Order in some families of trees

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Abtract

Let G be a simple graph on n vertices $\{x_1, x_2, \ldots, x_n\}$. The adjacency matrix of G is the 0-1 matrix $A = A(G) = (a_{ij})$, with $a_{ij} = 1$ if there is an edge between x_i and x_j and $a_{ij} = 0$, otherwise. The spectrum of G is formed by the eigenvalues of A. It will be denoted by $\lambda_1, \ldots, \lambda_n$ and λ_1 is called the index of the graph. The problem of ordering families of trees by spectral parameters appears constantly in the literature, mainly after the indication of Cvetkovic in [1] of twelve directions in further investigations of graph spectra. In [4] and [5] we see trees ordered by their indexes and, in [2] and [3] trees are ordered by their algebraic connectivity. The aim of this paper is to use a graph-theoretical approach to establish an order in some families of trees of fixed diameter. We define a total order based on the index of the graph, as well as on the energy. Specifically, we deal with the following families: trees of diameter 3, and four families of trees of diameter 4. The first family of diameter 4 is formed by caterpillars consisting of double starlike trees such that the vertices with degree greater than two are non adjacents. The second and the third families are also composed by caterpillars, formed by $P3$, with $p+r$ pendent vertices in each extremity and $p-2r$ in the other vertex, for one case and $P3$, with $p-r$ pendent vertices in one extremity, $p+r$ in the other and p pendent vertices attached to the central vertex of $P3$. Finally, we consider generalized Bethe trees, which are rooted trees in which vertices at the same level have the same degree.

References

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