What we know and what we need to know about ordering of graphs via algebraic connectivity

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Many approaches to ordering graphs by their spectra are contained in the literature. We can find ordering graphs either via the greatest eigenvalues relative to their adjacency matrices (spectral radius or index), as the absolute sum of the adjacency eigenvalues (the energy of a graph) or the second smallest eigenvalues of their Laplacian matrix (the algebraic connectivity) among others. By considering the fact that the algebraic connectivity seems to say something on the connectedness of graphs, several structural properties on graph relative to it have been studied. So, a large number of papers about ordering graphs via algebraic connectivity, mainly about trees and graphs with few cycles, have been published. This paper survey the significative results concerning these topics, trying to focus possible points to be investigated in order to understand the difficulties to obtain a maximum partial ordering (perhaps even a total order) via algebraic connectivity for a class of trees.