DIFFICULTIES AND REMEDIES IN SOLVING A BULK PRODUCT LOGISTIC SYSTEM MODEL

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RESUMO

O propósito deste artigo é alertar novatos sobre obstáculos no uso de programação matemática na análise de sistemas logísticos. Dificuldades são discutidas no contexto de estudo de viabilidade de sistema para distribuição de um derivado de petróleo no Brasil. O sistema é constituído de nove plantas, mais de 2.800 pontos de demanda e quinze localizações potenciais para bases. A sazonalidade da demanda e a necessidade de flexibilidade na programação da produção conduziram a uma formulação matemática multiperiodal. O modelo de programação linear inteira mista resultante tinha mais de um milhão de variáveis e milhares de restrições; grande demais para ser resolvido em tempo aceitável e a custo compatível com o orçamento. Foram utilizadas técnicas para agregar dados e eliminar arcos não essenciais, assim como artifícios para eliminar ou avaliar erros introduzidos na redução do modelo. Questões como a representação e interpretação dos resultados e do significado de imprecisões também são discutidos.

PALAVARAS CHAVE. Sistemas logísticos, Localização de instalações, Redução de modelos.

Área principal. L&T – Logística e Transportes

ABSTRACT

The purpose of this paper is to caution newcomers against pitfalls found in mathematical programming applications to logistic systems analysis. Difficulties are discussed in the context of a real feasibility study regarding a system for nation-wide physical distribution of a petroleum derivative in Brazil. The system comprised nine plants, over 2,800 demand points, and fifteen potential warehouse locations. Demand seasonality and the need for flexibility in the production plan called for a multi-period mathematical formulation. The resulting mixed integer linear programming model-instance of over a million variables and thousands of constraints was too large for acceptable running time and to fit the cost budget. Techniques for data aggregation and elimination of extraneous arcs were employed. Devices for eliminating or simply assessing errors introduced by model reduction were also used. Issues such as representation and interpretation of results and the significance of inaccuracies are also discussed.

KEYWORDS. Logistic systems, Facilities location, Model size reduction.

Main area. L&T – Logistics and Transportation.

1. Introduction

Mathematical programming is increasingly more affordable to logistic systems analysts thanks to advances in computer-based modeling systems, solvers, data management and geographic information systems (GIS). In some cases, particularly in facilities location analyses, the model formulation is only a small part of the total effort spent in the study and the resulting model may be surprisingly simple. Assembling model instances, and solving them, may bring surprises. The seasoned location analyst certainly has already faced many challenging implementation problems and developed effective remedies. However, for the novice, except for a few accounts like Conceição et al. (2012), the existing literature on such pitfalls and tricks may not provide him the expected assistance. The novice will certainly find good help and advices in some quite popular texts, such as Geoffrion (1975), Geoffrion and Powers (1995), but though the proposed solutions are usually well conceived, effective, general and elegant, they are often beyond the time, budget and expertise available to the project. With some creativity the troubled analyst may be better served with quick-and-dirty ad hoc solutions to practical hindrances. Even if not the best in terms of accuracy or computation time, simple approaches may produce satisfactory results and be winners if time and cost budgets are taken into account.

In this paper, we have no intention to provide generally applicable artifices to overcome frequent implementation troubles, nor do we suggest that quick-and-dirty tricks are as good as precise well-conceived and scientifically tested methods. Our intention is solely to highlight often overlooked practical considerations and encourage the troubled novice to use creativity and caution to face implementation challenges, when the literature search provides no viable help.

To this purpose, we will present problems found and solutions forwarded in a feasibility study regarding a system for nation-wide physical distribution of a petroleum derivative in Brazil. Obtaining and processing data, and reducing the size of the model-instance were the main hassles faced.

The multi-period distribution planning system was modeled as a mixed integer linear programming model-instance of over a million variables, requiring an unacceptably large amount of time to be solved. Other observed difficulties were related to the numerous and spread out demand points, to the determination of production capacities, which depended on the demand of other derivatives, and to the way the distances were originally computed in the distribution network.

Among other remedies, to make the model practically solvable model size-reduction was sought by aggregating demand points and eliminating “almost surely” infeasible arcs. In some cases the difficulty could be eliminated at some cost, in others cases, reduced. How much precision is enough and how much effort to spend in improving the precision of each aspect are also difficulties rarely considered and discussed in the literature. In spite of the sensitivity analysis techniques available, it remains a thorny issue that will merit some discussion in this paper. Other important issues concerning the project management and relations with the clients will not be addressed here.

In the rest of the paper we will describe the problem and the model (section 2), discuss the more relevant difficulties encountered and solutions adopted (section 3), and conclude summarizing the main lessons learned and making general comments (section 4).

2. The Problem and its Model

The feasibility study was motivated by a general feeling among the producers and distributors of a petroleum derivative that the present multi-company distribution system was inefficient and was not meeting the customers’ needs fully. The study of a new venture specialized network of stocking bases was commissioned by a consortium of distributors. The logistic system was comprised by nine producing refineries (all with some storage capacity, and two of them capable of receiving imports), over 2,800 demand points scattered all over the country, and fifteen candidate locations for stocking bases. Highway transportation was virtually the only mode, and the investment and operating cost of the truck fleet and bases was to be considered together with the inventory costs. Besides location, the size of the bases and fleet, as well as the storage capacity at each stocking point were to be determined. Clearly, the complexity and dimensions of the problem called for a mathematical programming model. Given the objective of economic feasibility study, it was agreed that the analysis should ignore the fact that different and autonomous competing companies would be participating in its operation. The commercial arrangements to guarantee efficient operation within the market governance would be arranged latter in agreement with the industry-regulating agency.

Some of the main initial questions were the following: a) How to best meet the customers demand using the existing infrastructure? b) What are the ideal storage capacities, base locations and fleet size required to meet the geographically dispersed demands? c) Where are the bottlenecks? d) What is the potential savings in operating expenses? e) How much investment will be required? f) What are the product flow patterns in each period?

An interesting aspect present in the problem under study is that the cargo transshipment points (bases) bring about several benefits as follows: a) cheaper inbound transportation cost by using long haul mode for larger inbound cargo and more flexible mode for outbound, b) smoother production requirements in the face of seasonal demand by building anticipation inventories, c) better service due to shorter delivery lead-times, whenever a base is closer to the customer than a refinery, and d) better utilization of the fleet.

The last benefit is obtained by using (otherwise idle) trucks during the low demand season for advancing cargo to a base closer to the customer than the refinery. Then, later on at the peak season, when transportation needs are higher, the cargo will be closer to these customers lessening the need for trucks. Though not explicitly considered in the model (necessary data was not available), the inventory at the bases would serve as a pooled safety stock for the nearby customers, thus further enhancing the service level (benefit *c* above).

Given the significant demand seasonality, and the benefits *b* and *d* above, the study called for a multi-period mixed integer linear programming model, with a planning horizon no shorter than one year. Consequently, the model size will grow linearly with the number of periods in the planning horizon. Therefore, the model of the system can be seen as a single product warehouse location model. The warehouses have a fixed investment cost, and a variable capacity with constant marginal total cost. Operational transportation costs, as well as the fixed and variable costs of the truck fleet are considered.

2.1. The model

The model developed for the feasibility study is quite simple and is briefly commented below. Due to space limitation details are not relevant for the purpose of this short paper and a full description is omitted. As usual, the model served to deepen and structure the discussion and the accumulated knowledge about the problem, becoming itself the expression of this understanding. In fact the model was developed *pari passu* with the problem definition and validation also done progressively at project seminars.

The optimization model was implemented in the Lingo language whose similarity with the standard mathematical notation was quite handy for making alterations, and keeping documentation update when needed. The main features of the model are summarized below.

The objective function to be minimized corresponds to the total logistic cost of the system within the planning time span as described below:

total logistic cost =
total cost of
[shipping and handling charge +
+ carrying stock at refineries and bases +
+ imports (only the increment over the domestic production cost) +
+ maintaining the bases (investment and operating) +
+ truck fleet (investment and operating)].

Note that the production costs (and of imported material) are not explicitly considered, because constraints impose complete demand fulfillment, making them irrelevant for the decisions.

The fleet cost is composed of fixed cost (investment cost, maintenance, insurance and driver's wages, among others) and operating cost. Investment cost is proportional to the fleet size, while the operating cost is the sum of the cost of all round trips performed.

The cost of a trip is also composed of a fixed cost that includes cargo handling and a variable cost proportional to the travelled distance. The necessary fleet size is internally determined as the largest truck-requirement in all periods. Knowing the time a truck (with its driver) is available for travelling in each period, the number of trucks required in each period is simply the total time travelled in the period divided by the available hours of a truck. The time spent in a round trip is based on the origin-destination distances, on the average travel speed, and on the cargo handling time. Since all trucks have the same capacity, the number or trips necessary for hauling the required material from a given origin to a given destination is evaluated simply by dividing the total weight to be moved by the truck capacity.

Other constraints in the model are the usual ones in facility location, such as demand fulfillment and material balance at the refineries and bases. Imports were viable only at the refineries at the coast, and though virtually unlimited, an upper bound at each refinery was imposed for ease of building "no imports" scenarios.

As production planning at the refineries considers the oils available and the demand of all products, refinery capacity is left as a bounded variable in each period, due to the limited freedom that a refinery has to decide on its product mix. Therefore, the model features a flexible capacity for each refinery in each period.

To keep the model linear (though mixed-integer), for each period and facility, two constraints are considered: (a) the ending inventory should not exceed its (variable) storage capacity and, (b) with the use of a binary variable, the capacity must be zero if the additional storage or basis is not built.

The finite-horizon model was intended to study long-term performance of the logistic system. This makes the initial condition (inventory at the facilities) irrelevant in the long run. However, setting initial inventories to zero poses a problem, because if production and import capacities are not large enough the problem may result infeasible. Additionally, if left loose, the optimization will yield final zero inventories, because it is a finite horizon model and there is a cost to hold inventories of no use.

For a more realistic representation of the problem it was decided to leave the initial and final inventories in the model free to vary, and add a constraint requiring that the value of the final inventory be the initial value increased by an annual growth factor. Assuming that the current demand pattern will repeat endlessly this produces something like a "steady state" solution. This solution should provide information realistic than an empty-start and empty-end solution.

3. Difficulties and Remedies

Modeling the problem turned out to be easy and with no surprises once the problem essence and details were well understood. The hard part – implementing the model and solving it – was aggravated by time and budget constraints.

3.1. Software Choices and Integration

As already mentioned, for ease of perusal and alteration, it was decided to use an algebraic language similar to the standard mathematical language. The modeling environment and solver used was the Lingo 14 Industrial, capable of solving problems with up to 16,000 constraint, 32,000 variables of which 3,200 integer. The possibility of interfacing the solver with spreadsheets and database management allowed moving data input from "in model", to the spreadsheet, and to the database, as the model became increasingly more complex, reliable and stable. More complete modeling environments and powerful solvers could make it easier to implement the model and meet the pre- and post-processing needs, but were discarded on grounds of cost and also because model system was not for frequent use and was to be run by technical people with no need of sophisticated interfaces. The choice seemed adequate in spite of the limitations of Lingo's algorithmic language, its relatively slow solver and limited interface with databases. Building the interfaces in a high level general purpose language such as C^{++} could remove these inconveniences at the cost of substantial programming effort. Database management was implemented in Access, and the spatial data presented in ArcView. Lingo does not offer simple direct data exchange with ArcView, and though interchange between ArcView and Access is possible, it was not considered worthwhile the effort, since the desired maps depended very much on the ever changing aspects of interest. Consequently, all data interchange with ArcView was made manually using Access queries and ArcView data handling features.

3.2. Implementation Stumbling Blocks

Developing the model required less than one third of the effort. The bulk of the difficulties came from the data requirements and the large size of the model.

Data requirements.

Spatial location data were available from the Brazilian Institute of Geography (IBGE) and Statistics consumption data from the Brazilian National Petroleum Agency (ANP), the petroleum-regulating agency. Consistency checks indicated that the quality of the data was satisfactory, though they exposed some accuracy problems that were readily resolved. Because it was known that product shortages occurred in some periods, consumption data would probably underestimate the actual demand. There was no satisfactory way to correct this problem, and to avoid questionable corrections, it was decided to make no correction and leave the problem to be discussed with the help of sensitivity analysis.

Evaluating millions of highway distances was difficult, because most of the itinerary calculators available at reasonable cost were not reliable. This issue was more acute in areas of scant highway network and when the viable paths involved inland navigation often of unsatisfactory conditions. The difficulty was most severe in the undeveloped Amazonian and in the Central-West regions, where itinerary calculators produced distance errors of a few thousands percent. However, statistical regression analysis of a sample from other regions has shown a reasonably uniform relationship (close to 1.25) between highway distances and "as the crow flies" distances. It was realized that, in these areas, the use of true highway distance did not worth the effort, because the small gain in precision over adjustments to the as the crow flies distances would be inconsequential in face of the uncertainties in other data estimates. Moreover, these itinerary calculators are for automobiles, not for trucks, and did not consider that road conditions and the topography may have substantial influence on the cost and duration of the trip. Fortunately, the problematic itineraries were few and could be handled manually by routing experts.

The sponsors (mostly based in previous cost and performance surveys) provided costs and operational performance data.

Model-size Reduction.

The crude model instance with 9 refineries, 15 bases, and some 2,800 demand points (customers) would involve around 50,000 constraints, and over a million continuous variables and 21 binary variables. Solving such a problem with the computation resources available was quite difficult. The multi-period structure of the problem suggests the application of the Dantzig-Wolfe decomposition (Dantzig and Wolfe, 1960), but its implementation would require unaffordable time and effort, with no guarantee of reasonable computation time due to its known slow convergence.

It was decided to reduce the model size through spatial aggregation of demand points and elimination of nonessential arcs (*i.e.* arcs that cannot be in the optimal solution). Spatial aggregation was done, as usual, by concentrating the demand of clusters of demand points in their demand-weighted medians by means of the *p*-median problem. This, in turn created two other difficulties, namely: (a) how to solve a *p*-median problem with 2,800 demand nodes and candidate locations with more than a hundred medians, and (b) how to deal with errors introduced by the aggregation (Hillsman and Rhoda, 1978).

The easy-to-implement Maranzana (1964) heuristic method, starting with clusters formed by the 137 Brazilian Mesorregiões (neighboring cities geographically akin) was used with reasonable results. A more reliable algorithm, such as the ones based on the Teitz and Bart (1968) method, is still to be tried, because their efficiency depends on clever data structures of time consuming implementation.

Total elimination of the aggregation error seems impracticable, due to the possibility of holding stock from for future periods. However, it was possible to reduce the error in the total transportation cost from an origin to the clients of a cluster by using a demand-adjusted distance as proposed by Current and Schilling (1990).

Aggregation did not suffice; with the consideration of several periods, the storage, and transportation capacities the model was still too large for comfortable reasonable running times. Model reduction techniques are found in the literature (e.g. Letchford and Miller, 2014), but they are meant to simple static location and p-median problems. Adaptation to the complexities of the current model would not be time-viable.

Just by visual inspection it becomes clear that a client in the Southeast of Brazil – where most of the production capacity is concentrated – will never be served by a refinery with small capacity and far away in the north of Brazil. This suggested the application of an *ad hoc* method for eliminating nonessential arcs (usually called “extraneous variables”, see for example, Thompson et al., 1966). The method considers a single period version of the model and its main idea is to eliminate from consideration any arc that cannot be in the optimal solution of the multi-period model. Thus, for one source (refinery or base) at a time, this method minimizes the total cost leaving unbounded the capacity of one of the current refinery, while holding the capacities of all other refineries at their lower bound. Clearly, no non-optimal arc emanating from this refinery can become optimal, if the capacity of any other refinery is increased from its lower bound. Hence, any arc not in the solution set of any of the problems solved can be in the optimal solution of the complete single-period problem. This elimination method was very effective achieving a reduction to almost a half. Given that stocks may exist at refineries and at bases, it seems hard to prove formally that the method does not eliminate arcs that could be in the optimal solution of the multi-period model. However, since the production capacity range of all refineries was quite wide, it very unlikely that it eliminates any arc that could be in the optimal solution.

3.3. Interpreting the solutions

For management it is crucial to know how stable the optimal solution of a problem is, since a very unstable solution adds little to the decision process. It is thus important that sensitivity analysis and other post solution analyses be performed in a process that involves the decision makers. In the current case, because the model has many “degrees of freedom” (*i.e.* many different ways of satisfying the customers demand) some significant alterations in the model had little impact on the value of the objective function, but produced quite different solutions (in terms of number and location of the bases, in imports or in fleet size, among others relevant aspects). This came with some surprise, and quoting Allan Manne (1961): “… is fortunate from the viewpoint of the business executive [but] may be disastrous, however, from the viewpoint of an economist trying to forecast investment choices on the basis of an optimizing model …”. Indeed “shallow objective functions” that do not produce a sharply defined optimal solution are frequent and require extensive investigation taking into account important considerations too hard, or impossible, to model. It also means that though model parameters, such as costs and operational performance metrics, may not affect much the total logistic cost but may produce radically different solutions. To discuss with managers which are the more critical parameters a "tornado diagram" (Howard, 1988) involving the model parameters of more disputable precision may prove helpful.

As put by Geoffrion (1976) paraphrasing Hamming (1962): "The purpose of mathematical programming is insight, not numbers." Indeed, running many diverse versions (with the demands aggregated in respectively 6, 27 and 137 medians), scenarios (especially different demand growth rates) and cases (including with and without imports, with and without bases, with different candidate location for the bases) of the problem we managed to uncover some important aspects of the problem. Organized interaction with the decision makers was necessary to raise the more relevant questions, while keeping within bounds the effort required for running, analyzing and interpreting the results. As a corollary, the model implementation must be efficient enough, both in running time and in setup effort to make model exploration feasible.

4. Discussion and Conclusions

In this paper, we have presented some practical problems faced in the economic analysis of a bulk material distribution system. Our intention was to expose in simple terms some aspects seldom present in the literature, hoping that this will encourage newcomers to logistic systems design creatively look for practical, though *ad hoc*, solutions that can produce satisfactory results.

When first face a real problem, it is natural that we associate it to something we have seen before especially similar, and often more general, situations reported in the literature. Our argument here is that peculiarities of the case may open opportunities to simplify the formulation and the solution of the problem. A deep understanding of the actual situation includes the objectives of the study. It was the long long-term viewpoint and the recognition that the historical data available was not sufficient for a meaningful projection of the demand at each location that led us to study the steady state solution rather trying to extend the analysis horizon. The fact that the plant capacities could vary within a range was an additional complexity element in relation to the classic facilities location problems. However, an *ad hoc* heuristic for model size reduction was possible by recognizing that the lower bound on these capacities could help dropping arcs that could never be in an optimal solution.

Of course, one should be careful to assess possible inaccuracies resulting from heuristics, but the point is that, given a limited time and cost budget, a good balance between accuracy of methods and depth of the analysis is essential. To achieve this balance, adequate error bounds are often not too difficult to assess by means of limited experimentation.

Among the issues discussed along this work, there are some considerations resulting from our experience that merit reinforcement.

Powerful mathematical programming software and hardware makes the analyst life much easier, but not always affordable. Nonetheless, besides affordability other practical aspects must be considered. (a) Is the project team familiar with the software? (b) Does the modeling language and structure facilitate model perusal and documentation? (c) Is the mathematical programming software amenable to interfacing with data management systems and other software used in the project? The use of some GIS system in location problems is a quite a must in every phase of the project. Without visualization a clear spatial interpretation of the massive input and output clear data would be all but impossible. Software integration is important for enabling extensive analyses but must be balanced with the cost, effort and time required to build. If the many model runs (different tests, versions, scenarios, and cases) required are to be performed by team members, a fully integrated system with a very easy to operate and user-friendly interface may not be the more cost effective choice.

Careful interpretation of results is certainly a very important task. A model cannot and should not represent all aspects of the real problem. Rather than helping, too much detail wastes effort and obliterates the essence of the problem. A progressive refinement approach to modeling with frequent interaction with the decision makers is very convenient. It allows both, the analyst and the management to continuously refine their understanding of the problem with the model playing a central role in guiding the discussions, pointing out flawed conceptions and guiding the investigation telling the important from the irrelevant.

Large problems of location analysis in networks usually present geographically dispersed elements (demand points, potential locations for facilities). This sparsity suggests that the influence of an element upon others tends to diminish as the elements are farther apart. Our literature search revealed that this spatial attenuation has not yet received the due attention by researchers looking for efficient ways of decomposing large location analysis problems.

References

Conceição S.V., Pedrosa L.H.P., Neto A.S.C., Vinagre M. and Wolff E. (2012), The facility location problem in the steel industry: a case study in Latin America. *Production Planning & Control*, 23, 26-46.

Current, J. R. and Schilling, D. A. (1990), Analysis of errors due to demand data aggregation in the set covering and maximal covering location problems, *Geographical Analysis*, 22, 116-126.

Dantzig, G. B. and Wolfe, P. (1960), Decomposition principle for linear programs, *Operations Research*, 8, 101-111.

Geoffrion, A. M. (1975), A guide to computer-assisted methods for distribution systems planning, *Sloan Management Review*, 16, 17-41.

Geoffrion A. M. (1976), The purpose of mathematical programming is insight, not numbers, *Interfaces*, 7, 81-92.

Geoffrion A. M. and Powers R. F. (1995), Twenty years of strategic system design: An evolutionary perspective, *Interfaces*, 25, 105-127.

Hamming, R.W., *Numerical methods for scientists and engineers*, McGraw-Hill, New York, 1962.

Hillsman E. L. and Rhoda (1978), Errors in measuring distances from populations to service centers, *Annals of the Regional Science Association*, 12, 74-88.

Howard, R. A. (1988), Decision analysis: practice and promise, *Management Science*, 34, 679-695.

Letchford, A. N. and Miller, S. J. (2014), An aggressive reduction scheme for the simple plant location problem, *European Journal of Operational Research*, 234, 674–682.

Maranzana (1964), On the location of supply points to minimize transport costs, *Operational Research Quarterly*, 15, 261-270.

Teitz M. B. and Bart P. (1968) Heuristic methods for estimating the generalized vertex median of a weighted graph, *Operations Research*, 16, 955-961.

Thompson, G. L., Tonge, F.M. and Zionts, S. (1966), Techniques for removing nonbinding constraints and extraneous variables from linear programming problems, *Management Science*, 12, 588-608.