Spectra of two families of generalized Bethe trees and trees of diameter ${\rm five}^1$

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Abstract

A generalized Bethe tree is a rooted unweighted tree in which vertices at the same distance from the root have the same degree. Let $u_1 \left\{ \mathcal{B}_i^{(1)} : 1 \leq i \leq r_1 \right\}$ and $u_2 \left\{ \mathcal{B}_j^{(2)} : 1 \leq j \leq r_2 \right\}$ be trees of generalized Bethe trees having in common the root vertex u_1 and u_2 , respectively. We characterize the eigenvalues of the Laplacian and adjacency matrices of the tree obtained from $u_1 \left\{ \mathcal{B}_i^{(1)} : 1 \leq i \leq r_1 \right\}$, $u_2 \left\{ \mathcal{B}_i^{(2)} : 1 \leq i \leq r_2 \right\}$ and edge $u_1 u_2$ joining the roots u_1 and u_2 . Special attention is given to the algebraic connectivity and spectral radii. Finally, we apply the above results to trees of diameter 5.

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