Signless Laplacian of weighted complete graphs and clustering

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Let G be a simple weighted graph on n vertices. We write $A = A(G)$ for the adjacency matrix of a graph and $D = D(G)$ for the diagonal matrix of its degrees, where the degrees are given by the sum of the weights to its vertices. Also, we shall write $L = D - A$ for the Laplacian and $Q = D + A$ for the signless Laplacian of G. A comprehensive survey about $Q-$theory can be found in [2]. We shall denote the second smallest eigenvalue and the smallest eigenvalue of L and Q as $\mu_{n-1}(G)$ and $q_n(G)$, respectively.
It is well-known that the second smallest eigenvalue of the Laplacian matrix, L, is used as a measure of connectivity of the graph since $\mu_{n-1}(G) = 0$ if and only if G is disconnected and an eigenvector related to μ_{n-1} can be used to partition the vertices set into two clusters, [1]. Motivated by computational experiments on complete weighted graphs, we have observed that $q_n(G)$ and its eigenvector have given more information about clustering process comparing to the Laplacian matrix. Completed weighted graphs arise naturally as instances from the Multidimensional Scaling Problem (see [3]), where one of the problems is just the clustering identification. Our current research is related to find out theoretical reasons that could explain our computational results, as it was done in [4] related to Markov Chains.

Keywords: spectral clustering, signless Laplacian matrix, smallest eigenvalue, Multidimensional Scaling Problem.

References

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