DECISION MAKING IN A FUZZY ENVIRONMENT AND MULTICRITERIA POWER ENGINEERING PROBLEMS

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ABSTRACT

This paper presents results of research into the use of models and methods of multicriteria decision making in a fuzzy environment for solving power engineering problems. Two major classes of situations requiring the use of a multicriteria approach are identified and, correspondingly, two general classes of models ($< X, M >$ and $< X, R >$ models, respectively) are considered. The analysis of $< X, M >$ models is based on the applying the Bellman-Zadeh approach to decision making in a fuzzy environment. Its use provides constructive lines in obtaining harmonious solutions on the basis of analyzing associated *maxmin* problems. Several techniques based on fuzzy preference modeling are considered for the analysis of $< X, R >$ models. The authors' experience in the development of these models and methods and their use for solving diverse classes of problems of power engineering is described. It demonstrates the advantages of applying fuzzy mathematics in optimization and decision making problems of power industry.

KEYWORDS. Multicriteria decision making. Fuzzy sets, Power systems.

ADM - Apoio à Decisão Multicritério

1. Introduction

Various types of uncertainty are commonly encountered in a wide range of optimization and decision making problems related to power system planning and operation. It should be considered as natural and unavoidable in the context of power industry problems. Considering this, the incorporation of the uncertainty factor in the construction of mathematical models serves as a vehicle for increasing their adequacy and, as a result, the credibility and factual efficiency of decisions based on their analysis. Investigations of recent years, in particular, in the power engineering area, show the benefits of applying fuzzy set theory (for instance, (Zimmermann, 1996)) to deal with diverse types of uncertainty. Its use offers advantages of both the fundamental nature (the possibility of validly obtaining more effective, less "cautious" solutions as well as the possibility of considering simultaneously different manifestations of the uncertainty factor) and the computational character (Ekel, 2001; Pedrycz et al., 2011).

The uncertainty of goals is an important kind of uncertainty, related to a multicriteria character of many problems of power system planning and operation. Some experts in the field of operational research and systems analysis agree that, from the substantial point of view, this type of uncertainty is the most difficult to overcome because "we simply do not know what we want". This type of uncertainty cannot be effectively captured only on the basis of using formal models, as sometimes the unique information sources are the individuals who make decisions.

The questions related to the necessity of setting up and solving multicriteria problems as well as the classification of situations, which need the application of a multicriteria approach, are discussed, for example, in (Zopounidis and Pardalos, 2010). However, from the general point of view, it is possible to identify two major classes of situations (Ekel, 2001; Pedrycz et al., 2011) which demand the use of a multicriteria approach:

• problems whose solution consequences cannot be estimated with a single criterion: these problems are related to analyzing models with economic as well as physical indices (when alternatives cannot be reduced to comparable form) and also by the need to consider indices whose cost estimations are hampered (for example, many power engineering problems are to be considered on the basis of criteria of technological, economical, ecological, and social nature);

• problems that may be solved on the basis of a single criterion or several criteria. However, if the uncertainty of information does not permit one to derive unique solutions, it is possible to solve these problems, utilizing additional criteria, including criteria of a qualitative character (whose use is based on knowledge, experience, and intuition of involved experts).

According to this, two classes of models, so-called $< X, M >$ models (as multiobjective models) and $< X, R >$ models (as multiattribute models) may be constructed. The present work is briefly describes the results of long-standing authors' studies in the analysis of these models and their application to solving diverse classes of problems of power system planning and operation. This experience convincingly demonstrates the advantages of applying fuzzy mathematics in optimization and decision making problems of power engineering.

2. $< X, M >$ models, Their Analysis and Applications

When analyzing $< X, M >$ models, a vector of objective functions $F(X) = \{F_1(X),...,F_q(X)\}$ is considered, and the problem consists of simultaneous optimizing all objective functions, i.e.,

$$F_p(X) \rightarrow \underset{X \in L}{\text{extr}}, \quad p = 1,...,q \tag{1}$$

where L is a feasible region in R^n.

The first general step in analyzing (1) is associated with determining a set of Pareto optimal solutions (Ehrgott, 2005); the corresponding concept of optimality was proposed in (Edgeworth, 1881) and generalized in (Pareto, 1886). This step is useful; however, it does not permit one to derive unique solutions. It is necessary to choose a particular Pareto solution on the basis of information provided by a decision maker (DM). There exist three approaches to using this information (Coelho, 2002; Pedrycz et al., 2011): *a priori*, *a posteriori*, and adaptive. When using the last one, the procedure of successive improving the solution quality is realized as steps of transitions from $X^0_\alpha \in \Omega \subseteq L$ to $X^0_{\alpha+1} \in \Omega \subseteq L$ with considering information I_α of the DM.

In the statement and the solution of multiobjective problems, it is necessary to elaborate answers to specific questions related to normalizing objective functions, choosing principles of optimality, and considering priorities of objective functions. The resolution of these questions and, consequently, the development of multiobjective methods, are heading in several directions, for instance, (Ehrgott, 2005; Pedrycz et al., 2011): scalarization methods; methods based on placing constrains on objective functions, including lexicographic techniques; methods of goal programming and of a global criterion; methods based on the use of the principle of guaranteed result. Without discussing these directions, it is necessary to indicate two fundamental points.

The first one is associated with the ability of methods based on placing constraints on levels of objective functions and methods of goal programming to produce solutions that are not Pareto optimal. This violates the basic concept of multicriteria decision making.

An important question in multiobjective analysis is the quality of obtained solutions. It is considered high if the levels of satisfying objectives are equal or close to each other (giving rise to so-called harmonious solutions) when the importance levels of the objective functions are equal (Ekel, 2001; Ekel, 2002). It is not difficult to extend this concept for the case when the importance levels of the objective functions are different (Pedrycz et al., 2011). From this point of view, it should be recorded the validity and advisability of the direction related to the principle of guaranteed result (Ekel, 2002). Other directions may lead to solutions with high levels of satisfying some criteria that is reached when assuring low levels of satisfying some other criteria. This situation could be completely unacceptable, for example, (Ekel and Galperin, 2003).

The lack of clarity of the concept of "optimal solution" is the basic methodological complexity in analyzing multiobjective problems. When applying the Bellman-Zadeh approach to decision making in a fuzzy environment (Bellman and Zadeh, 1970) to solve multiobjective problems, this concept is defined with reasonable validity: the maximum degree of implementing goals serves as a criterion of optimality. This conforms to the principle of guaranteed result and provides constructive lines in obtaining harmonious solutions. The application of the Bellman-Zadeh approach allows one to realize an effective (from the computational standpoint) as well as rigorous (from the standpoint of obtaining solutions $X^0 \in \Omega \subseteq L$) method of analyzing multiobjective models. Finally, its use allows one to preserve a natural measure of uncertainty in decision making and take into account indices, criteria, and constraints of qualitative character.

When applying the Bellman-Zadeh approach, each $F_p(X)$ is replaced by a fuzzy objective function or a fuzzy set $A_p = \{X, \mu_{A_p}(X)\}, X \in L$. It permits one to construct a fuzzy solution $D = \bigcap_{p=1}^{q} A_p$ with a membership function

$$\mu_D(X) = \bigwedge_{p=1}^{q} \mu_{A_p}(X) = \min_{p=1,..q} \mu_{A_p}(X), \quad X \in L \tag{2}$$

to obtain a solution providing the maximum degree of belongingness to D

$$\max \mu_D(X) = \max_{X \in L} \min_{p=1,..,q} \mu_{A_p}(X). \tag{3}$$

It reduces the problem (1) to a search for

$$X^0 = \arg\max_{X \in L} \min_{p=1,...,q} \mu_{A_p}(X). \tag{4}$$

The solution (4) requires to build membership functions $\mu_{A_p}(X)$, $p = 1,...,q$ reflecting a degree of achieving "own" optima by $F_p(X)$, $p = 1,...,q.$ As it is shown in (Pedrycz et al., 2011), it is rational to utilize the membership functions

$$\mu_{A_p}(X) = \left[\frac{\max F_p(X) \; - \; F_p(X)}{\max\limits_{X \in L} F_p(X) - \min\limits_{X \in L} F_p(X)} \right]^{\lambda_p} \tag{5}$$

for minimized objective functions and

$$\mu_{A_p}(X) = \left[\frac{F_p(X) \quad - \min\limits_{X \in L} F_p(X)}{\max\limits_{X \in L} F_p(X) \ - \min\limits_{X \in L} F_p(X)} \right]^{\lambda_p} \tag{6}$$

for maximized objective functions. In (5) and (6), λ_p, $p = 1,\ldots,q$ are importance factors.

Since X^0 is to belong to $\Omega \subseteq L$, it is necessary to utilize

$$\overline{\mu}_D(X) = \bigwedge_{p=1}^{q} \mu_{A_p}(X) \wedge \mu_\pi(X) = \min\{\min_{p=1,..q} \mu_{A_p}(X), \mu_\pi(X)\} \tag{7}$$

where $\mu_\pi(X) = 1$ if $X \in \Omega$ and $\mu_\pi(X) = 0$ if $X \notin \Omega$.

Finally, the existence of additional conditions (indices, criteria, and/or constraints) of a qualitative character, defined by linguistic variables (Zimmermann, 1996), reduces (4) to

$$X^0 = \arg\max_{X \in L} \min_{p=1,\ldots,q+s} \mu_{A_p}(X) \tag{8}$$

where $\mu_{A_p}(X), X \in L, p = q+1,\ldots,s$ are membership functions of fuzzy values of linguistic variables which reflect the additional conditions.

There is some theoretical justification behind the validity of applying *min* operator in (2)-(4), for example, (Bellman and Giertz, 1974). Considering this, it is necessary to note that there exist many families of aggregation operators that may be used in place of the *min* operator. An important question emerges: among aggregation operators, how to select the one, which is adequate for a particular problem at hand? Although some selection criteria are suggested in (Zimmermann, 1996), for instance, the majority of investigations is focused on choosing the operators on the basis of some available experimental evidence. Thus, the selection of the operators, to a significant extent, is experience-based. Considering this, below we discuss an example showing the use of not only the *min* operator but also the *product* operator which has found a wide use in decision making problems.

2.1. Multicriteria Allocation of Resources

The statement of the problem of multicriteria allocation of resources or their shortages (these problems are equivalent from the conceptual and mathematical points of view) among consumers (enterprises, regions, projects, etc.) supposes the possibility to use diverse types of objective functions in (1) defined in a feasible region

$$L = \{X \in R^n \,|\, 0 \le x_i \le A_i, \ \sum_{i=1}^{n} x_i = A\} \tag{9}$$

where $X = (x_1,\ldots,x_n)$ is a vector of limitations (for the sake of our considerations) for consumers, A_i is the permissible value of limitation for the *i*th consumer, A is a total value of limitations for all consumers considered in this planning or control problem.

The general scheme for solving the problem formalized by (1) and (9), is presented in (Pedrycz et al., 2011). This scheme assumes the availability of a procedure for building a term-set $T(Q)$ of the linguistic variable Q - *limitation for consumer* and membership functions for its fuzzy values to provide the DM with the possibility to consider conditions that are difficult to formalize. Besides, if the solution X_α^0 with $\mu_{A_p}(X_\alpha^0)$, $p = 1,\ldots,q$ is not satisfactory, the DM has to have the possibility to correct it, passing to $X_{\alpha+1}^0$ by changing the importance of one or more objective functions. Thus, the general scheme also assumes the availability of the procedure for constructing and correcting a vector of importance factors $\Lambda = (\lambda_1,\ldots,\lambda_q)$.

The general scheme for solving the problem described by (1) and (9) has been used for implementing an adaptive interactive decision making system AIDMS1 (Pedrycz et al., 2011). This system includes procedures for maximizing (3) on the basis of a non-local search that comes as a modification of the Gelfand's and Tsetlin's "long valley" method (Pedrycz et al., 2011).

Multicriteria power and energy shortage allocation

The existing conceptions of load management are united by the following: the elaboration of control actions is performed on the two-stage basis. At the level of energy control centers, optimization of allocating power and energy shortages (natural or associated with the economic advisability of load management) is carried out at different levels of territorial, temporal, and situational hierarchy of planning and operation. It allows one to draw up tasks for consumers to realize the corresponding control actions. Thus, the problems of power and energy shortage allocation are of a fundamental importance in a family of load management problems. They are to be analyzed not only as technical and economical tasks, but as ecological and social as well. In addition, when solving them, it is necessary to account for considerations of forming incentives to consumers. Considering this, it should be pointed out that methods based on fundamental principles of resource allocation exhibit drawbacks. They can be overcome by casting the problems within the framework of multiobjective models (Berredo et al., 2011).

The substantial analysis of problems of power and energy shortage allocation, systems of economics management as well as readily available reported information has permitted the construction of a general set of goals to solve these problems in the multicriteria statement. The complete list includes 17 types of goals. Some of them are given below:

1. Primary limitation of consumers with more low cost of produced production or given services on consumed 1 kWh of energy;

12. Primary limitation of consumers with a more high level of the coefficient of energy possession of work on consumed 1 kWh of energy;

15. Primary limitation of consumers with a more low value of the demand coefficient;

16. Primary limitation of consumers with a more low duration of using maximum load in twenty-four hours.

Consider the solution of problems of power shortage allocation formalized within the framework of the model (1) and (9) for six consumers with A^1=20000 kW and A^2=50000 kW with using *min* operator and in comparison with using *product* operator, taking into account the goals indicated above, described by the linear objective functions

$$F_p(X) = \sum_{i=1}^{6} c_{pi} x_i, \quad p = 1, 15, 16 \tag{10}$$

that are to be minimized and

$$F_{12}(X) = \sum_{i=1}^{6} c_{12i} x_i \tag{11}$$

that is to be maximized. Here $x_i, i = 1,\ldots,6$ are limitations of power supply for consumers. The coefficients $c_{pi}, p = 1, 12, 15, 16,\ \ i = 1,\ldots,6$ are determined by specific characteristics of consumers. Table 1 provides initial information for the problems.

The results of the solution on the basis of applying *min* operator (X^0) and *product* operator (X^{00}) are presented in Table 2 and Table 3.

Table 1. Initial information

I	1	2	3	4	5	6
$c_{1,i}$ (monetary units/kWh)	1.50	4.10	1.40	2.20	1.20	2.13
$c_{12,i}$	5.40	6.20	5.80	5.30	4.20	4.70
$c_{15,i}$	0.63	0.33	0.28	0.21	0.26	0.36
$c_{16,i}$ (hours)	15.30	17.20	21.10	18.50	17.40	19.60
A_i (kW)	14000	6000	4000	7000	19000	14000

Table 3 brings out that $X^0 \succ X^{00}$ (the use of *min* operator leads to solutions more harmonious than solutions obtained on the bases of the utilization of *product* operator).

Table 2. Power shortage allocation

I	1	2	3	4	5	6
$X^{1,0}$	5398	2515	2399	950	6738	0
$X^{1,00}$	5804	1104	870	6898	5324	0
$X^{2,0}$	13020	5076	3986	6223	19000	12695
$X^{2,00}$	14000	5731	4000	7000	19000	10269

Table 3. Power shortage allocation

P	1	12	15	16
$\mu_{A_p}(X^{1,0})$	0.604	0.605	0.605	0.606
$\mu_{A_p}(X^{1,00})$	0.615	0.590	0.633	0.630
$\mu_{A_p}(X^{2,0})$	0.428	0.431	0.428	0.428
$\mu_{A_p}(X^{2,00})$	0.366	0.700	0.353	0.714

Multicriteria operation of power systems considering an environmental impact

The use of the results described above stipulates that it is possible to apply the multicriteria approach to power system operation to realize dispatch with several objectives. This is illustrated by a case study with the standard IEEE 30-bus system presented in Fig. 1 (bus 1 is a slack bus) when considering objectives of minimizing losses $L(X)$, and reducing sulfur oxide emissions $E_{SOx}(X)$, and nitrogen oxide emissions $E_{NOx}(X)$.

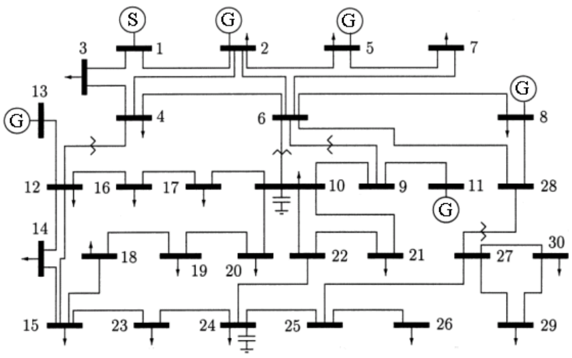

Fig 1. System diagram

The details of the characteristics of the generators are listed in Table 4. It includes the coefficients for estimating levels of *SOx* and *NOx* emissions on the basis of the relationships

$$E_{SOx,i}(x_i) = a_{SOx,i}x_i^2 + b_{SOx,i}x_i + c_{SOx,i} \tag{12}$$

and

$$E_{NOx,i}(x_i) = a_{NOx,i}x_i^2 + b_{NOx,i}x + c_{NOx,i}\,. \tag{13}$$

Table 4. Generator characteristics

i	x_1 (MW)	x_2 (MW)	x_3 (MW)	x_4 (MW)	x_5 (MW)
Bus	2	5	8	11	13
Fuel	hydro	Gas	oil	coal	Hydro
$a_{SOx,i}$ (kg/MW^2)	0	0	0.010	0.015	0
$b_{SOx,i}$ (kg/MW)	0	0	0.800	1.200	0
$a_{NOx,i}$ (kg/MW^2)	0	0.010	0.015	0.030	0
$b_{NOx,i}$ (kg/MW)	0	0.200	0.300	0.600	0

The consideration of (12) and (13) creates no difficulties at all. At the same time, the presentation of the function $L(x)$ in an explicit form gives rise to some difficulties. One way around this problem is using procedures of sequential optimization, applying sensitivity models to reflect the loss change occurring at each optimization step (Ekel et al., 2013).

Tables 5 and 6 include results of the successive multicriteria optimization steps.

Table 5. Results obtained in successive steps of multicriteria optimization

m	$x_1^{(m)}$ [MW]	$x_2^{(m)}$ [MW]	$x_3^{(m)}$ [MW]	$x_4^{(m)}$ [MW]	$x_5^{(m)}$ [MW]
0	85.50	66.92	66.76	58.91	48.67
1	85.86	70.26	67.25	56.67	46.71
2	84.34	73.78	66.36	53.91	48.38
3	82.44	77.47	65.20	51.31	50.35
4	83.56	81.35	64.94	48.74	48.18

Table 6. Levels of objective functions

m	$L(x^{(m)})$[MW]	$E_{SOx}(x^{(m)})$[kg]	$E_{NOx}(x^{(m)})$[kg]
0	9.04	220.72	284.52
1	8.91	215.23	281.81
2	8.78	205.41	274.69
3	8.60	195.73	268.60
4	8.55	188.25	265.70

2.2. Multicriteria Decision Making in Distribution System Planning and Operation

The most important functions of Distribution Management Systems are realized on the basis of solving the problems of optimizing network configuration and optimizing voltage control in distribution systems. Their statement and solution within the framework of <*X*, *M*> models are briefly discussed below. The present Subsection also includes the consideration of the problem of allocating reactive power sources in distribution systems applying <*X*, *M*> models.

Multicriteria optimization of network configuration in distribution systems

The problems of optimizing network configuration in distribution systems are associated with altering topologies of distribution networks by changing the state of their switches. Many works have been focused on developing solutions to these problems. Almost all these works "compete" in aspiration for obtaining "more optimal" solutions under the monocriteria statement. However, this aspiration, considering that the combination of the information uncertainty and relative stability of optimal solutions produces decision uncertainty regions, is not convincing. At the same time, network reconfiguration problems are inherently multicriteria in nature because they have impact on reliability, service quality, and economical feasibility of power supply. Considering this, the developed computing system DNOS (Berredo et al., 2011) permits one to consider and to minimize objective functions of power losses, energy losses, system average interruption frequency index (SAIFI), system average interruption duration index (SAIDI), undersupply energy, poor quality energy consumption, and integrated overload of network elements in diverse combinations. Examples of solving the reconfiguration problems are given in (Berredo et al., 2011). They show that the use of the multicriteria approach leads to the harmonious solutions with small deviations from locally optimal solutions.

Optimization of voltage control in distribution systems

The techniques for optimal voltage control, which are implemented within the framework of the computing system VCOS (Berredo et al., 2011) are directed at minimizing poor quality energy consumption. However, in accordance with a situational hierarchy, it may be necessary to realize an energetically efficient control, considering static characteristics of loads. Thus, it becomes necessary to speak about the second statement directed at the minimization of poor quality energy consumption and the reduction of peak load and/or energy consumption. This statement is also implemented (on the basis of the results discussed in Section 2) as a function of VCOS. Examples of the application of VCOS are given in (Berredo et al., 2011).

Reactive power compensation in distribution systems

Traditionally, problems of reactive power compensation in distribution systems are associated with the selection of the locations, sizes, and types of capacitors to minimize the objective function of an economical nature, while the constraints on upper and lower voltage limits at different load levels are satisfied.

Considering the discrete nature of such problems, the generalized algorithms (Ekel and Schuffner, 2006) have been applied for their solution. However, the experience shows that the necessity to simultaneously observe constraints on upper and lower voltage limits creates essential obstacles. It is not uncommon to face situations when these constraints induce empty feasible regions. It can be avoided by minimizing the objective function of an economical nature as well as the objective function reflecting a volume of poor quality energy consumption. Besides, if the problem is associated with determining capacitor types (fixed or switched), the quantity of objectives should be increased.

Considering this, the algorithms of (Ekel and Schuffner, 2006) have been modified to solve discrete multicriteria problems using the results of Section 2. The modified algorithms have served for developing the computing platform EPODIAN to solve reactive power compensation problems in monocriteria and multicriteria settings for large-scale distribution networks. Examples of its utilization are given in (Araujo et al., 2013).

Although the presented results do not take into account the uncertainty of initial information, EPODIAN allows its consideration within the framework of a general scheme of multicriteria analysis under information uncertainty (Ekel et al., 2011) with the evaluation of particular (monocriteria) and aggregated (multicriteria) risks in multiple scenarios. This scheme combines the construction and analysis of $< X, M >$ as well as $< X, R >$ models. Considering this, it should be noted that many instances of decision making problems related to power systems may be resolved with the application of the results of (Ekel et al., 2011).

3. $< X, R >$ models, Their Analysis and Applications

The problem of multiattribute analysis of alternatives in a fuzzy environment can be formulated as follows.

Assume we are given a set X of alternatives coming from the decision uncertainty region and/or predetermined alternatives, which are to be examined by q criteria of a quantitative and/or qualitative nature. The problem of decision making may be presented as pair $< X, R >$ where $R = \{R_1,\ldots,R_q\}$ is a vector of nonstrict fuzzy preference relations (Orlovsky, 1981; Pedrycz et al., 2011) which can be presented as follows:

$$R_p = [X \times X, \mu_{R_p}(X_k, X_l)], \quad p = 1,\ldots,q,\ X_k, X_l \in X \tag{14}$$

where $\mu_{R_p}(X_k, X_l)$ is a membership function of the *p*th fuzzy preference relation.

In (14), R_p is defined as a fuzzy set of all pairs of the Cartesian product $X \times X$, such that the membership function $\mu_{R_p}(X_k, X_l)$ represents the degree to which X_k weakly dominates X_l, i.e., the degree to which X_k is at least as good as X_l for the *p*th criterion.

A convincing and natural approach to obtaining matrices R_p is presented in (Ekel et al., 1998; Pedrycz et al., 2011). In particular, the availability of fuzzy or linguistic estimates of alternatives $F_p(X_k)$, $p = 1,\ldots,q$, $X_k \in X$ with $\mu[F_p(X_k)]$, $p = 1,\ldots,q$, $X_k \in X$ permits one to construct R_p, $p = 1,\ldots,q$ as follows:

$$\mu_{R_p}(X_k, X_l) = \sup_{\substack{X_k, X_l \in X \\ F_p(X_k) \le F_p(X_l)}} \min\{\mu[F_p(X_k)], \mu[F_p(X_l)]\}; \tag{15}$$

$$\mu_{R_p}(X_l, X_k) = \sup_{\substack{X_k, X_l \in X \\ F_p(X_l) \le F_p(X_k)}} \min\{\mu[F_p(X_k)], \mu[F_p(X_l)]\} \tag{16}$$

when F_p has a minimization character. When F_p is to be maximized, the relationships (15) and (16) are to be written for $F_p(X_k) \ge F_p(X_l)$ and $F_p(X_l) \ge F_p(X_k)$, respectively.

The fuzzy preference relations are not a unique form of preference representation. For instance, the authors of (Zhang et al., 2007) indicate eight formats which can be used to establish preferences among alternatives. It is natural that their application demands a conversion of all formats to a unique format which can be processed and analyzed. Considering the advantages and rationality of the application of fuzzy preference relations for this objective, the results of (Herrera-Viedma et al., 2002; Pedrycz et al., 2011) permit one to construct so-called transformation functions to convert different preference formats to fuzzy preference relations.

Let us consider the situation of setting up a single fuzzy nonstrict preference relation R. It can be processed (Orlovsky, 1981) to build a fuzzy strict preference relation $R^S = R \setminus R^{-1}$ with

$$\mu_R^S(X_k, X_l) = \max\{\mu_R(X_k, X_l) - \mu_R(X_l, X_k), 0\}. \tag{17}$$

The expression (17) serves as the basis for the choice procedure analyzed in (Orlovsky, 1981). In particular, $\mu_R^S(X_l, X_k)$ is the membership function of the fuzzy set of all X_k which are strictly dominated by X_l. Its complement by $1-\mu_R^S(X_l, X_k)$ gives the fuzzy set of alternatives which are not dominated by other alternatives. To choice the set of all alternatives which are not dominated by other ones, it is necessary to find the intersection of all $1-\mu_R^S(X_l, X_k), X_k \in X$ on all $X_l \in X$. This intersection is the set of nondominated alternatives with

$$\mu_R^{ND}(X_k) = \inf_{X_l \in X}[1-\mu_R^S(X_l, X_k)] = 1 - \sup_{X_l \in X} \mu_R^S(X_l, X_k). \tag{18}$$

The use of (18) allows one to evaluate the level of nondominance of each alternative X_k. Considering that it is natural to choose alternatives providing the highest level of nondominance, one can choose alternatives X^{ND} in accordance with the following expression:

$$X^{ND} = \{X_k^{ND} \mid X_k^{ND} \in X, \mu_R^{ND}(X_k^{ND}) = \sup_{X_k \in X} \mu_R^{ND}(X_k)\}. \tag{19}$$

The expressions (17)-(19) may be used to solve choice problems as well as other problems related to the evaluation, comparison, choice, prioritization, and/or ordering of alternatives with a single criterion. These expressions may also be applied when R is a vector of fuzzy preference relations, under different approaches for multicriteria analysis discussed in (Ekel et al., 1998; Pedrycz et al., 2011). Among them, we highlight: a lexicographic approach, a compensatory approach with adjustment of the trade-off rates among criteria as well as a flexible approach with adjustment of degree of optimism.

The lexicographic approach is based on step-by-step application of criteria for comparing alternatives to construct a sequence $X^1, X^2, \ldots, X^q$ so that $X \supseteq X^1 \supseteq X^2 \supseteq \ldots \supseteq X^q$. This is accomplished by using the following expressions:

$$\mu_{R_p}^{ND}(X_k) = \inf_{X_l \in X^{p-1}}[1-\mu_{R_p}^S(X_l, X_k)] = 1 - \sup_{X_l \in X^{p-1}} \mu_{R_p}^S(X_l, X_k), \quad p = 1,\ldots,q; \tag{20}$$

$$X^p = \{X_k^{ND,p} \mid X_k^{ND,p} \in X^{p-1}, \mu_{R_p}^{ND}(X_k^{ND,p}) = \sup_{X_l \in X^{p-1}} \mu_{R_p}^{ND}(X_k)\}. \tag{21}$$

The compensatory approach with adjustment of the trade-off rates among criteria is based on the use of the weighted arithmetic mean as given by the following expression:

$$\mu^{ND}(X_k) = \sum_{p=1}^{q} \lambda_p \mu_{R_p}^{ND}(X_k) \tag{22}$$

where the importance factors are to satisfy the conditions $\lambda_p > 0$, $p = 1,\ldots,q$ and $\sum_{p=1}^{q} \lambda_p = 1$.

Finally, the flexible approach with adjustment of degree of optimism is accomplished by using the ordered weighted average (OWA) operator, originally introduced in (Yager, 1988), in accordance with the following expressions:

$$\mu^{ND}(X_k) = \mathrm{OWA}(\mu_{R_1}^{ND}(X_k),\ldots,\mu_{R_q}^{ND}(X_k)) = \sum_{i=1}^{q} w_i B_i(X_k) \tag{23}$$

where $B_i(X_k)$ is the ith largest value among $\mu_{R_1}^{ND}(X_k),\ldots,\mu_{R_q}^{ND}(X_k)$.

The set of weights in (23) satisfies the conditions $w_i > 0$, $i = 1,\ldots,q$ and $\sum_{i=1}^{q} w_i = 1$. These weights can be determined by the DM using the expressions of (Liu and Han, 2008). They include a parameter for regulating the optimism degree inherent to the decision attitude.

The results described above as well as, based on these results, models and methods for multicriteria group decision making (Parreiras et al., 2010, 2012; Pedrycz et al., 2011) have served for developing two computing tools: a computing system for prioritization in distribution system maintenance planning GIMPRIS and a web-based decision support center DSC for electrical energy companies which are briefly described below.

Prioritization in distribution system maintenance planning

The problem of prioritization in maintenance planning aims at assuring reliability and service quality of power supply through network and equipment preventive maintenance. The Energy Company of Minas Gerais (CEMIG) implements a strategy to realize the maintenance according to the plans suggested by manufacturers of network elements and equipment.

The prioritization techniques, based on the use of the results described above, allow one to take into account not only parameters of network elements and equipment, but also factors related to the conditions of their operation. In addition to the failure risks, which can be accessed through the statistical analysis, factors associated with the impact of these failures are considered in the prioritization process. Among them it is possible to mention quantitative factors gas well as qualitative factors (political impact, maintenance complexity, environmental impact, etc.).

The computing system for prioritization in maintenance planning GIMPRIS implemented for CEMIG provides group decision making environment in the web-based platform (Fig. 2). The flexible tools for preference and aggregation modeling are designed to extend the capabilities of $< X, R >$ models for prioritization of more than 50000 equipment items. The dynamic group management scheme implemented within the system allows a supervisor to efficiently control the process of convergence to consensus, while moderating up to 10 experts simultaneously.

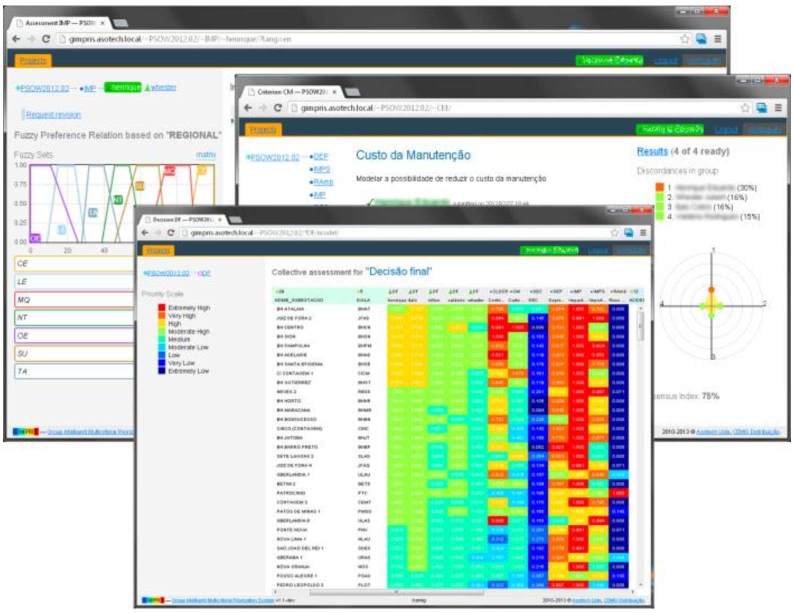

Fig. 2. Web-based group decision making environment for prioritization in distribution system maintenance planning

A web-based decision support center for electrical energy companies and its application

A web-based decision support center was created to aid various interrelated decision making situations, which emerge from planning and operation processes in electrical energy companies. This center permits one to support small collaborative groups working in an asynchronous way, in an environment where a single DM, who centralized the responsibility for a final decision, can be aided by a group of experts, who contribute with their opinion to that decision. It is based on the construction and the analysis of $< X, R >$ models. Two preference formats, namely value functions and fuzzy (linguistic) estimates, are made available to the input of preference information. The corresponding transformation functions for dealing with preference measures on interval scales are utilized to construct fuzzy preference relations. The availability of different aggregation operators allow the DM to reproduce different attitudes: pessimistic, optimistic, compensatory with adjustment of the trade-off rates among criteria as well as lexicographic with prioritization of criteria. When the DM cannot choose a unique attitude to analyze a problem, DSC recommends a generalized solution, which considers all attitudes simultaneously. The efficiency of the application of DSC is demonstrated in (Kokshenev et al., 2014) by the solution of the problem of power system expansion planning.

4. Conclusions

The problems of power system planning and operation requiring the application of a multicriteria approach can be adequately and effectively formulated within the framework of two general classes of models of multiobjective ($<X, M>$ models) and multiattribute ($<X, R>$ models) decision making. Their analysis based on the utilization of fuzzy set theory (in particular, on the use of the Bellman-Zadeh approach to decision making in a fuzzy environment and the techniques of fuzzy preference modeling) offers advantages of both the fundamental nature (the possibility of validly obtaining more effective, less "cautious" solutions as well as the possibility of considering simultaneously different manifestations of the uncertainty factor) and the computational character. It was illustrated by the presented results on the solution of diverse classes of problems of power engineering.

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