A vertex removal and its effects on the algebraic connectivity

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Let G be a simple graph on n vertices. We write $A(G)$ for the adjacency matrix of a graph, $D(G)$ for the diagonal matrix of its vertices degree and $L = D - A$ for the Laplacian of G. We denote $a(G)$ as the second smallest eigenvalue of L, that is, the algebraic connectivity of G, [1]. Consider the graph $G\backslash v$ obtained from G after removing a vertex v and the following function

$$\phi_G(v) = a(G) - a(G\backslash v).$$

In 2010, Kirkland [2] established necessary and sufficient under G such that $\phi_G(v) = 1$. As a consequence of this result, he showed that if v has degree $n-1$, then $\phi_G(v) = 1$. Also, he raised the question:

"What can we say about $\phi_G(v)$ if vertex degree of v is $n-2$?"

In this paper, we studied this problem in the cases where there is a vertex of degree $n-2$ that generates $\phi_G(v) = 0$.

Keywords: vertex removal, Laplacian matrix, algebraic connectivity, Fiedler vector.

References

[1] M. Fiedler, *Algebraic connectivity of graphs*, Czechoslovak Mathematical Journal 98, 298–305, 1973.

[2] S. Kirkland, *Algebraic connectivity for vertex–deleted subgraphs, and a notion of vertex centrality*, Discrete Mathematics 310, 911–921, 2010.