

## On the algebraic connectivity of classes of trees with given diameter

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We introduce a class  $\mathcal{P}_{n,k}$  of rooted trees of order  $n$  and diameter  $2k$ . We prove that all trees in this class have the same algebraic connectivity and equal to the algebraic connectivity of the path  $P_{2k+1}$  of  $2k+1$  vertices. Moreover, we study subfamilies of trees obtained from  $T_1, T_2, \dots, T_m \in \mathcal{P}_{n,k}$  and the path  $P_m$  identifying the root vertex of  $T_i$  with the  $i$ -th vertex of  $P_m$ . Finally, we characterize the extremal trees, that is, the trees having the maximal and the minimal algebraic connectivities among all trees in each of these subfamilies.

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