Covering $\Omega_n(T)$ by T-components

Liliana Costa[a], Enide Martins[a], and Nair M.M. Abreu[b]

[a] Universidade de Aveiro, Portugal

[b] COPPE - UFRJ, Brazil

A real square matrix $A = [a_{ij}]$ such that $a_{ij} \geq 0$ and $\sum_{i=1}^{n} a_{ij} = \sum_{j=1}^{n} a_{ij} = 1$ is known as *doubly stochastic matrix*. In 1946, Birkhoff asserted that a matrix in the polytope of $n \times n$ doubly stochastic matrices, Ω_n, is a vertex if and only if it is a permutation matrix. In fact, Ω_n is the convex hull of all permutation matrices of order n. Lately, some subclasses of Ω_n have been studied, namely the one that consists of the acyclic doubly stochastic matrices, [1], the so-called *acyclic Birkhoff polytope*:

$$\Omega_n(T) = \{A \in \Omega_n : A \text{ is acyclic}\}.$$

The authors in [1] gave an interpretation of the faces of $\Omega_n(T)$ in terms of graph theory. Given an usual graph G it can be considered a subgraph of G with the property of assigning to each of its vertices two possible colors. It was introduced the concept of bicolored (vertex) subgraph of a graph G (*i.e.*, a subgraph $G' = (V(G'), E(G'))$ of $G = (V(G), E(G))$ such that $E(G') \subseteq E(G)$ and $V(G') \subseteq V(G)$, and the vertex set $V(G')$ can be partitioned into $V_{\bullet} \dot{\cup} V_{\circ}$. Some of these subgraphs play a very important role, in particular the T-components.

Using T-components we cover a tree, we introduce the definition of complementary covers by vertices and edges and we count its numbers. It is related the dimension of the acyclic Birkhoff polytope with the dimensions of the faces corresponding to these complementary covers, [2].

References

[1] L. Costa, C. M. da Fonseca, E. A. Martins, The diameter of the acyclic Birkhoff polytope, Linear Algebra Appl. **428** (2008), 1524–1537S.

[2] L. Costa, E. A. Martins, Abreu, N.M.M., On complementary coverage of $\Omega_n(T)$, *manuscript*