

SIZING AND SCHEDULING RESOURCES: A DECISION SUPPORT SYSTEM APPLIED TO OIL RIGS SCHEDULING

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ABSTRACT

In the competitive and growing oil and gas industry, efficiency plays a crucial role. Activities and its associated resources must be efficiently scheduled in order to maintain satisfactory production, reducing operational costs. However, this is still a critical and difficult task. The present paper presents a decision support system including a mathematical programming model designed to facilitate and automatize activities' resources scheduling. The model estimates the optimal size of a homogeneous resources fleet for a set of activities and generates schedules for each activity and its resources. A successful application of this model is shown by a case study of activities executed by oil rigs over 200 wells and over 10 years of time horizon. With oil rigs sizing and scheduling results currently implemented in the company, this case study deals with investments in the order of billions of dollars.

KEYWORDS. Sizing. Scheduling. Optimization. Resources. Oil and Gas.

Main area: P&G - PO na Área de Petróleo e Gás



1. Introduction

In Brazil, the oil and gas industry assumes an important and dynamic role in the national economy and represents a huge amount of investments in the country. Between 2011 and 2014, expected investments are approximately US\$ 162 billion, which represents 61.5% of total investments in Brazil's industrial segments, according to Puga and Borça Jr. (2011). In addition, estimates of total investments until 2027 vary between US\$ 480 billion and US\$ 1,350 billion, as stated by De Almeida and Accurso (2013).

Due to its importance and its high investments, this industry presents an enormous potential of growth. However, in order to fully develop this potential, efficient planning in this industry is considered vital. Within this planning there are some critical and complex processes, such as oil and gas activities scheduling and its resources sizing and allocation, which are considered difficult decisions.

Sizing and scheduling problems have been long discussed in the literature. Although addressing similar problems, some authors propose different considerations and solving methods, such as mathematical programming, different algorithms and heuristics.

Eagle (1996) used simulated annealing algorithm to schedule homogeneous drilling rigs with the objective of maximizing net present value. Da Costa et al. (2005) addressed scheduling homogeneous rigs through the heuristic maximum priority tricriteria in order to minimize the oil production loss. Aloise et al. (2006) dealt with a similar problem, but considered a heterogeneous fleet of workover rigs through another heuristic, the variable neighborhood search. Still with the same objective of reducing oil production loss, Ribeiro et al. (2011) solved the workover rig scheduling problem considering time windows by using simulated annealing and comparing to the metaheuristics GRASP, Genetic Algorithm and Scatter Search. Gharbi (2011) analyzed different mathematical programs to optimize the scheduling of a heterogeneous fleet of drilling rigs to serve a set of heterogeneous wells. The author considered different forms of schedule optimization, such as optimizing time, production, number of rigs, moves and cost.

Most works, as the ones previously mentioned, treated onshore oil fields whereas only few works treated the offshore environment where the costs are significantly higher than in onshore operations. Iyer et al. (1998) solved the scheduling of heterogeneous drilling rigs in offshore oil fields through a mathematical model integrating facility allocation, production planning and scheduling. Also considering offshore fields, Bassi et al. (2012) addressed the workover rigs scheduling problem, a simplification of the rig scheduling problem as it deals only with workover rigs, which are homogeneous. Bassi et al. (2012) contributed to research on scheduling rig problems as it presents, to the best of our knowledge, the first stochastic approach of the problem, where the service time is assumed to be uncertain.

From the performed literature review about oil rig sizing and scheduling problems, no specific work suggests a methodology that deals with both sizing and scheduling problems at the same time, which is the main contribution of the present work.

Given the importance of this industry, this paper presents then a methodology to size and schedule oil rigs, which are expensive resources, for offshore oil projects. As these resources can represent daily rates of up to US\$600,000, evaluating all aspects of rig productivity, including efficient planning of their use, is of great importance according to Osmundsen, Roll and Tveterås (2010).

In order to optimize rig utilization in this industry, this paper addresses the problem of sizing and scheduling a homogeneous fleet of workover rigs. It deals with decisions at strategic level, such as fleet sizing, at the same time as tactical level decisions such as scheduling. To the best of our knowledge, this is the first academic approach that deals with sizing workover rigs in the oil and gas industry and also with scheduling, integrating the solution into a single mathematical programming model. The proposed



model minimizes the total number of workover rigs necessary to cover all demand, at the same time that it maximizes the utilization of each rig when scheduling them to specific wells. In this sense, it is crucial to determine which rig should visit which well at which instant, as well as to estimate the number of resources needed to execute all activities.

The next sections in this paper present the definition of the problem, followed by an explanation of the mathematical model used and a case study with its results. Finally, a conclusion is made with proposals of future works.

2. Problem Description

Operations in oil and gas companies are divided into five main sequential phases: exploration, drilling, appraisal, completion and production. The exploration phase consists in detecting potential oil and gas deposits through geological and geophysical methods. When positive evidence of oil is found during exploration, the well is drilled by rigs to determine the presence or absence of oil and gas. Due to its processes and characteristics, exploration and drilling phases have costly and risky operations as even when positive evidence of oil presence is found during exploration, it can highly likely be absent or not commercially profitable when drilling is made. In case oil or gas is confirmed in a well, adjacent wells are drilled during the appraisal phase to collect additional data in order to infer oil and reservoir characteristics. Upon a company's decision of developing a well according to economic analysis, the completion is done. This phase regards a set of operations designed to equip the well to produce oil and gas safely and economically feasible. After completion, the production phase starts, involving the direct production of oil and gas and the indirect production of by-products such as water. These products are drained through strings and ducts from the well to the platform located at sea level. Besides these phases, during a well's productive life, it may need interventions or workovers to control and maintain a satisfactory well productivity or to correct well problems such as oil flow loss and mechanical failures.

For a successful operation in an oil well, each phase previously described must operate with its specific resources, which can include rigs, equipment and others. Due to its high costs, limited number and vital importance for most operations, rigs can commonly be considered critical resources in the oil industry. A good planning of these resources allocation into wells is essential to guarantee a competitive and sustainable oil and gas production cycle. An inefficient schedule or an unnecessary quantity of rigs could cause delays, reduce productivity and increase costs. Planning schedules for each phase of the cycle is thus a difficult and challenging task faced by oil and gas companies.

Given that, this paper presents a tool to help decision-makers in sizing and scheduling a homogeneous fleet of Light Workover (LWO) rigs to execute the installation of wet Christmas tree, which are sets of valves, connectors and adaptors situated over the wellhead. This subsea equipment must be installed in every well after completion is done. It aims to control fluids flow to the surface safely. After this equipment is installed, an operation called tie-in is executed to connect the wet Christmas tree to the platform through ducts in order to lift oil to the surface. Only then the well is enabled to start production. Figure 1 schematizes this problem activities sequence from completion until production.



Figure 1. Problem Activities Sequence.

For this type of problem, there are given a set of wells, their possible time



windows for the wet Christmas tree activity and this activity duration estimate for each well. Through mathematical modeling, a fleet of rigs is then allocated to execute each well's activity with the intent of maximizing rigs utilization and minimizing the number of resources needed. This way a homogeneous fleet is sized to attend demand in each one of the wells, according to the rig schedule generated by the modeling, which allocates activities to rigs respecting time windows. This problem deals, therefore, with strategic level decisions when sizing a fleet at the same time as with tactical level decisions while scheduling these resources into activities.

3. Model Formulation

In this section, we present a formulation of the mathematical model proposed by this paper along with a description organized by sets, parameters, variables, objective function and constraints. This is a model with a discrete planning horizon with time intervals that have the same length. Although all variables are binary, some may be relaxed because of the model's mathematical structure.

3.1 Sets

- *r* Rig: number of the rig.
- *t* Time: time period.
- *w* Well: wells.

3.2 Parameters

$\mathrm{D}_{\mathrm{w}} \in \mathrm{Z}_+$	Duration:	Number of time periods a rig activity lasts in well <i>w</i> .
$PV_r \in Z_+$	Priority Value:	Rig utilization priority value.
$\mathrm{TW}_{\mathrm{w},\mathrm{t}} \in \{0,1\}$	Time Window:	1, if a rig can start an activity in well <i>w</i> at time period <i>t</i> ; 0, otherwise.
3.3 Variables		
$\mathbf{e}_{w,t,r} \in \{0,1\}$	Execution:	1, if rig <i>r</i> executes activity in well <i>w</i> at time period <i>t</i> ; 0, otherwise.
$f_{w,t,r} \in \{0,1\}$	Finish:	1, if rig <i>r</i> finishes activity in well <i>w</i> at time period <i>t</i> ;

$s_{w,t,r} \in \{0,1\}$ 0, otherwise. $s_{w,t,r} \in \{0,1\}$ 1, if rig *r* starts activity in well *w* at time period *t*;0, otherwise.

3.4 Objective Function

Minimize $\sum_{w,t,r} PV_r s_{w,t,r}$

(1)

The objective function (1) minimizes the total number of rigs used by adding all activities executions weighted by given priority values. Due to the definition of priority values, the objective function maximizes, at the same time, the utilization of each rig.

The decision variable $s_{w,t,r}$ is a binary variable that equals one for the period in which the rig starts to be used in a well and zero in all other periods. By multiplying this variable to the priority value PV_r, the objective function grows by the value of the priority for every well that the rig attend. By minimizing $\sum_{w,t,r} PV_r s_{w,t,r}$ the model certifies that one rig is used up to its capacity before considering a second rig to satisfy demand. Therefore, it maximizes the utilization of each rig while minimizing the total number of rigs necessary to operate all the wells within the given time windows.



3.5 Constraints

The following constraints represent the problematic of sizing and scheduling LWO rigs.

$s_{w,t,r} \leq TW_{w,t}$	$\forall w, t, r$	(2)
$f_{w,t,r} \leq TW_{w,t}$	$\forall w, t, r$	(3)

$$\sum_{t r} e_{w,t,r} = D_w \quad \forall w \tag{4}$$

$$\sum_{t=r} S_{w,t,r} = 1 \quad \forall w \tag{5}$$

$$\sum_{t,r} f_{w,t,r} = 1 \qquad \forall w \tag{6}$$

$$\mathbf{e}_{w,t,r} = \sum_{t' \le t} \mathbf{s}_{w,t',r} - \sum_{t' \le t-1} \mathbf{f}_{w,t',r} \quad \forall w, t, r$$
(7)

$$f_{w,(t+D_w-1),r} = s_{w,t,r} \quad \forall w, t, r$$
 (8)

$$\sum_{w} e_{w,t,r} \le 1 \quad \forall t,r \tag{9}$$

Constraint (2) indicates that a rig can only start an activity in a well at periods according to the time window. If there is no time window for the rig to operate the well, it is not possible for the rig to start the activity. Constraint (3) is equivalent for the end of a rig's activity on a well. It indicates that a rig can only finish an activity in a well at periods within its time window. Constraint (4) indicates that, for each well and rig, the sum of periods of an activity execution must equal exactly the duration of the rig activity in that well, D_w . Constraint (5) implies that a rig's activity must start only once in each well, at a given period with only one rig. It assures that a rig cannot start activities more than once in the same well and that it must start in some period. Constraint (6) is the equivalent for the end of the activity. assuring that a rig cannot finish activities more than once in the same well. Constraint (7) sets the execution variable $e_{w,t,r}$ as 1 for every period between $s_{w,t,r} = 1$ and $f_{w,t,r} = 1$, inclusive, for each well and rig. The activity of a rig is executed in all periods from the activity start until its end, including both starting and finishing periods. Constraint (8) assures that the variable $f_{w,t,r}$ assumes the value 1 exactly $D_w - 1$ periods after the variable s_{w,t,r} does. It defines that the time period passed between start and finish periods of a rig's activity is exactly the duration of the activity. Constraint (9) indicates that each rig can execute activity in only one well at a time.

4. Case Study

A case study was held considering one of the largest oil companies in Brazil. It comprehended data from over 200 wells located offshore. Within the data, each well is classified into test wells or normal wells. Fernández et al. (2009) defines a test well as a well drilled with the objective of determining oil presence and its productivity and collecting data for economic analysis. Thus, test wells activities will assume priority in execution as normal wells development depend on test wells results. Additionally, each well is associated with a time window, in which the proposed activity wet Christmas tree installation must occur. This time window start date is determined by the completion phase finish date, when the well is equipped to start production. On the other hand, this time window finish date is determined by the tie-in process start date, when ducts are installed to connect the well to the platform and allow oil elevation. Figure 2 shows a general time window in which a wet Christmas tree installation activity must be allocated.





Figure 2. Wet Christmas Tree Installation Time Window.

Furthermore, each well has a wet Christmas tree installation duration time estimate. Due to relative proximity between wells, rigs movement time between their scheduled wells may be considered small in comparison to activities duration. Thus, as a simplification, total activity duration considers the wet Christmas tree installation duration plus a constant moving time between the well being served and any other subsequent well where the rig must move to.

This case study time horizon was defined by the data span and delimited from the earliest to the latest date registered in the data, which extends for 128 months. Although activities duration vary according to each well's specificity, within the data set considered, this value ranges from 14 to 51 days.

To guarantee priority in their activity execution over normal wells, test wells time window is shortened to only fit the activity within the time window. This approach fixes test wells activities start at the earliest possible date and prevents the model to rearrange this allocation, securing their priority. Other option is to consider a small flexibility to test wells, in which their time window accommodates the activity duration plus a small flexibility. This approach also concedes priority to test wells, but add some flexibility in activity allocation, what may improve the proposed model solution. According to the company, a plausible flexibility for test wells activity execution is 14 days.

Even after applying these data treatments, due to the mathematical model size and the number of input data, the problem would not be computationally feasible if solved with data in a daily basis. Therefore, a conversion of time dependent parameters into a weekly basis was made necessary.

4.1 Decision Support System

To solve this problem, a decision support system was developed. Input data was provided by the company in Microsoft Excel spreadsheet format. This data, including wells, their classification, time windows and activities durations, was exported to a database stored at Microsoft Access, where the data treatment was done. Due to the database management system and the considerably large size of the proposed model, treated data including all model sets and parameters was inputted in AIMMS, a mathematical programming software, where the optimization model was developed and implemented. The model was solved by CPLEX 12 software and involved 76,497 variables, from which 23,032 were integers, and 104,905 constraints. This model decision variables were returned as results to the same database in Microsoft Access, which transformed the results into queries and analysis. These were then exported to Microsoft Excel to generate a final solution composed by analysis, graphs, schedules and Gantt charts in an accessible and understandable way to the users. The architecture of this decision support system is represented in Figure 3.





Figure 3. Decision Tool System Architecture.

4.2 Results

The present case study analyzed three scenarios, numbered 1 to 3, which included data from all wells considered in this study. Scenario 1 is considered the base case and treats all wells as normal wells, i.e., disregards test wells priority characteristics. Scenario 2 contemplates test wells nature and applies its specific cuts, restraining its time window to the exact duration of the activity. Scenario 3 also considers test wells differences, but consider 14 extra days of flexibility to each test well time window. Table 1 summarizes each scenario main characteristics, similarities and differences.

Table 1. Scenarios Summary.

Scenario	Are test wells treated differently from normal wells?	If considered in the scenario, do test wells have flexibility?				
Scenario 1	No	Not considered				
Scenario 2	Yes	No				
Scenario 3	Yes	Yes				

These three scenarios were implemented and the optimization model was run to generate LWO rigs fleet sizing and scheduling solutions for the wet Christmas tree installation activity. Figure 4 shows a comparison between each scenario's solutions. Each one of the graphs demonstrates the total number of rigs utilized in the time horizon considered and their utilization by year, following the color code presented in the same figure, where dark blue denotes high utilization values while light red denotes low utilization values.

According to the mathematical model objective function, rigs have their utilization value maximized in order to avoid rigs idleness. By considering a crescent priority value as a penalty, rig 1 is chosen to be allocated as much as possible before rig 2 is allocated. Similar thought is applicable to higher number rigs. For this reason, rig 1 is more utilized than the others in any scenario, followed by rig 2, rig 3 and so forth.

According to Figure 4, Scenario 1 presents a total number of four rigs to attend all demand during time horizon. Scenarios 2 and 3, however, present a total number of six rigs needed. This increase is caused by the consideration of test wells priority nature. In Scenario 2, test wells are fixed in the earliest possible date, what prevents the model to optimize their allocation within the time window. The result of this is a higher number of rigs needed to attend demand due to activities overlap. In Scenario 3, test wells assume flexibility and are, therefore, optimally allocated within their short time window rather than fixed. As a result, this scenario presents a less overlapped scheduling solution than Scenario 2. Even though in Scenario 3, six rigs are also needed, these assume slightly higher utilization values.

Scenario 1	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7	Year 8	Year 9	Year 10
Rig 1	100%	100%	100%	100%	100%	100%	100%	29%	17%	32%
Rig 2	88%	100%	100%	100%	100%	96%	21%	8%		
Rig 3	88%	100%	100%	100%	88%	23%				
Rig 4	81%	98%	96%	85%	50%					
Scenario 2	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7	Year 8	Year 9	Year 10
Rig 1	100%	100%	100%	100%	100%	92%	88%	21%	21%	32%
Rig 2	88%	100%	100%	100%	92%	73%	29%	6%		
Rig 3	88%	100%	96%	96%	79%	31%				
Rig 4	81%	96%	88%	81%	69%					
Rig 5	19%	29%	46%	25%	15%					
Rig 6	15%	10%	8%	13%	15%					
Scenario 3	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7	Year 8	Year 9	Year 10
Rig 1	100%	100%	100%	100%	100%	96%	98%	21%	2 1%	32%
Rig 2	88%	100%	100%	100%	100%	79%	21%	6%		
Rig 3	88%	100%	100%	100%	87%	19%				
Rig 4	88%	100%	98%	96%	79%			0%	25%	
Rig 5	3.5%	23%	29%	2.1%	8%			26% 51%	50% 75%	
Rig 6		- I	8%	8%				76%	100%	

Figure 4. Scenario's Fleet Size and Rigs Utilization.

As the proposed scenarios assume different premises, an analysis must be held to determine which one is more realistic and should therefore be adopted. The different scenarios above were able to be analyzed due to the developed system. Through an advantages and disadvantages analysis, given by Table 2, the best scenario was defined.

Scenario	Advantages	Disadvantages
Sconorio 1	Simple	Do not consider test well
Scenario 1	 Less rigs needed 	difference from normal well
	Consider test well difference	• Do not consider flexibility for
Scenario 2	from normal well	test wells
		 More rigs needed
	Consider test well difference	Higher complexity
Scenario 3	from normal well	More rigs needed
	Consider flexibility to test wells	

Table 2. Scenarios Advantages and Disadvantages Analysis.

According to this analysis, Scenario 3 was chosen to be the most robust. Test wells



by nature must be executed in priority in order to provide valuable information about other wells, and therefore, must be differentiated over normal wells. Furthermore, considering flexibility to test well activities allocation within its time window rather than fixing its activities allow the optimization model to best rearrange activities schedule and reduce rigs' idle time. As a drawback, the chosen scenario assumes a higher complexity in its solution as more data treatment is needed and it presents a higher number of rigs needed when compared to the first scenario. Nevertheless, Scenario 3 was considered the most adherent to the company's policies.

As previously shown in Figure 4, Scenario 3 needs a total number of six rigs to attend all wells during the time horizon. However, due to low utilization values during the last years, the number of rigs owned by the company could be lower, if an option to rent rigs at low cost is available. During years 1 through 5, when utilization is higher, extra rigs could be rented by the specific time needed, incurring less costs than possessing an extra rig, which would be often idle. Also, after year 6, idle rigs could be released to be used for other purposes within the company. Decision-makers should then size the rig fleet upon their own plausible utilization values and costs, which are beyond the scope of this paper. As final results, the decision support system developed not only gives a LWO fleet sizing solution as presented above, but also a schedule for each rig as shown by Figure 5. Through the following Gantt chart, each well, identified from A to H, has its wet Christmas tree installed according to the proposed schedule, where each rig is represented by a different color.



Figure 5. Scheduling Solution Example.

5. Conclusion

With high investments, the Brazilian oil and gas industry is an important and dynamic sector of the country and presents an undoubtedly potential of growth. In order to successfully develop this potential, a good planning and scheduling of activities from the oil and gas production cycle is essential to make production feasible and satisfactory. More than scheduling activities, resources must also be associated and scheduled to execute each activity, what is currently a critical, tough and time-consuming process. In addition, as some resources can represent expensive assets, they may be shared among activities, what increases even more difficulty in scheduling.

To overcome these difficulties, we propose a decision support system to help decision-makers estimate the number of homogeneous resources needed to execute all activities considered and to plan a schedule for each activity and its resources. Within this tool, a mathematical programming model was developed to optimize the allocation of resources into activities. It aimed to determine the size of a homogeneous fleet of resources to execute a single activity. At the same time, this model was also designed to create schedules of activities and its associated resources, minimizing their idle time, what may be undesired due to resources high daily cost.

A case study presented in this paper showed a successful application of the proposed model into a fleet of rigs. The case analyzed the problem of sizing and scheduling



a homogeneous fleet of Light Workover (LWO) rigs to execute an activity called wet Christmas tree installation in a given set of wells. Considering over 200 wells and a 128 months' time horizon, the case study involved investments in the order of US\$ 4 billion, as rigs daily cost is up to US\$600,000.

As its main contribution, this paper provides a mathematical programming model to size a fleet of homogeneous resources minimizing their idle time at the same time as to schedule activities to the available resources within time windows.

Future works may consider a heterogeneous fleet of resources, where not all resources are capable of executing all activities but must instead respect operational limitations. In addition, one-activity scheduling may be extended to multi-activity scheduling. Other suggestions of future works are to consider activity probabilistic duration rather than fixed duration and to add eventual operational constraints that must be met.

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