OPTIMAL ENERGY MATRIX THROUGH KRIGING METHOD: AN ANALYSIS OF DIFFERENT RISK MEASURES

Marcelo Rosário da Barrosa
Escola Politécnica da Universidade de São Paulo
Av. Professor Almeida Prado, Travessa 2, nº 128
Cidade Universitária
São Paulo, SP, CEP 05508-070
dabarrosa@uol.com.br

Celma de Oliveira Ribeiro
Escola Politécnica da Universidade de São Paulo
Av. Professor Almeida Prado, Travessa 2, nº 128
Cidade Universitária
São Paulo, SP, CEP 05508-070
celma@usp.br

Arthur Valle Salles
Escola Politécnica da Universidade de São Paulo
Av. Professor Almeida Prado, Travessa 2, nº 128
Cidade Universitária
São Paulo, SP, CEP 05508-070
arthur.salles@usp.br

Erik Rego
Escola Politécnica da Universidade de São Paulo
Av. Professor Almeida Prado, Travessa 2, nº 128
Cidade Universitária
São Paulo, SP, CEP 05508-070
erikrego@usp.br

RESUMO

Este artigo apresenta uma nova metodologia para solucionar o problema de portfólio de energia utilizando o desvio padrão, o Valor-em-Risco e o Valor-Em-Risco Condicional como medidas de risco. Esta nova metodologia, baseada na Krigagem, permite a obtenção de uma solução aproximada para o problema, utilizando-se qualquer medida de risco disponível na literatura, e relaxando as hipóteses restritivas inerentes às metodologias disponíveis.

ABSTRACT

This article presents a new approach to solve the energy portfolio problem utilizing the standard deviation, the Value-at-Risk and the Conditional Value-at-Risk as risk measures. This new methodology, based on the Kriging Method, allows for approximating the solution to the problem utilizing virtually any risk measure available in literature, relaxing the usual restrictive hypothesis inherent to the available methodologies.

Key-words: Operational Research, Optimization, Portfolio, Energy Matrix, VaR, CVaR, Kriging

Área principal: EN - PO na Área de Energia, AD&GP - PO na Administração e Gestão da Produção

1. INTRODUCTION

Modern Portfolio Theory (MPT) introduced by MARKOWITZ (1952) studies the problem of composing an optimal portfolio of financial assets, assessing the conflicted objectives between minimizing risk and maximizing investment return. The same conflicting objectives are observed when composing electric energy production portfolios, or the energetic matrix, with different technologies (ie gas, coal, biomass, hydro, wind, etc.). In the latter, the objective is to minimize the average unit cost of energy production, minimizing the uncertainties, or risk, on this cost. BAR-LEV and KATZ (1976), AWERBUCH and BERGER (2003), AWERBUCH (2006) and KREY and ZWEIFEL (2006), DELARUE et al (2011) and LOSEKANN et al (2013), among others, compose a long list of recent literature developments on the application of MPT principles to the energetic matrix composition problem, invariably utilizing the variance (σ^2) of the unit production cost as a risk measure. The resulting optimization problem minimizes a quadratic function subject to linear constraints, to which well known optimization techniques are applied.

The application of the variance as a risk measure, however, does not provide useful information about the tail of the probability distribution of the unit cost of energy production, being weak for stress scenarios. Thus, the application of different risk measures, such as the Value at Risk -VaR and the Conditional Value at Risk - CVaR to the energy portfolio problem configures a solid contribution to policymakers and professionals of the sector.

RIBEIRO (2012) proposed a methodology based on the Kriging Method to smooth the response surface (risk surface) of the CVaR and approximate the optimal solution to the investment portfolio problem. This article presents the same approach, but to optimize the energy matrix problem utilizing both the VaR and the CVaR as risk measures.

The text is divided in five sections other than the introduction. Section 2 presents a literature review on the application of MPT principles to the electric energy portfolio problem, re-writing it to facilitate the application of different risk measures. Section 3 presents the proposed methodology, based on the Kriging Method. Section 4 presents the basic structure of this research and the set of utilized data, followed by the results presented on section 5, including the optimization of the energetic matrix for the variance, the VaR and the CVaR as risk measures. The paper is concluded by showing that the proposed methodology may be applied to the energy portfolio problem utilizing virtually any risk measure as the objective function, and relaxing the usual restrictive hypothesis inherent to the available methods.

2. PORTFOLIO THEORY IN ELECTRICITY SECTOR – A REVIEW

The fundaments of the Modern Portfolio Theory lie on the fact that a portfolio composed of assets with negatively correlated returns offers better return-to-risk composition than that of individual assets. The use of portfolio theory in the electricity sector is not new. AWERBUCH and BERGER (2003), AWERBUCH (2006) and KREY and ZWEIFEL (2006) presented the application of the principles behind the MPT to the European and American Electricity Sector. The objective is to find the optimal mix of electric energy production technologies as a percentage of the total production. The authors consider the inverse of the unit cost of producing energy [kWh/$] as the “return” of the electric energy portfolio and the standard deviation of this “return” as a risk measure, following the pioneer introduction of MPT application to the electricity sector of BAR-LEV and KATZ (1976).

A different approach to the problem considers the unit cost of energy production [$/kWh] as the objective function, and the standard deviation of the unit cost, expressed as a percentage of the average cost, as the risk associated to each technology. The general form of the problem can be written as follows (LOSEKANN et al, 2013 and DELARUE et al, 2011).

Min $COST(\boldsymbol{x}) = \sum_i x_i UTCO_i$

Subject to (1) $[\boldsymbol{x}^t \sum \boldsymbol{x}]^{1/2} \leq R$

(2) $\sum_{i=1}^{n} x_i = 1$

(3) $x_i \geq 0, i = 1, \dots, n$

The objective function $COST(\boldsymbol{x})$ is the total unit cost of electric energy production in terms of the decision vector $\boldsymbol{x}$, which represents the weight or allocation of each technology in the energy portfolio. Notably, the total unit cost is expressed as the sum of the average unit cost of each technology, with $UTCO_i$ standing for the unit production cost of technology i, as defined by DELARUE et al (2011). The *first constraint* represents the standard deviation, or risk, of the portfolio as a function of $\boldsymbol{x}$, with $\sum$ the covariance matrix between the historical values of the unit cost of each technology. This constraint is parameterized in R, the greatest acceptable portfolio risk. The *second constraint* assures full allocation of the necessary energy supply within the available technologies, and the *third constraint* assures only null or positive allocation of each technology in the portfolio, a physical restriction.

The presented problem is clearly a simplified representation, since other important constraints, not considered here, are usually applied to the energy portfolio problem. These include, but not limited to, (1) restrictions on the actual installed capacity of each technology, (2) policy limitations given environmental issues (in case of Nuclear energy, for example), (3) reliability of supply throughout the entire year, considering the intermittence of wind supply, for example, among others (LOSEKANN, 2013). For the purposes of this article, however, this simplified representation of the problem is sufficient, since it does not directly affect the application of the proposed methodology.

To express the unit cost of producing electric energy, let *I* be the set of available technologies (index *i*), and *K* the set of cost categories (index *k*). Following DELARUE et al (2011), the unit production cost of technology *i* is expressed as:

$$UTCO_i = \sum_k C_{i,k} = INVe_i + FU_i + FOMe_i + VOM_i$$

Where,

$C_{i,k}$ is the cost component *k* of technology *i*
$INVe_i$ is the investment cost of technology *i*, via technology specific load factor expressed in [$/kWh], per year
FU_i is the fuel cost of technology *i* [$/kWh]
$FOMe_i$ is the fixe O&M cost of technology *i* [$/kWh]
VOM_i is the variable O&M cost of technology *i* [$/kWh]

Clearly, the average production cost of the energy matrix (or portfolio) is expressed as $COST(\boldsymbol{x}) = \sum_i x_i UTCO_i$, and the standard deviation $[\boldsymbol{x}^t \sum \boldsymbol{x}]^{1/2}$, or, algebraically, $\sqrt{\sum_i \sum_j X_i X_j \rho_{ij} \sigma_i \sigma_j}$, with ρ the Pearson Correlation Factor between costs of technology i and j.

LOSEKANN et al. (2013) present a scatter plot of the average unit cost for producing energy *versus* the standard deviation of the unit energy production cost, expressed as a percentage of the average, for multiple technologies in the Brazilian market. The results are presented in **Figure 1**.

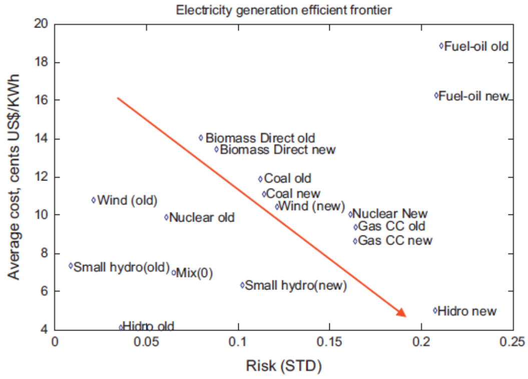

Figure 1: Average Unit Production Cost of Energy *versus* Standard Deviation of Unit Production Cost of Energy for Multiple Technologies. (LOSEKANN et al, 2013).

Figure 1 shows that there is a negative correlation between the average unit cost and the standard deviation of the unit cost of producing electric energy, for different technologies. In fact, AWERBUCH and YANG (2007) present an estimated correlation matrix between the costs of electric energy production for multiple technologies.

The conflict between minimizing both (1) the average unit production cost of a portfolio composed of multiple technologies and (2) its standard deviation (risk) is well represented by a Pareto Optimality Frontier for conflicted objectives (PAPALAMBROS, 2000), as exemplified in **Figure 2**, extracted from LOSEKANN et al (2013) and references therein.

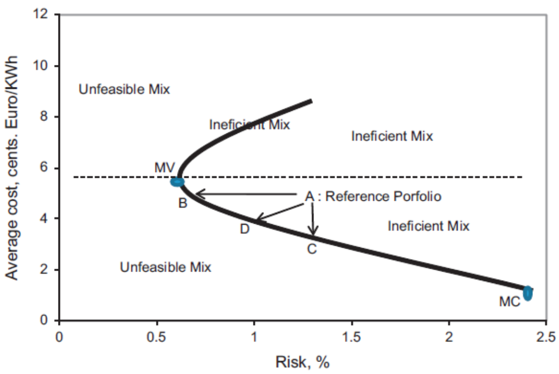

Figure 2: Example of energy portfolio efficient frontier (LOSEKANN et al, 2013).

Alternatively to the optimization problem presented above, it is equivalent to re-write it with the uncertainties around the unit cost of producing energy as the objective function, $Risk(\boldsymbol{x})$, subject to a parameterized restriction on the greatest acceptable average unit cost of production, C. Thus,

Min $RISK(\boldsymbol{x})$

Subject to

$$(1)\ \sum_i UTCO_i \leq C$$

$$(2)\ \sum_{i=1}^{n} x_i = 1$$

$$(3)\ x_i \geq 0, i = 1, \ldots, n$$

This form allows the use of different risk measures to the problem in a more convenient fashion. The latest developments in literature on investment risk introduced, among others, two vastly discussed risk measures:

(1) **Value at Risk – VAR**, defined as the greatest loss that might occur with α% of probability in a fixed time horizon (G-30, 1994, JORION, 1997), and

(2) **Conditional Value at Risk – CVAR**, a coherent risk measure (ARTZNER, 1999) defined as the average of values that exceed the VAR, for a fixed confidence level, or $\mathrm{E}\{\mathrm{y}|\mathrm{f}(\mathrm{x},\mathrm{y}) \leq \nu\}$, with ν the VaR (ROCKAFELLAR and URYASEV, 2000; QUARANTA and ZAFFARONI, 2008).

Both the VaR and the CVaR measure different aspects of risk when compared to the standard deviation. Among others, the most important to mention includes information about the tail of the probability distribution of the unit production cost, being more suitable to stress scenarios. As such, the ability to optimize energy production portfolios, or the energetic matrix, using different interpretations of risk represents a sound contribution to policymakers and professionals of the energy sector. The following section presents a methodology that is applicable to approximate the solution to the presented problem using virtually any known risk measure as the objective function, and relaxing all usual restrictive hypothesis inherent to the available methodologies.

3. A NEW APPROACH TO THE PROBLEM

RIBEIRO et al (2012) proposed the application of the Kriging Method to optimize investment portfolios utilizing the CVaR as a risk measure. Instead of considering an analytical expression for CVAR in terms of probability distribution of returns, as presented by ROCKAFELLAR and URYASEV (2002), the authors considered estimates of CVAR calculated in a grid of R^N, representing different portfolios. These estimates, obtained by a simulation process that covers the entire dominium of the CVaR function, are used to construct a response surface - the risk surface - which is the objective function of the optimization model. The response surface is approximated by a polynomial interpolation process in which the error is a function of the distance between the simulated values in space (RIBEIRO, 2004, YIN, 2011). The same approach may be applied not only for the CVaR, but for any risk measure available in literature.

Given q pairs $\left\{\left(w^{(j)}; y^{(j)}\right)\right\}_{j=1}^{q}$, with $w^{(j)} \in R^N$ and $y^{(j)} = f(w^{(j)})$, the Kriging Model creates a polynomial approximation of a function f(.), $\sum_{j=1}^{q} \alpha_j f^{(j)}(w) + e(w)$. The random errors $e(w)$ are correlated, normally distributed, with mean equal to zero and variance σ^2. The covariance between the errors is given by $cov\left(e(X^i), e(X^j)\right) = \sigma^2 \sum_{ij}$, where $\sum_{ij}$ is the correlation between two errors ($\sum_{ij} \ = R(\theta, d_h) = corr\left(X^j, X^j\right)$).

An unbiased estimator of $f(X^*)$ is given by RIBEIRO (2004) and LOPHAVEN (2002): $\hat{f}(X^*) = \sum_{j=1}^{m} \beta_j^* f^j\,(X^*) + r'\Sigma^{-1}(y - F\beta^*)$ with $\beta = (F^T\Sigma^{-1}F)^{-1}\, F^T\Sigma^{-1}y.$ Vector r is the correlation vector between errors related to X^* and the other points in the sample, $\sum$ is the correlation matrix between the points in the sample, y is the vector of the observed values for the objective function, and F is the matrix with the values calculated in the points of the sample.

The Gaussian correlation is given by $R(\theta, d_h) = \exp(-\theta_h d_h^2)$ with the distance measure between two points depending on parameters θ_h and p_h:

$$d(X_i, X_j) = \sum_{h=1}^{n} \theta_h \, |x_h^i - x_h^j|^{p_h}$$

According to JONES (1998), the θ_h parameter measures the influence or "activity" of the variable x_h. The exponent p_h is related to the smoothness of the function in relation to the points h. Similarly to QUEIPO (2002), the parameters adopted in this work are $\theta_h = 1$ and $p_h = 2$.

The most appropriate sample to the experiment is the Deterministic Generation (DABARROSA et al, 2014), in which each face of the hypercube $[0,1]^n$ is subdivided in a defined number of intervals, which generate other cubes which vertices are the points of the sample (RIBEIRO et al, 2012).

Once the sample generation technique applied, the correlation and the regression are defined, and the model is constructed as follows (RIBEIRO et al, 2012):

1. Generate a sample of points $\{X^i\}_{i=1}^{q}$, satisfying the constraints $X^i \in R^n$ and $0 \leq x_j^i \leq 1$, $i = 1, \dots, n$;
2. For each vector X^i, calculate $y^i = RISK(X^i)$
3. Construct the approximating function $(X^i; Y^i)$;
4. Solve the resulting optimization model

4. RESEARCH METHODS

To illustrate the application of the proposed methodology, the energy matrix composed of five different technologies in the Brazilian energy sector is considered: gas, coal, hydro, biomass and wind. For simplification purposes, all studied technologies are defined as "new", thus not considering the already installed capacity, in consonance with the simplified presentation of the problem in **Section 2**. One may interpret this assumption as if policies are to be developed from green field. Additionally, nuclear and fuel-oil technologies were not considered in the analysis, since their representation in the Brazilian actual energy matrix accounts for less than 5% of total, as per the 2020 Decennial Plan for Energy Expansion (DPEE 2020).

Since data in the Brazilian energy sector is not vastly available and the primary focus of this article is to present the methodology, this study bases itself mainly on the data provided by LOSEKANN et al (2013). Moreover, for simplification purposes but without loss of generality, it considers neither the $UTCO_i$ breakdown per cost category nor the effect of CO_2 emission cost. Instead, a hypothetical historical series of 2.000 observations preserving the main statistics (ie standard deviations and average unit costs, as per **Figures 3 and 4**) presented in the above reference is created, randomly altering the skewness and the kurtosis of each series probability distribution. As a result, the covariance matrix utilized is hypothetical and the results should not be interpreted as real, serving solely for the purpose of exercise. The exercise does not diminish, though, the applicability and generality of the proposed methods.

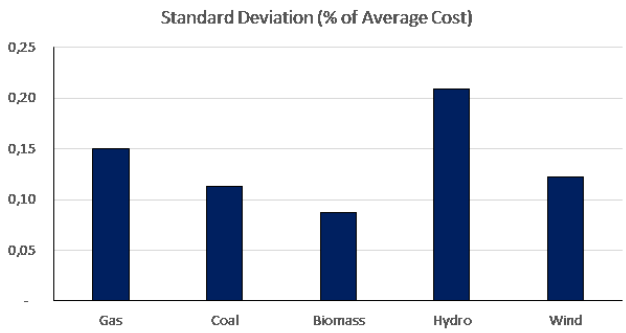

Figure 3: Standard Deviation of Unit Cost of Energy Production Sample (LOSEKANN et al, 2013).

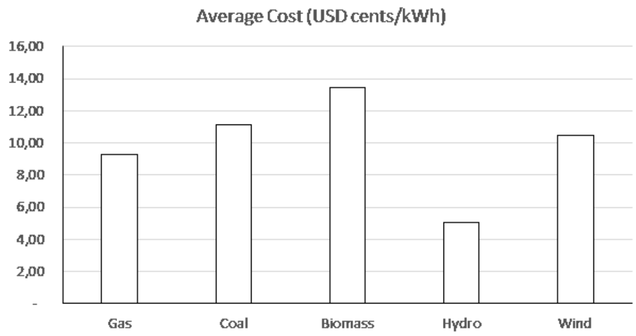

Figure 4: Average Unit Cost of Energy Production Sample (LOSEKANN et al, 2013).

The straightforward analysis of the real data provided in **Figures 3 and 4** emphasizes the trade-off between minimizing the unit production cost and its risk, since the low cost technologies usually present higher volatility (ie standard deviation), and vice-versa.

The portfolio optimized using the standard deviation as a risk metric is calculated both using Kuhn-Tucker technique and the Kriging Method, serving as a control of the experiment. The optimizations utilizing the VaR and the CVaR as risk metrics are performed with the proposed method.

Results, in the following section, are presented both in form of efficient frontier (ie Paretto's Optimality Frontier for conflicted objectives) and as the optimal energy matrix composition for different risk levels.

5. RESULTS

This section presents the results of the research, divided in three sub-sections: Standard Deviation, VaR and CVaR. The first sub-section is the control of the experiment, in which the exact solution obtained by the optimization of a quadratic function (variance) subject to linear constraints is compared to the results obtained by the proposed methodology. The following sub-sections present the results obtained by the application of the proposed methodology to the VaR and the CVaR, in which simulated values of Risk are obtained using order statistics and relaxing all restrictive hypothesis usually considered in other available methods.

5.1 Standard Deviation

As mentioned before, the variance (σ^2) of a portfolio is a quadratic function, and the presented optimization problem subjects it to linear constraints. Thus, the problem can be easily solved through the application of Kuhn-Tucker approach, which will serve as a control for the proposed methodology. This sub-section presents the results obtained by the classic method (control), as well as by the proposed methodology.

Figure 5, below, presents the efficient frontiers for the energy matrix calculated by the Kuhn-Tucker and the proposed method.

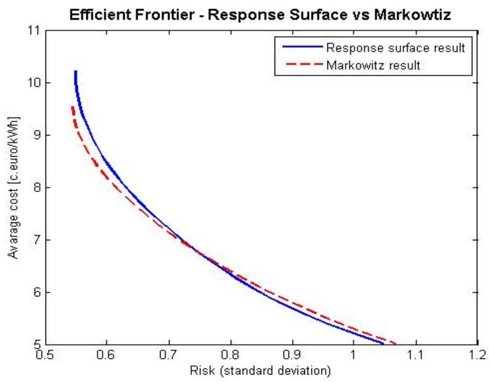

Figure 5: Efficient Frontier for Energy Matrix using Standard Deviation as risk metric, calculated both through Kuhn-Tucker and Kriging methods.

Figures 6 and 7, below, show the optimal portfolio compositions for different risk levels, using the standard deviation as risk metric.

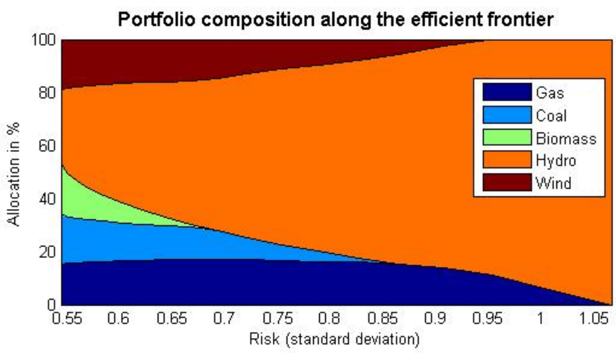

Figure 6: Optimal Energy Matrix composition calculated with Standard Deviation as risk metric, using Kuhn-Tucker technique.

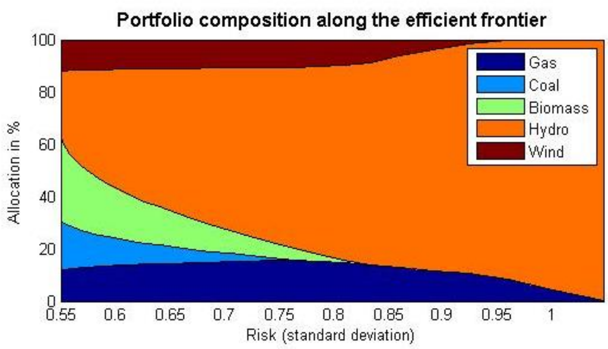

Figure 7: Optimal Energy Matrix composition calculated with Standard Deviation as risk metric, using Kriging Method.

By comparing the results presented in **Figures 6 and 7**, it becomes clear that the proposed methodology (Kriging Method) offers a sound approximation to the exact solution to the problem, as presented in **Figure 6**. Main differences are represented by the trade-off between the utilization of Biomass and Coal energy in mid risk levels, but the general form of the curves are quite similar. This strongly indicates that the proposed methodology approximates the optimization of the energetic matrix appropriately.

Even though the data utilized in this article is hypothetical, the average unit energy production cost and its standard deviation are real. This allows for a discussion on the results presented in **Figures 6 and 7**, in which a low allocation to hydro energy is observed for low risk levels. AWERBUCH and YANG (2007) estimate that the investment risk for new hydropower plants is very high (43%), and LOSEKANN (2013) attributes this fact to delayed building schedules, mainly related to environmental and labor issues, which impact directly the planned cash flow. This problem is potentiated by the high interest rates of Brazil. Indeed, the three large hydro plants under construction in Brazil (JIRAU, RO, BELO MONTE, PA and SANTO ANTONIO, RO) are facing these problems. On the high risk end, new hydro represents 100% of the portfolio, as expected.

5.2 The VAR Model

This section presents the results of the energy portfolio optimization problem using the VaR as a risk measure. The presented results were obtained by the application of the proposed methodology (Kriging Method). Results are presented both in form of efficient frontier (**Figure 8**) and in form of optimal portfolio compositions for various risk levels (**Figure 9**).

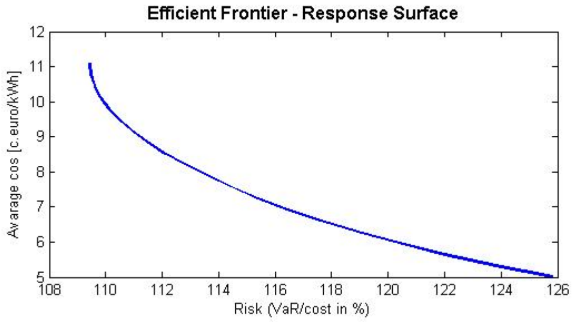

Figure 8: Efficient Frontier for Energy Matrix using Value-at-Risk as risk metric, calculated through Kriging Method.

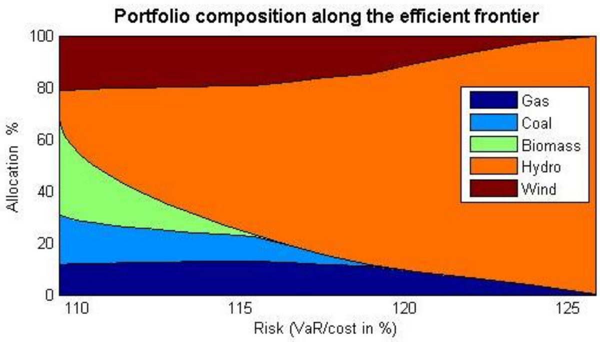

Figure 9: Optimal Energy Matrix composition calculated with the Value-at-Risk as risk metric, using Kriging Method.

Discussion on the results follows at the end of the next sub-section.

5.2 The CVAR Model

This section presents the results of the energy portfolio optimization problem using the CVaR as a risk measure. The presented results were obtained by the application of the proposed methodology (Kriging Method). Results are presented both in form of efficient frontier (**Figure 10**) and in form of portfolio compositions for various risk levels (**Figure 11**).

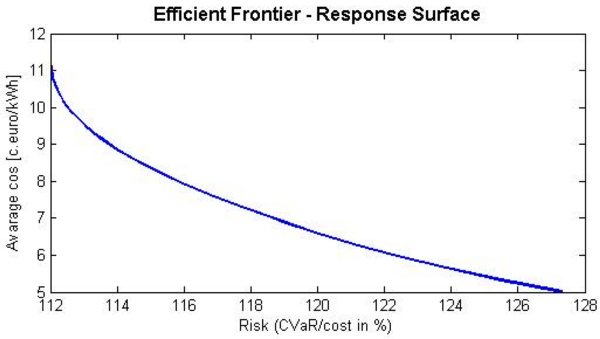

Figure 10: Efficient Frontier for Energy Matrix using Conditional Value-at-Risk as risk metric, calculated through Kriging Method.

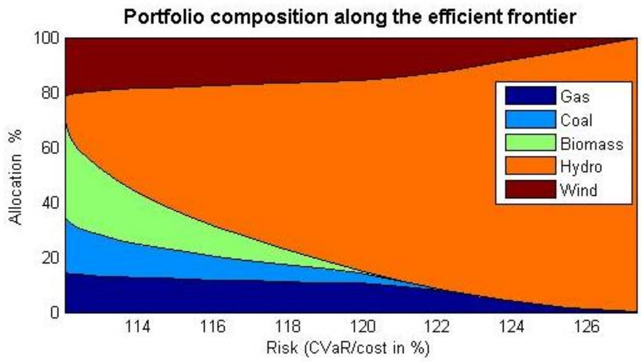

Figure 11: Optimal Energy Matrix composition calculated with the Conditional Value-at-Risk as risk metric, using Kriging Method.

Even with hypothetical data, it is possible to analyze the results presented in **Figures 9 and 11**, in which the VaR and the CVaR are applied as risk measures, respectively. Comparing these figures with **Figure 7** (standard deviation as risk measure), it is noticeable that the allocation of new hydropower to minimal risk levels is lower in the former than in the latter. Environmental and labor issues, as discussed, are sporadic events of supposedly low chance of occurrence, which cause delays in construction schedules. One may interpret these as tail events, captured more efficiently by the VaR and, even more effectively, by the CVaR. The preliminary conclusions on this analysis reinforces the importance of utilizing different risk measures to the energy matrix problem. On the high risk (and low cost) end, all risk measures appoint hydropower dominating the portfolio, as expected.

6. CONCLUSIONS

This paper presented a new methodology for optimizing the energetic matrix utilizing multiple risk measures as the objective function of the problem. The methodology includes first the generation of simulated scenarios in a grid of R^N, in which the real values of the calculated risk are obtained, thus relaxing the restrictive hypothesis inherent to each risk metric in other available methodologies. Second, the smoothened approximation of the response surface – risk surface – is obtained through the application of the polynomial interpolation, as per the Kriging Method. Lastly, the optimization of the smoothened risk surface is performed.

The control of the experiment was performed by comparing the results obtained in the optimization problem using the standard deviation as a risk metric, applying both the conventional Kuhn-Tucker technique and the proposed methodology. The similarity of the obtained results confirms the robustness of the application of the Kriging Method for optimizing energy portfolios. Further applications include the utilization of the Value-at-Risk (VaR) and the Conditional Value-at-Risk (CVaR) as risk measures, characterizing a solid contribution to policymakers and professionals of the energy sector.

Further extensions on the research include the application of the proposed methodology to real data of the energy sector, since the application herein presented includes hypothetical data simulated respecting the primary parameters of the Brazilian electric energy market (namely the mean and the standard deviation of the unit cost for various technologies). Other extensions include the application of the methodology to the problem considering additional restraints related to actual installed capacity, supply guarantee and environmental issues, expressing reality more precisely.

References

ARTZNER, P., DALBAEN, F., EBER, J.-M., & HEATH, D. (1999). Coherent mesures of risk. *Mathematical Finance* , *9*, 203-228.

AWERBUCH, S., BERGER, M. (2003). Applying portfolio theory to EU Electricity planning and policy making. IEA/EET Working Paper, EET/2003/03.

AWERBUCH, S. (2006). Portfolio-Based Electricity Generation Planning: Policy Implications for Renewables and Energy Security, *Mitigation and Adaptation Strategies for Global Change*, Vol. 11, n. 3, 2006.

AWERBUCH, S., YANG, S. (2007). Efficient Electricity Generating Portfolios for Europe: Maximizing Energy Security and Climate Change Mitigation. EIB Papers v.7, 2007.

BAR-LEV, D., KATZ, S. (1976). A Portfolio Approach to fossil fuel procurement in the Electric Utility Industry, *The Journal of Finance*, vo. 31, Issue 3, p. 933-947, 1976.

DA BARROSA, M., RIBEIRO, C., ARAUJO, P. (2014), Risk Surface Approximations for Optimal Portfolios, *Business Analytics in Finance and Industry (BAFI)*, Santiago, Chile, 2014.

DELARUE, E., DE JONGHE, E., BELMANS, R., D'HAESELEER, W., (2011). Applying portfolio theory to the electricity sector: Energy versus power. Energy Economics ,v.33, p.12–23, 2011.

Group of Thirty (1994), Derivatives: Practices and Principles, Group of Thirty, Washington D.C., 1994.

JORION, P. (1997) Value at Risk: the new benchmark of managing financial risk. 3th. ed. New York: McGraw-Hill, 1997.

KREY, B., ZWEIFEL, P. (2006), Efficient Electricity Portfolios for Switzerland and the United States. *Socioeconomic Institute of the University of Zurich*, 2006.

LOPHAVEN, S., NIELSEN, H., & SONDERGAARD, J. (2002). *DACE: a matlab kriging tollbox.* Denmark: Informatics and Mathematical Modelling, Technical University of Denmark.

LOSEKANN, L., MARRERO, G. A.,RAMOS-REAL, F. J., ALMEIDA, E. L. F. (2013). Efficient power generating portfolio in Brazil: Conciliating cost, emissions and risk. Energy Policy ,v.62, p.301–314, 2013.

LUENBERGER, D. G. (1998). *Investment Science.* New York: Oxford University Press.

MARKOWITZ, H. (1952). Portfolio Selection. *The Journal of Finance , v.7*, pp. p.77-91.

PAPALAMBROS (2000), P.Y. e D.J. WILDE, Principles of Optimal Design, *Cambridge University Press*, 2000.

QUEIPO, v. N., PINTOS, S., RÍNCON, N., CONTRERAS, N., & COLMENARES, J. (2002). Surrogate Modeling-based optimization for the integration of statistic and dynamic data into a reservoir description. *Journal of Petroleum Science and Engineering , v.37, n.3-4*, pp. p.167-181.

REGO, E.E. (2012) Proposta de aperfeiçoamento da metodologia dos leilões de comercialização de energia elétrica no ambiente regulado: aspectos conceituais, metodológicos e suas aplicações. 2012. *Tese (Doutorado em Energia) - Energia, Universidade de São Paulo*, São Paulo, 2012.

RIBEIRO, C., CASTRO, R., DA BARROSA, M. (2012), Application of kriging method for estimating the conditional value at risk in asset portfolio risk Optimization, *CLAIO/SBPO*, 2012.

ROCKAFELLAR, R. T., & URYASEV, S. (2013). The Fundamental Risk Quadrangle in Risk Management, Optimization and Statistical Estimation. *Surveys in Operations Research and Management Science, V. 18, Issues 1–2, October 2013, Pages 33-53*

ROCKAFELLAR, R. T., & URYASEV, S. (2002). Conditional Value-at-Risk for general loss distributions. *Journal of Banking and Finance , v.26*, pp. 1443-1471.

ROCKAFELLAR, R. T., & URYASEV, S. (2000). Optimization of Conditional Value-at-Risk. *Journal of Risk , v. 2*, pp. 21-41.

SZOLGAYOVÁ, J., FUSS, S., KHABAROV, N. and M. OBERSTEINER (2011), A dynamic CVaR-portfolio approach using real options: an application to energy investments, *European Transactions on Electrical Power*, Volume 21, Issue 6, pages 1825–1841

YIN, J., Ng, S., & Ng, K. (2011). Kriging metamodel with modified nugget-effect: The heteroscedastic variance case. *Computers & Industrial Engineering , 61*, pp. 760–777.