The least eigenvalue of graphs with convex-qp stability number

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A stable set of a graph is a set of mutually non-adjacent vertices. The determination of a maximum size stable set (the maximum stable set problem) and/or the determination of its size (the stability number) in a graph are central combinatorial optimization problems. In general, given a nonnegative integer k, to determine if a graph G has a stable set of size k is NP-complete [3]. A convex quadratic programming upper bound on the stability number of graphs was introduced in [4] and the graphs for which the stability number attains this upper bound were called in [1] *graphs with convex-qp stability number*, where *qp* means quadratic programming. There are infinite families of graphs with convex-*qp* stability number [1]. Usually, the graphs with convex-*qp* stability number are denoted $\mathcal{Q}$-graphs. In this presentation, the properties of the least eigenvalue of $\mathcal{Q}$-graphs are surveyed and their extensions to the maximum size k-regular induced graph problem are presented. Some open problems are also proposed. Notice that the least eigenvalue of a graph plays a central role in the characterization of $\mathcal{Q}$-graphs. In fact, if λ_n is the least eigenvalue of a graph G, then G is a $\mathcal{Q}$-graph if and only if there exists a stable set $S \subset V(G)$ such that $-\lambda_n \leq \min_{v \in V(G)\setminus S} |N_G(v) \cap S|$ [2], where $N_G(v)$ is the neighborood of v in G.

MSC2000: 05C50, 05C69, 90C27, 90C35.

Keywords: Graph spectra, stability number, programming involving graphs.

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