Is the algebraic connectivity a good measure for telecommunication network robustness?

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Abstract

A telecommunication network with N nodes and M links can be represented by a simple graph $G = G(V, E)$ of order $N = |V|$ and size $M = |E|$. Some papers use the algebraic connectivity for evaluating network robustness. In this work we discuss robustness definition and evaluation, emphasizing particularly the applicability of the algebraic connectivity for robustness evaluation of telecommunication networks.

Suppose initially that we want to build a tree of order N, and that robustness is a major concern of the design. So, we search for a tree maximizing some robustness measure.

The node connectivity $NConn$ is the fundamental measure we use to evaluate network robustness, since it provides the valuable information about how many failures the network can support, remaining connected. However, it is not a very discriminative measure. From the viewpoint of node connectivity, all trees are equally robust. So, this measure does not help us to make a choice and our question remains open.

We then consider the algebraic connectivity $AlgConn$, a measure from specral graph theory that is able to spread the robustness of the trees into the range $(0, 1]$. The algebraic connectivity is a more sensitive measure in the sense that it measures not only the number of nodes that can disconnect a network, but also how difficult is to disconnect it. Regarding the algebraic connectivity, a path network, which is also called a bus, is the less robust tree of order N, since there are $N - 2$ ways to disconnect it, and the star network is the more robust one, since there is only one way to disconnect it. So, algebraic connectivity answers our question by choosing the star.

From the application viewpoint we observe that, if there is just one way of disconnect a network, this also means that there is a critical node whose removal completely destroys the network. Indeed, the star totally collapses by removing the maximum degree node, while in any other tree some communication functionality could be maintained (some pairs of nodes remain connected). If the star seems to be a good choice supposing uniform node failure probability, it is precisely the most vulnerable tree in case of attacks. This example points out that a robustness measure for uniform random failures may not be suitable for deliberate attacks.

After this illustrative example, we note that the above reasoning can be generalized. Since algebraic connectivity is a lower bound on node connectivity, our question can be translated into searching a graph G such that $AlgConn(G) = NConn(G) = k$. Using the characterization of these graphs provided by the literature, we conclude that, if G is a non-complete graph of order N such that $AlgConn(G) = NConn(G) = k$, where $1 \leq k \leq N/2$, then G becomes a set of $N - k$ isolated nodes after removing a particular set of k nodes. To see this, consider for instance the complete bipartite graph $K_{k,N-k}$.

We conclude that, in the context of telecommunication networks, algebraic connectivity can not be used alone, we must use additional constraints for designing robust networks, such as maximum degree for instance. Finally, we would like to discuss how to formalize, in terms of graph theory, a robustness definition incorporating the desired features, and how to improve the available robustness measures.