Indices for special classes of trees

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In this paper we investigate the index (greatest eigenvalue of the adjacency matrix) of special kinds of trees: the *generalized broom* and the *double broom*. The generalized broom $P(a;i;r)$ is a graph on $n = a + r$ vertices, obtained by attaching $a \geq 1$ pendent vertices to the vertex v_i of the path P_r, with $r \geq 2$. The double broom $D(a;r;b)$ is a graph on $n = a + r + b$ vertices, with $r \geq 2$, obtained by attaching $a \geq 1$ pendent vertices to one extremity of the path P_r and $b \geq 1$ pendent vertices to the other. If the double broom has diameter 3, we say that it is a *double star*.
For each of these classes, we determine conditions for the index to be an integer. Furthermore, we construct infinite families of non integral trees with integer index. The integral case (where all eigenvalues are integers) has been discussed in [1,2]. We also determine a new upper bound for the index λ as a function of the maximum degree Δ, in the theorems below:

Theorem 1. *Let $P(a;i;r)$ be a generalized broom with maximum degree Δ, for $r \geq 2$ and $a \geq 1$. Then its index λ satisfies*

$$\sqrt{\Delta} \leq \lambda \leq \sqrt{\Delta + 1}\ ,$$

if $P(a;i;r) \neq P_{r+1}$, for $r \geq 5$, and $P(a;i;r) \neq P(1;4;8)$ or $P(a;i;r) \neq P(1;i;r)$, for $r \geq 9$ and $3 \leq i \leq r-2$.
Moreover, $\lambda = \sqrt{\Delta}$ if only if $P(a;i;r)$ is a star; $\lambda = \sqrt{\Delta+1}$ if only if $P(a;i;r) = P_5$, or $P(1;4;7)$ or $P(1;3;8)$.

Theorem 2. *Let $D(a;r;b)$ be a double broom with maximum degree Δ, for $r \geq 3$ and $a \geq b \geq 2$. Then its index λ satisfies*

$$\sqrt{\Delta} < \lambda \leq \sqrt{\Delta + 1}\ .$$

Morever, $\lambda = \sqrt{\Delta+1}$ if only if $a = b = 2$ or $a = b \geq 3 = r$.

Key words: double broom, generalized broom, integral graph, integer index.

[1] A.E. Brouwer, R.R. Del-Vecchio, D.P. Jacobs, V. Trevisan, and C.T.M. Vinagre, *Integral trees homeomorphic to a double star*, Bulletin of the ICA **61**, 77-80, 2011.

[2] L. Wang, *Integral trees and integral graphs*, PhD thesis, University of Twente, 2005.