Characterization of Laplacian eigenvalues of caterpillars

DOMINGOS M. CARDOSO (Universidade de Aveiro, Portugal)
MARIA AGUIEIRAS A. DE FREITAS (Universidade Federal do Rio de Janeiro, Brazil)
ENIDE ANDRADE MARTINS (Universidade de Aveiro, Portugal)
MARÍA ROBBIANO (Universidad Católica del Norte, Chile)
BERNARDO SAN MARTÍN (Universidad Católica del Norte, Chile)

Consider k disjoint graphs, $G_1, \ldots, G_k$, and a graph H on $1, \ldots, k$ vertices. Each vertex j of H is assigned to the graph G_j. The H-join of $G_1, \ldots, G_k$ [1] (generalized composition in [2]) is the graph G obtained by connecting each vertex of G_i to each vertex of G_j whenever the vertices i, j are adjacent in H. A caterpillar is a tree of order $n \geq 5$ such that removing all the pendant vertices produces a path with at least two vertices. In particular, the caterpillar $T(q_1, \ldots, q_k)$ is obtained from a path P_k, with $k \geq 2$, attaching the central vertex of the star S_{q_i} $(1 \leq i \leq k)$ to the ith vertex of the path P_k. In this work, we characterize completely the nonzero and the non ones-Laplacian eigenvalues of $T(q_1, \ldots, q_k)$ by means of the eigenvalues of the line graph $\mathcal{L}(T(q_1, \ldots, q_k))$ using the concept of H-join. Some additional consequences are explored, namely regarding the largest Laplacian eigenvalue and the algebraic connectivity.

Keywords: Laplacian eigenvalues, caterpillar, algebraic connectivity, H-join.

References

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