ON THE DISTRIBUTION OF LAPLACIAN EIGENVALUES OF TREES

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Trevisan, Carvalho, Del Vecchio and Vinagre [5] conjectured that for any tree T with $n > 1$ vertices, the number of Laplacian eigenvalues smaller than the average degree of the vertices, $2 - \frac{2}{n}$, is at least $\lceil \frac{n}{2} \rceil$. In [5] this conjecture is proved for diameter 3 trees and some special cases. Note that it is also true for any star S_n, with $n \geq 2$, since its Laplacian Spectrum is $\{0, 1^{n-2}, n\}$, so it has $n-1$ Laplacian eigenvalues smaller than the average degree.

We prove that a path P_n, with $n \geq 2$ vertices, has exactly $\lceil \frac{n}{2} \rceil$ Laplacian eigenvalues smaller than the average. We also show that the Conjecture holds for any tree that contains, at some extremity, a star S_l, with $l \geq 4$ (that is, a tree that contains a vertex of degree $d \geq 4$ with $d-1$ pendants). In addition, applying an algorithm due to Jacobs and Trevisan [3], we show that it only remains to prove the Conjecture for trees in which every quasi-pendant vertex, no matter what degree, is adjacent to exactly one pendant.

Jacobs and Trevisan's algorithm computes, for any given tree T with n vertices and interval (α, β), how many eigenvalues of T lie within the interval. It is an $O(n)$ algorithm that can be executed directly on the tree, so that the adjacency matrix is not needed explicitly. Their method can readily be adapted to estimate the Laplacian eigenvalues of any given tree. We also show how Jacobs and Trevisan's adapted algorithm can be applied to derive bounds on the number of Laplacian eigenvalues in some chosen intervals. We obtain new bounds and also bounds that were previously known due to Faria [1], Grone, Merris & Sunder [2] and Merris [4], in a straightforward manner.

References

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