set of run-modes in refining process.
$t \in T$: set of periods.

Parameters

$\alpha_{ap\tilde{p}g}$: maximum proportion between the amount of saleable products p and $\tilde{p}$ possible to assign to seller a from location g.
β_a: fraction of the revenue that seller a receives as premium.
δ_{akpt}: demand of customer k to seller a for product p in period t.
η_{irt}: amount of crude oil i incoming to refinery r in period t.
γ_{bs}: amount of basic oil b necessary for producing one unit of speciality oil s.
λ_{pgjt}: maximum amount of product p that the company can sell from location g to region j in period t.
ρ_{bim}: amount of basic oil b generated from one unit of crude oil i at run-mode m.
θ_{pkt}: sale price of one unit of product p to customer k in period t.
ζ_{pgkt}: value chain cost of one unit of product p if it is ordered from location g to be sold to customer k in period t.
ψ_{pkt}: cost for unsatisfied demand of customer k for product p in period t.
C^i_{rmt}: cost of refining one unit of crude oil i in mode m at refinery r in period t.
C^s_{ht}: cost of producing one unit of speciality oil s at hub h in period t.
C_{fgt}: unitary transport cost from location f to location g in period t.
C_{gkt}: unitary transport cost from location g to customer k in period t.
C^i_r: inventory cost from storing one unit of crude oil i in refinery r.
C^b_f: inventory cost from storing one unit of basic oil b in location f.
C^s_g: inventory cost from storing one unit of speciality oil s in location g.
I^i_{r0}: initial inventory of crude oil i at refinery r.
I^b_{f0}: initial inventory of basic oil b at location f.
I^s_{g0}: initial inventory of speciality oil s at location g.
$\bar{Y}^i_{rm0}$: initial amount of crude oil i refined in mode m in refinery r.
$\bar{Y}^h_{s0}$: initial amount of speciality oil s produced at hub h.

APPENDIX B: SALES SUB-MODEL

Decision variables

w^p_{agkt}: amount of saleable product p ordered by seller a from location g to be shipped to customer k in period t.

Max-premium objective function

$$\max Premium = \sum_{a \in A} \sum_{p \in P} \sum_{g \in G} \sum_{k \in K} \sum_{t \in T} \beta_a w^p_{agkt} (\theta_{pkt} - \zeta_{pgkt}) \tag{1}$$

Constraints

$$\sum_{g\in G} w^p_{agkt} \leq \delta_{akpt} \ \ \forall a \in A, p \in P, k \in K, t \in T. \tag{2}$$

$$w^p_{agkt} \geq 0 \ \ \forall a \in A, p \in P, g \in G, k \in K, t \in T. \tag{3}$$

APPENDIX C: COORDINATION CONSTRAINTS

$$\sum_{k\in K} w^p_{agkt} \leq \alpha_{ap\tilde{p}g} \sum_{k\in K} w^{\tilde{p}}_{agkt} \ \ \forall a \in A, p \in P, \tilde{p} \in P, g \in G, t \in T. \tag{4}$$

$$\sum_{k\in K} \sum_{a\in L_j} w^p_{agkt} \leq \lambda_{pgjt} \ \ \forall j \in J, p \in P, g \in G, t \in T. \tag{5}$$

APPENDIX D: OPERATIONS SUB-MODEL

Decision variables

v^p_{agkt}: amount of saleable product p sold from location g to customer k through seller a in period t.
x^p_{fgt}: amount of saleable product p transported from location f to location g in period t.
y^i_{rmt}: amount of crude oil i refined at refinery r in mode m in period t.
y^s_{ht}: amount of speciality oil s produced at hub h in period t.
z^i_{rt}: amount of crude oil i stored in refinery r at the end of period t.
z^b_{ft}: amount of basic oil b stored in location f at the end of period t.
z^s_{gt}: amount of speciality oil s stored at location g at the end of period t.

Min-cost objective function

$$\begin{aligned} \min Cost \quad = \quad & \sum_{m\in M}\sum_{r\in R}\sum_{i\in I}\sum_{t\geq 1} C^i_{rmt} y^i_{rmt} + \sum_{h\in H}\sum_{s\in S}\sum_{t\geq 1} C^s_{ht} y^s_{ht} \\ & + \sum_{p\in P}\sum_{f\in F}\sum_{g\in G}\sum_{t\geq 1} C_{fgt} x^p_{fgt} + \sum_{r\in R}\sum_{i\in I}\sum_{t\geq 1} C^i_r (z^i_{r,t-1} + z^i_{rt})/2 \\ & + \sum_{f\in F}\sum_{b\in B}\sum_{t\geq 1} C^b_f (z^b_{f,t-1} + z^b_{ft})/2 + \sum_{g\in G}\sum_{s\in S}\sum_{t\geq 1} C^s_g (z^s_{g,t-1} + z^s_{gt})/2 \\ & + \sum_{a\in A}\sum_{p\in P}\sum_{k\in K}\sum_{t\geq 1} \psi_{pkt}(\delta_{akpt} - \sum_{g\in G} v^p_{agkt}) \end{aligned} \tag{6}$$

Constraints

$$z^i_{r0} = I^i_{r0} \ \forall r \in R, i \in I; \ z^b_{f0} = I^b_{f0} \ \forall f \in F, b \in B; \ z^s_{g0} = I^s_{g0} \ \forall g \in G, s \in S. \tag{7}$$

$$y^i_{rm0} = \bar{Y}^i_{rm0} \ \forall m \in M, r \in R, i \in I. \tag{8}$$

$$y^s_{h0} = \bar{Y}^s_{h0} \ \forall b \in B, h \in H, s \in S. \tag{9}$$

$$z^i_{r,t-1} + \eta_{irt} = z^i_{rt} + \sum_{m\in M} y^i_{rmt} \ \forall r \in R, i \in I, t \geq 1. \tag{10}$$

$$z^b_{r,t-1} + \sum_{i\in I}\sum_{m\in M} \rho_{bim} y^i_{r,m,t-1} = z^b_{rt} + \sum_{h\in H} x^b_{rht} + \sum_{d\in D} x^b_{rdt} \quad \forall r \in R, b \in B, t \geq 1. \tag{11}$$

$$z^b_{h,t-1} + \sum_{r\in R} x^b_{rht} = z^b_{ht} + \sum_{d\in D} x^b_{hdt} + \sum_{a\in A}\sum_{k\in K} v^b_{ahkt} + \sum_{s\in S} \gamma_{bs} y^s_{ht} \quad \forall h \in H, b \in B, t \geq 1. \tag{12}$$

$$z^b_{d,t-1} + \sum_{r\in R} x^b_{rdt} + \sum_{h\in H} x^b_{hdt} = z^b_{dt} + \sum_{a\in A}\sum_{k\in K} v^b_{adkt} \quad \forall d \in D, b \in B, t \geq 1. \tag{13}$$

$$z^s_{h,t-1} + y^s_{h,t-1} = z^s_{ht} + \sum_{d\in D} x^s_{hdt} + \sum_{a\in A}\sum_{k\in K} v^s_{ahkt} \quad \forall h \in H, s \in S, t \geq 1. \tag{14}$$

$$z^s_{d,t-1} + \sum_{h\in H} x^s_{hdt} = z^s_{dt} + \sum_{a\in A}\sum_{k\in K} v^s_{adkt} \ \forall d \in D, s \in S, t \geq 1. \tag{15}$$

$$v^p_{agkt} \leq w^p_{agkt} \quad \forall a \in A, p \in P, g \in G, k \in K, t \in T. \tag{16}$$

$$\begin{gathered} v^p_{agkt} \geq 0 \ \forall a \in A, g \in G, k \in K, t \in T;\ x^p_{fgt} \geq 0 \ \forall p \in P, f \in F, g \in G, t \in T; \\ y^i_{rmt} \geq 0 \ \forall i \in I, r \in R, m \in M, t \in T;\ y^s_{ht} \geq 0 \ \forall s \in S, h \in H, t \in T; \\ z^i_{rt} \geq 0 \ \forall i \in I, r \in R, t \in T; z^b_{ft} \geq 0 \ \forall b \in B, f \in F, t \in T;\ z^s_{gt} \geq 0 \ \forall s \in S, g \in G, t \in T. \end{gathered} \tag{17}$$

APPENDIX E: INTEGRATED MODEL

We maintain the notation and definitions from the previous section for all parameters, sets and variables, but for explicit differentiation in the decision variable on demand fulfilment between this integrated model and the previous cases, instead of using v^p_{agkt} we use the notation $\bar{v}^p_{agkt}$ as decision variable for the amount of saleable product p sold from location g to customer k through seller a in period t.

Objective Function

$$\begin{aligned} \max Contribution \ = \ & \sum_{a\in A}\sum_{p\in P}\sum_{g\in G}\sum_{k\in K}\sum_{t\geq 1} \bar{v}^p_{agkt}\theta_{pkt} - \sum_{m\in M}\sum_{r\in R}\sum_{i\in I}\sum_{t\geq 1} C^i_{rmt} y^i_{rmt} \\ & - \sum_{h\in H}\sum_{s\in S}\sum_{t\geq 1} C^s_{ht} y^s_{ht} - \sum_{p\in P}\sum_{f\in F}\sum_{g\in G}\sum_{t\geq 1} C_{fgt} x^p_{fgt} \\ & - \sum_{a\in A}\sum_{g\in G}\sum_{k\in K}\sum_{p\in P}\sum_{t\geq 1} C_{gkt} \bar{v}^p_{agkt} - \sum_{r\in R}\sum_{i\in I}\sum_{t\geq 1} C^i_r (z^i_{r,t-1} + z^i_{rt})/2 \\ & - \sum_{f\in F}\sum_{b\in B}\sum_{t\geq 1} C^b_f (z^b_{f,t-1} + z^b_{ft})/2 - \sum_{g\in G}\sum_{s\in S}\sum_{t\geq 1} C^s_g (z^s_{g,t-1} + z^s_{gt})/2 \end{aligned} \tag{18}$$

Constraints

The constraints of the integrated model are the same as constraints (7) - (11); constraints (12) - (17) are also considered, but now the variables v in the formulations are changed to $\bar{v}$ and Constraint (16) is re-formulated as $\sum_{g\in G} \bar{v}^p_{agkt} \leq \delta_{akpt} \ \ \forall a \in A, p \in P, k \in K, t \in T$.