PRICING OF AIR TRAFFIC CONTROL SERVICES THROUGH LEVELING

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RESUMO

Nesta comunicação, um quadro global, baseado na teoria dos jogos e modelagem de redes é considerado para modelizar e analizar o problema da tarificação dos serviços de controle e de gerência do trafégo aéreo-ATC/ATM com um operador. Isso leva à formulação de um problema bi-nível de otimização, onde o líder é o prestador do serviço ATC responsável pela fixação das taxas destes serviços para as companhias aéreas e o seguidor é o setor aéreo que fornece o serviço de transporte aéreo para os usuários finais, os passageiros. A solução analítica do problema inferior leva, através de um processo dito de de nivelamento á resolução de um problema non convexo para o líder. Um algoritmo baseado em redução elíptica é proposto.

PALAVARAS CHAVE: Programação bi-nivel, fluxos em redes, otimização.

Área principal : Programação matématica

ABSTRACT

In this communication a global framework, based on game theory and networks modeling is adopted to define and analyze the Air Traffic Control /Air Traffic Management (ATC/ATM) charges definition problem for a public service provider. This leads to the formulation of a bi-level optimization problem where the leader is the ATC service provider in charge of setting ATC charges to airlines and the follower is the airline sector which provides the air transportation service to end users, air travelers and freight companies. The explicit solution of the lower quadratic linear programming problem is fed back to the upper problem (leveling process). This leads to the formulation of a non convex optimization problem for which a solution scheme based on ellipsoid reduction is proposed.

KEYWORDS. Bi-level programming, flows in networks, optimization

Main area : Mathematical programming

1. Introduction

Along the last decades, many studies in the fields of Operations Research, Systems Management and Applied Economics have been devoted to air transportation planning , tariffs and operations related issues. In the case of the pricing of ATC/ATM services this issue has been linked to the question of the privatization of this activity(G.A.O., 2005), Kuhlman(2005), (G.A.O., 2007) while in general the studies which have been performed are limited to the analysis of the direct effects so that the scope of the adopted models are in general too much limited. This implies that feedback phenomena between the different actors and involved air transportation activities cannot be fully taken into account by the models to perform a comprehensive analysis and to design efficient plans and policies. In this study a global framework, based on game theory and networks modeling (Assad, 1978) is developped to analyze the ATC/ATM charges definition problem in the case of a public ATC/ATM service provider. This situation as well as the situation of a private ATC/ATM service provider have been already considered in the mono dimensional case (Guettaf et al., Part I and Part II, 2009) while a general framework composed of a set of network models associated to the different perceptions of each interacting organization inside the air transportation system has been already proposed (Oumarou, 2006). To insure coherency between these models, compatibility constraints have been introduced. These constraints are mainly relative to interactions between physical flows and available capacities at different levels, as well as flows conservation equations.

This analysis leads to the formulation of a bi-level optimization program (Bard, 1998), (Dempe, 2002). Since the follower problem, which corresponds to the airline sector profit maximization, is a quadratic-linear programming problem, its linear solution is introduced in the formulation of the leader problem, which corresponds to the maximization of final users air transportation demand. Then the leader problem is a linear criterion optimization problem with quadratic constraints. A solution algorithm based on the LMI formalism (Brotcorne et al., 2000) is then proposed for the latter.

2. Pricing for an ATC/ATM public service provider

In the case of a public ATC/ATM service provider, it is considered that the main objective is to promote air transportation for end users, i.e. the passengers (freight is not considered explicitly in this study), through a safe and efficient transportation supply by airlines and ATM authorities. Hence a passenger demand model, reactive to airlines tariffs is introduced to take into account indirect influence of ATC/ATM charges on passengers demand levels over different air transportation markets. Another objective, which is taken into account through inequality constraints, is that the economic performance of the airline sector is not impaired by the retained levels of ATC/ATM charges. Also, ATC/ATM costs related directly with the current traffic situation (investments costs leading to enlarged ATC/ATM capacity to face future traffic situations are not considered) should be adequately covered.

Here to illustrate the proposed approach, we consider the case of a large low level traffic area with the following assumptions (see figure 1):

- some main airports are related by international airlines to external airports;
- local airlines operate air transportation between the different airports of this area;
- a unique ATC authority covers the whole area.

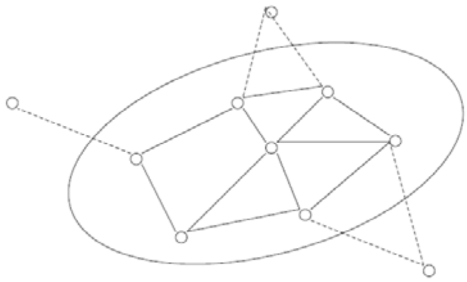

Figure 1. A large low air traffic area

According to the above assumptions, a bi-level program can be established for a unique period of time where the ATC/ATM sector solves the following problem:

$$\max_{v_u \geq 0} \sum_{u \in U} \phi_u + \sum_{e \in E} \phi_e \tag{1}$$

with:

$$\sum_{u \in U} ((v_u - \sigma_u^{\text{int}}) f_u^{\text{int}} + \alpha \lambda_u^{\text{int}} \pi_u^{\text{int}} \phi_u^{\text{int}}) + \sum_{e \in E} ((w_e - \sigma_e^{ext}) f_e^{ext} + \alpha \lambda_e^{ext} \pi_e^{ext} \phi_e^{ext}) - C_{ATC}^F \geq R_{ATC} \tag{2}$$

$$\sum_{u \in U} (\pi_u^{\text{int}} \phi_u - (c_u + v_u) f_u^{\text{int}}) - C_{ALN}^F \geq R_{ALN} \tag{3}$$

while taking into account the profit maximization behaviour of the airline sector and the final users air transportation demand described bellow.
Here ϕ_u is the effective demand for local link u and ϕ_e is the effective demand for border link e, f_u^{int} is the flow of aircraft along local link u, f_e^{ext} is the flow of aircraft along international link e, v_u is the ATC/ATM tariff rate applied over local link u and w_e is the ATC/ATM tariff rate applied to international link e, C_{ATC}^F is the fixed cost resulting from ATC/ATM services in the whole considered region, σ_u^{int} is the average variable cost of ATC/ATM services over local link u, σ_e^{ext} is the average variable cost of ATC/ATM services over international link e, λ_u^{int} is the tax rate applied to users of air transportation on a local flight along link u, λ_e^{ext} is the tax rate applied to users of air transportation on an international flight along link e, α is a subside rate allocated to the ATC/ATM service operator, π_u^{int} is the air ticket mean fare over a local link u, C_{ALN}^F is the total fixed cost of airlines, c_u is the average variable cost of a flight over link u for regional airlines. R_{ATC} is the minimum net result expected for the ATC/ATM service operator (for a public operator it can be taken negative if no feasible solution with positive net result is possible) and R_{ALN} is the minimum net result expected for the airlines sector (it is supposed positive).
In this situation, the budget of international airlines is not considered since they develop extra area activities which cannot be taken into account here. It will be only supposed that the ATC charges remain inferior to a given percentage of the revenue for each external link:

$$w_e f_e^{ext} \leq \eta_e \phi_e^{ext} \pi_e^{ext} \qquad e \in E \tag{4}$$

It is supposed here that the ATC charges are such that:

$$v_u = v_u^0 + l_u\,\delta \qquad u \in U \qquad \text{and} \qquad w_e = w_e^0 \qquad e \in E \tag{5}$$

where l_u is a mix of the length of the connecting route u and of the mean mass of the standard aircraft operating this link, while v_u^0 represents the airport, departure and approach charges for this connection. In the case of international connections, the local common rate δ does not apply.
The objectives assigned to airlines in this study are of a pure economic nature: profit maximization over the considered period of time. Since here planning and strategic issues are considered, the reference unit of time retained is chosen equal to one year. The main concern of this study being with the definition of efficient ATC/ATM charges, the airline sector is taken as a whole, so that market competition between airlines is not contemplated. This is a limitation of the study, which is done in sake of limited complexity, since ATC/ATM charges may have some influence on the equilibrium state of different air transportation markets. However, it is also worth to observe that these airlines are in general represented by a unique entity during negotiations with ATC/ATM authorities.
Here π_u and f_u are given by the solution of the airlines sector problem :

$$\max_{\pi_u, f_u} \quad \sum_{u\in U}(\pi_u\,\phi_u - ((c_u + v_u)\,f_u) - C_{ALN}^F \tag{6}$$

with a potential demand (assumption of concurrency between links with common destinations):

$$D_u = D_0^u - \rho_{uu}\;\pi_u(1+\lambda_u^{\text{int}}) + \sum_{v\neq u}\rho_{uv}\;\pi_v(1+\lambda_v^{\text{int}}) \qquad u \in U \tag{7}$$

and a satisfied demand given by:

$$\phi_u = \max\{0, \min\{q_i\;\; f_u, D_0^u - \rho_{uu}\;\pi_u(1+\lambda_u^{\text{int}})\}\;\} \quad \text{where} \quad u \in U_i, i \in I \tag{8}$$

and

$$0 \le \sum_{u\in U_i} L_u\,f_u \le (\sum_{u\in U} L_u)\; f_{\max}^i \qquad i \in I \tag{9}$$

with

$$U = \bigoplus_{i\in I} U_i \tag{10}$$

Here i$\in I$ is the set of links operated with aircraft of the same capacity q_i, so that the U_i form a partition of U.
Constraint (9) is a fleet constraint showing the limits of the assignment of the fleet of type i to the local air transportation network. It is assumed also that the potential demand on international links is such that (no concurrency between them):

$$D_e = D_0^e - \rho_e\;\pi_e(1+\lambda_e^{ext}) \qquad e \in E \tag{11}$$

3. Supply optimization by airlines

Given the ATC charges, to the airlines sector is supposed to solve the network optimization problem (6) to (9). Relation (7) and (11) can be rewritten as:

$$\underline{D}^{\text{int}} = \underline{D}_0^{\text{int}} - \Delta^{\text{int}} \Lambda^{\text{int}} \underline{\pi}^{\text{int}} \text{ and } \underline{D}^{ext} = \underline{D}_0^{ext} - \Delta^{ext} \Lambda^{ext} \underline{\pi}^{ext} \quad (12)$$

where

$$\Delta^{\text{int}} = [\Delta_{uv}^{\text{int}}] \text{ with } \Delta_{uu}^{\text{int}} = \rho_{uu} \text{ and } \Delta_{uv}^{\text{int}} = -\rho_{uv} \text{ if } v \neq u \quad (13)$$

$$\Delta^{ext} = [\Delta_{uv}^{ext}] \text{ with } \Delta_{uu}^{ext} = \rho_{uu} \text{ and } \Delta_{uv}^{\text{int}} = 0 \text{ if } v \neq u \quad (14)$$

In general there will be:

$$\rho_{uu} >> \rho_{uv} \geq 0 \quad \forall u \in U, \forall v \in U \quad (15)$$

and it is assumed that:

$$\partial \underline{D}^{\text{int}} / \partial \underline{\pi}^{ext} = 0 \text{ and } \partial \underline{D}^{ext} / \partial \underline{\pi}^{\text{int}} = 0 \quad (16)$$

Here

$$\Lambda^{\text{int}} = \underset{u \in U}{diag}\{1 + \lambda_u^{\text{int}}\} \qquad \Lambda^{ext} = \underset{e \in E}{diag}\{1 + \lambda_e^{ext}\} \quad (17)$$

Let :

$$Q = \underset{i \in I}{diag}\{q_i\} \quad (18)$$

then if the following conditions are satisfied ($i \in I$):

$$\underline{L}_i' Q^{-1}(\underline{D}_0^{\text{int}} - \Delta^{\text{int}} \Lambda^{\text{int}} \underline{\pi}^{\text{int}}) \leq \underline{L}_i' \underline{I}_i f_{\max}^i \quad (19)$$

the solution is such that:

$$\underline{f} = Q^{-1}(\underline{D}_0^{\text{int}} - \Delta^{\text{int}} \Lambda^{\text{int}} \underline{\pi}^{\text{int}}) \quad (20)$$

Then the profit of the local airlines sector is given by:

$$P_{ALN} = \underline{\pi}^{\text{int}}{}'(\underline{D}_0^{\text{int}} - \Delta^{\text{int}} \Lambda^{\text{int}} \underline{\pi}^{\text{int}}) - (\underline{c} + \underline{v})' Q^{-1}(\underline{D}_0^{\text{int}} - \Delta^{\text{int}} \Lambda^{\text{int}} \underline{\pi}^{\text{int}}) - C_{ALN}^F \quad (18)$$

The first order optimality conditions become now:

$$\frac{\partial P_{ALN}}{\partial \underline{\pi}^{\text{int}}} = \underline{D}_0^{\text{int}} - (\Delta^{\text{int}} \Lambda^{\text{int}} + \Lambda^{\text{int}} \Delta^{\text{int}}{}') \underline{\pi}^{\text{int}} + \Lambda^{\text{int}} \Delta^{\text{int}}{}' Q^{-1}(\underline{c} + \underline{v}) = \underline{0} \quad (19)$$

while the second order optimality condition becomes:

$$\frac{\partial^2 P_{ALN}}{\partial \underline{\pi}^{\text{int}\,2}} = -(\Delta^{\text{int}} \Lambda^{\text{int}} + \Lambda^{\text{int}} \Delta^{\text{int}}{}') \leq 0 \quad (20)$$

In this situation, the optimal local air fares are given by:

$$\underline{\pi}^{\text{int}} * = P_d \underline{D}_0^{\text{int}} + P_c(\underline{c} + \underline{v})) \quad (21)$$

with

$$P_d = (\Delta^{\text{int}} \Lambda^{\text{int}} + \Lambda^{\text{int}} \Delta^{\text{int}}{}')^{-1} \quad (22)$$

and

$$P_c = (\Delta^{\text{int}} \Lambda^{\text{int}} + \Lambda^{\text{int}} \Delta^{\text{int}}{}')^{-1} \Lambda^{\text{int}} \Delta^{\text{int}}{}' Q^{-1} \quad (23)$$

Then:

$$\underline{f}^{\text{int}} * = Q^{-1}(F_d \underline{D}_0^{\text{int}} - F_c(\underline{c} + \underline{v})) \tag{24}$$

with

$$F_d = (I - \Delta^{\text{int}} \Lambda^{\text{int}} (\Delta^{\text{int}} \Lambda^{\text{int}} + \Lambda^{\text{int}} \Delta^{\text{int}}{}')^{-1}) \tag{25}$$

and

$$F_c = \Delta^{\text{int}} \Lambda^{\text{int}} (\Delta^{\text{int}} \Lambda^{\text{int}} + \Lambda^{\text{int}} \Delta^{\text{int}}{}')^{-1} \Lambda^{\text{int}} \Delta^{\text{int}}{}' Q^{-1} \tag{26}$$

Finally:

$$\underline{\phi}^{\text{int}} * = F_d \underline{D}_0 - F_c(\underline{c} + \underline{v}) \tag{27}$$

4. Reshaping the pricing problem for the public ATC/ATM operator

Taking into account the previous results, the public ATC service provider problem (1), (2) and (3) can be rewritten as:

$$\min_{\underline{v}, \underline{w}} \underline{m}' \underline{v} + \underline{n}' \underline{\pi}^{ext} + r \tag{28}$$

with

$$\underline{m} = \underline{I}^{\text{int}}{}' G \qquad \underline{n} = \underline{I}^{ext}{}' \Delta^{ext} \Lambda^{ext} \qquad r = -(\underline{I}^{\text{int}}{}'(F \underline{D}_0^{\text{int}} - G\underline{c}) + \underline{I}^{ext'} \underline{D}_0^{ext}) \tag{29}$$

under the constraints:

$$\underline{v}' N_{ATC} \underline{v} + \underline{M}_{ATC}' \underline{v} + \underline{P}_{ATC}' \underline{w} + z_{ATC} \geq 0 \tag{29}$$

$$\underline{v}' N_{ALN} \underline{v} + \underline{M}_{ALN}' \underline{v} + \underline{P}_{ALN}' \underline{w} + z_{ALN} \leq 0 \tag{30}$$

$$w_e \ f_e^{ext} \leq \eta_e \phi_e^{ext} \pi_e^{ext} \qquad e \in E \tag{31}$$

where

$$N_{ATC} = (Q^{-1} F_c + P_c' A' F_c) \tag{32}$$

$$M_{ATC} = -(\underline{D}_0^{\text{int}}{}' F_d' Q^{-1} + \underline{\sigma}' Q^{-1} F_c) + (\underline{D}_0^{\text{int}} P_d' + \underline{c}' P_c' A' F_c) - (P_c' A'(F_d' \underline{D}_0^{\text{int}} - F_c \underline{c})' \tag{33}$$

$$z_{ATC} = -\underline{\sigma}' Q^{-1}(F_c \underline{c} - F_d \underline{D}_0^{\text{int}}) - (\underline{D}_0^{\text{int}} P_d' A' + \underline{c}' P_c')(F_d \underline{D}_0^{\text{int}} - F_c \underline{c}) + C_{ATC}^F + R_{ATC} \tag{34}$$

where

$$A = \underset{u \in U}{diag}\{\alpha \lambda_u\} \tag{35}$$

and with

$$N_{ALN} = (Q^{-1} - P_c') F_c \tag{36}$$

$$M_{ALN} = -(\underline{D}_0^{\text{int}}{}' P_d' + \underline{c}' P_c' F_c) + (p_c'(F_d \underline{D}_0^{\text{int}} - F_c \underline{c})' - (\underline{c}' Q^{-1} F_c) + (F_d \underline{D}_0^{\text{int}} - F_c \underline{c})' Q^{-1} \tag{37}$$

$$z_{ALN} = -\underline{c}' Q^{-1}(F_d \underline{D}_0^{\text{int}} - F_c \underline{c}) + (\underline{D}_0^{\text{int}}{}' P_d' + \underline{c}' P_c')(F_d \underline{D}_0^{\text{int}} - F_c \underline{c}) - (C_{ALN}^F + R_{ALN}) \tag{38}$$

Considering that here it is supposed that the fares relative to international links are established exogenously through international agreements, and to alleviate local airlines, the ATC tariffs over these links can be chosen so that:

$$w_e = \eta_e \phi_e^{ext} \pi_e^{ext} / f_e^{ext} \qquad e \in E \tag{39}$$

In general, N_{ATC} will be definite positive and N_{ALN} will be definite negative, then figure 2 illustrates the two dimensional non convex case.

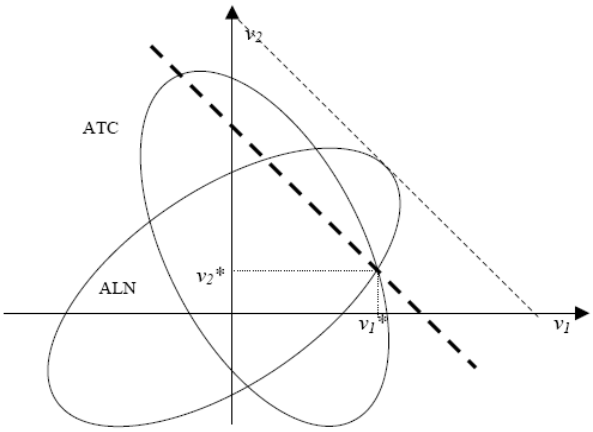

Figure 2. Example of solution in the two dimensional case for public ATC

This figure represents a two dimensional ATC charges space (first quadrant of R^2) where the profitable area for airlines is within ellipse ALN while the profitable area for ATC is outside ellipse ATC. The two lines correspond to constant levels of final demand for air transportation, the bold one corresponding to a maximum of demand.

5. General solution algorithm

The previous problem with a linear criterion and quadratic constraints can be considered to be a special case of a non convex linear program with LMI constraints (Boyd et al., 1994) such as:

$$\min \underline{c}^t \underline{z} \quad \text{under } M^1(\underline{z}) \geq 0 \text{ and } M^2(\underline{z}) \leq 0 \tag{40}$$

where $\underline{c} \in R^m$ is given and where :

$$M^j(z) = M_0^j + \sum_{i=1}^{m} z_i M_i^j \qquad j = 1,2 \tag{41}$$

$M_i^j{}_j, i = 0 \ \ to\ m, j = 1,2$ are symmetric matrices.

This non convex problem can be solved using an ellipsoid algorithm used in the field of LMI's (Dane, 2009).
At start it is supposed that an ellipsoid E_0 *in* R^m contains the feasible set and hence the optimal solution. A cutting plane crossing the center $z_c(0)$ of this ellipsoid is chosen so that the optimal solution lies in one of the half spaces of R^m given by:

$$\left\{ \underline{z} \in R^m : \underline{v}(0)^t (\underline{z} - \underline{z}_c(0)) \leq 0 \right\} \tag{42}$$

where $v(0)$ is a non zero vector of R^m . Then an ellipsoid E_1 with minimum generalized volume and containing the half ellipsoid given by :

$$E_0 \cap \{\underline{z} \in R^m : \underline{v}(0)^t(\underline{z} - \underline{z}_c(0))\} \tag{43}$$

is constructed. The size of this ellipsoid is smaller than the one of the previous ellipsoid and contains the solution. This process can be repeated until a required accuracy is achieved. Given an ellipsoid E_k given by :

$$\{\underline{z} \in R^m : (\underline{z} - \underline{z}_c(k))^t A_k^{-1}(\underline{z} - \underline{z}_c(k)) \le 1\} \tag{44}$$

where A_k is a symmetric definite positive matrix, the minimum volume ellipsoid E_{k+1} containing the half ellipsoid :

$$E_k \cap \{\underline{z} \in R^m : \underline{v}(k)^t(\underline{z} - \underline{z}_c(k)) \le 0\} \tag{45}$$

is given by

$$\{\underline{z} \in R^m : (\underline{z} - \underline{z}_c(k+1))^t A_{k+1}^{-1}(\underline{z} - \underline{z}_c(k+1)) \le 1\} \tag{46}$$

where:

$$\underline{z}_c(k+1) = \underline{z}_c(k) - \frac{1}{m+1} A_k \underline{w}_k \tag{47}$$

with
$$\underline{w}_k = \underline{v}_k / \sqrt{\underline{v}_k{}^t A_k \underline{v}_k} \tag{48}$$

and
$$A_{k+1} = \frac{m^2}{m^2 - 1}(A_k - \frac{2}{m+1} A_k \underline{w}_k \underline{w}_k{}^t A_k) \tag{49}$$

Then considering at step k a point $\underline{y}_k$ in R^m, two cases can be considered:

- either $M^1(\underline{y}_k) \ge 0$ and $M^2(\underline{y}_k) \le 0$, in that case one takes $\underline{v}_k = \underline{c}$ and the half space :

$$\{\underline{z} \in R^m : \underline{v}_k{}^t(\underline{z} - \underline{y}_k) > 0\} \tag{50}$$

can be deleted since there $\underline{c}^t\underline{z} > \underline{c}^t\underline{y}_k$ and points $\underline{z}$ cannot be solution of the optimisation problem.

- or $M^1(\underline{y}_k) < 0$ or $M^2(\underline{y}_k) > 0$, there exists a non zero vector u of R^n such that according to the case:

$$\underline{u}^t(M_0^1 + \sum_{i=1}^m \underline{z}_i M_i^1) < 0 \quad \text{or} \quad \underline{u}^t(M_0^2 + \sum_{i=1}^m \underline{z}_i M_i^j) > 0 \tag{51}$$

then choosing :

$$\underline{v}_{ki} = -\underline{u}^t M_i^j \underline{u}, \; i = 1 \, à \, m \tag{52}$$

we have for every $\underline{z} \in R^m$ such that $\underline{v}_k{}^t(\underline{z} - \underline{y}_k) \ge 0$:

$$\underline{u}^t M^1(\underline{z})\underline{u} = \underline{u}^t M^1(\underline{y}_k)\underline{u} - \underline{v}_k{}^t(\underline{z} - \underline{y}_k) < 0 \;\; \text{or} \;\; \underline{u}^t M^2(\underline{z})\underline{u} = \underline{u}^t M^2(\underline{y}_k)\underline{u} - \underline{v}_k{}^t(\underline{z} - \underline{y}_k) > 0 \tag{53}$$

The feasible set will be in the half space:

$$\{\underline{z} \in R^m : \underline{v}_k{}^t(\underline{z} - \underline{y}_k) < 0\} \tag{54}$$

and $\underline{v}_k$ allows to define the cutting plane at point $\underline{y}_k$. Then the whole process is repeated until the size of the ellipsoid becomes sufficiently small to insure accuracy of the solution. It can be shown (Bard, 1998) that convergence is exponential.

6. Conclusion

In this communication we have considered the problem of pricing ATC/ATM services from the point of view of a public operator while taking into account the behaviour and interests of the airline sector. This has led to the formulation of a bi-level programming problem. The adopted assumptions about the air traffic system, demand by end users and airlines costs structure lead for the lower level to a quadratic programming problem whose analytic solution which can be merged into the upper problem. This process, which is qualified here as leveling, has been considered already in other cases and situations (Mora-Camino, 1983). Finally a solution algorithm for the resulting non convex optimization problem (linear criteria and quadratic constraints), based on ellipsoid reduction, is displayed.

References

Assad, A.A. (1978), *Multicommodity network flows-a survey*, Networks, 8, pp. 37-91.
Bard, J.F. (1998) *Practical bi level optimization: algorithms and applications,* Kluwer Book Series: Non convex optimization and its applications, 30.
Boyd S., L. El Ghaoui, E. Feron and V. Balakrishnan (1994), Linear Matrix Inequalities in Systems and Control Theory, SIAM Studies in Applied Mathematics, Philadelphia.
Brotcorne L., M. Labbé, P. Marcotte and G. Savard (2000), *A bilevel model for toll optimization on a multicommodity transportation network*. Transportation Science 35, 1-14.
Dane B. (2009), *Improved ellipsoid algorithm for LMI feasibility problems*, Int. J. Control, Automation and Systems, vol. 7, nº6, pp. 1015-1019.
Dempe S. (2000), *Foundations of bilevel programming*. Kluwer Academic Publishers, Dordrecht.
G.A.O., (2005), *Characteristics and performances of selected international air navigation service providers and lessons learned from their commercialisation*, United States Government Accountability Office, Repport to Congressional requesters, July, 1-34.
G.A.O., (2007), *Assigning Air Traffic Control Costs to Users*, United States Government Accountability Office, Repport to Congressional requesters, July, 1-32.
Guettaf R., M. Larbani and F. Mora-Camino, (2009), *Pricing of ATC/ATM services with a public provider-part I*, VIII SITRAER, São Paulo, November.
Guettaf R., M. Larbani and F. Mora-Camino,(2009), *Pricing of ATC/ATM services with a private provide-part II*, VIII SITRAER, São Paulo, November.
Kuhlman R., A. Odoni (2005), Taxes fees resurrect debate over privatizing air traffic control, Airline Business Report ,vol.23, N°7, April.
Labbé M., P. Marcotte and G. Savard, (1998) *A bilevel model of taxation and its application to optimal highway pricing*, Management Science 44, 1595-1607.
Mora-Camino F., (1983), *Information Processing, hierarchical structures and management processing: a tentative study*, RAIRO série verte, vol.17, N°2, Mai ,137-156.
Oumarou A. A., F. Mora-Camino (2006), *Pricing in air transportation systems: a multilevel approach*, XIV Congreso Panamericano de Ingenieria de Transito y Transporte, September 20-23, Las Palmas de Gran Canaria, Spain.
Tretheway, M.W., (2009), *Cost Allocation Principles for ATC*, Conference on Air Traffic Economics, Belgrade, September.