# Static and Dynamics Maintenance Policies for Repairable Systems under Imperfect Repair Models

**Maria Luíza Guerra de Toledo**
Escola Nacional de Ciências Estatísticas - IBGE
Rua André Cavalcanti, 106 - Santa Teresa, Rio de Janeiro - RJ, 20231-050
malutoledo83@gmail.com

**Marta Afonso Freitas**
Universidade Federal de Minas Gerais
Avenida Presidente Antônio Carlos, 6627 - Pampulha, Belo Horizonte - MG, 31270-901
marta.afreitas@gmail.com

**Enrico Colosimo**
Universidade Federal de Minas Gerais
enricoc57@gmail.com

**Gustavo Gilardoni**
Universidade de Brasília
Campus Universitário Darcy Ribeiro, Brasília - DF, 70910-900
gilardon@gmail.com

**ABSTRACT**

In this paper, the problem of determining a maintenance policy for repairable systems is evaluated. It is proposed a preventive maintenance policy under imperfect repair models which takes into account the information provided by observing the failure history of a repairable system. The proposed policy is applied to a real situation involving maintenance of off-road engines owned by a Brazilian mining company. A simulation study compares the performance between the maintenance policy proposed and the static one.

**KEYWORDS. Reliability. Counting processes. Maintenance Policies.**

**Main Area: Estatística**

## 1. Introduction

Many equipments used in everyday life are not discarded after they fail. This means that, immediately after a failure, a repair action is performed which brings the equipment again to operating condition. On the other hand, also some prescheduled maintenance actions are performed periodically. Designing sound maintenance policies help to keep the equipments running longer and, hence, minimize operation costs.

Since the early work of Barlow and Hunter (1960), much of the literature regarding optimal maintenance of repairable systems has assumed both *minimal repair* (MR) and *perfect maintenance* (PM). MR, also called *as bad as old* in the engineering literature, is performed after each failure and, supposedly, returns the system to exactly the same condition it was immediately before it failed. On the other hand, prescheduled PMs or *as good as new* actions completely renew the system. However, since the MR assumption is too strong, attention has been given recently to *imperfect repair* (IR) models, which assume that the repair action after a failure is neither minimal nor perfect, in the sense that it restores the system to a condition which is somewhere in between the *as bad as old* and the *as good as new* conditions (see, for instance, Kijima *et al*. (1988), Doyen and Gaudoin (2004), Pan and Rigdon (2009).

Let $N(t)$ be the number of failures up to time $t$. Under MR the failure history of the system does not affect the failure intensity and hence, the process $N(t)$ has independent increments and is usually modelled as a nonhomogeneous Poisson process (NHPP). Therefore, the optimal PM policy should determine an optimal PM *periodicity* $\tau_P$, in the sense that PMs should be performed at times $\tau_P$, $2\tau_P$ and so on. More precisely, if $h(t)$ and $H(t) = \mathbb{E}N(t) = \int_0^t h(u)\,du$ are respectively the *intensity* and *mean funtions* of the process and $C_R$ and $C_M$ denote the fixed costs of the MR and PM actions. The total expected cost between two successive PMs performed $\tau$ units of time apart is $C(\tau) = C_M + C_R\,\Lambda(\tau)$. Deriving and equating to zero the expected cost *per unit of time* $H(\tau) = C(\tau)/\tau$, one obtains that the optimal period $\tau_P$ must be the solution of $B(\tau) := \tau h(\tau) - H(\tau) = \frac{C_M}{C_R}$, or $\tau_P = B^{-1}(C_M/C_R)$, where we have assumed that $h(t)$ is increasing and $\lim_{t\to\infty} B(t) > C_M/C_R$. This is the solution of Barlow and Hunter (1960). We note that a slightly more general set up is possible, in the sense that one could assume above (and also in the rest of the paper) that the costs of the PM and MR actions are random variables which are independent of the history of the system and have expected values $C_M$ and $C_R$.

When, as is often the case, one has access to the failure history of the equipment when deciding the maintenance policy, the previous solution will not be optimal for IR models. Hence, the objective of this paper is to understand how the information provided by the failure history of the system under IR models can be used to determine maintenance policies which improve the optimal periodic policy mentioned above.

The rest of the paper is organized as follows. The policies which take into account the history of the system are discussed in Section 2.. Maximum likelihood estimation for the Kijima *et al*. (1988) virtual age model and strategies to obtain bootstrap confidence intervals are discussed in Section 3. Section 4 describes the results of a Monte Carlo experiment designed to estimate the expected costs per unit of time of the periodic and dynamic policies under several scenarios. Section 5 contains an application regarding maintenance of off-road engines used in a mining company. Final remarks are given in Section 6.

## 2. Optimal policy under IR: The static and dynamic approaches

As before, suppose a failure process $N(t)$ subject exclusively to IR actions and define $H(t|s_0) = H(t|\Im(s_0^-)) = E[N(s_0 + t) - N(s_0)|\Im(s_0^-)]$, where $\Im(s_0^-)$ is the failure history up to the moment immediately before $s_0$. In other words, $H(t|s_0)$ is the expected number of failures during the next $t$ units of time given the history of the process up to $s_0$. We will consider the following assumptions: First, both PM and IR actions are performed instantaneously. Second, the costs of the PM and IR actions are independent of the history of the system and have expectation $C_{PM}$ and $C_{IR}$ respectively. Third, for each $s_0$, $H(t|s_0)$ is differentiable for every $t$ and its derivative $h(t|s_0) = h(t|\Im(s_0^-))$ is continuous and strictly increasing.

A *maintenance policy* for the time interval $(s_0, S)$ specifies the number of PMs $n$ and its moments $s_0 + \tau_1 < s_0 + \tau_1 + \tau_2 < \ldots < s_0 + \tau_1 + \ldots + \tau_n$, where $\tau_i > 0$ and $\sum_{i=1}^{n} \tau_i < S - s_0$. We will write $M(s_0, S) = (n; \tau_1, \ldots, \tau_n)$. Given a maintenance policy $M(s_0, S)$, its expected cost given $\Im(s_0^-)$ is

$$C[M(s_0, S)] = nC_{PM} + C_{IR}\{H(\tau_1|s_0) + \sum_{j=2}^{n} \Lambda(\tau_i|0) + H(S - s_0 - \tau_1 - \ldots - \tau_n|0)\}. \quad (1)$$

A maintenance policy $M_{OPT}(s_0, S)$ is optimal if $C[M_{OPT}(s_0, S)] \leq C[M(s_0, S)]$ for every other $M(s_0, S)$. Since the first PM action renews the system, it should be clear that, if $M_{OPT}(s_0, S) = (n, \tau_1, \ldots, \tau_n)$, then $M_{OPT}(0, S - s_0 - \tau_1) = (n - 1, \tau_2, \ldots, \tau_n)$. This shows that, to solve the general case, it is important to understand the problem with $s_0 = 0$.

- **The problem without information** $(s_0 = 0)$

  In order to obtain the optimal policy we will proceed in two stages. First, we will assume the number $n$ of PMs fixed and will obtain the optimal PM moments. Then, we will discuss how to obtain the optimal $n$ for an infinite horizon (i.e., for $S \to \infty$).

  Considering $n$ fixed in 1 and differentiating with respect to $\tau_i$ we get that $h(\tau_i|0) = h(S-\tau_1-\ldots-\tau_n)$ for $i = 1,\ldots,n$. Since $h(.|0)$ is strictly increasing, this implies that $\tau_i = S/(n+1)$. In other words, in this case the optimal policy specifies in fact an optimal period.

  Now, to obtain the optimal $n$, we substitute the previous times again in 1 to obtain $c(n) = nC_{PM} + (n+1)C_{IR}H(\frac{S}{n+1}|0) = S\frac{n+1}{S}[C_{PM} + C_{IR}H(\frac{S}{n+1}|0)] - C_{PM}$. Hence, to obtain the optimal $n$ one should minimize $c^*(n) = \frac{n+1}{S}[C_{PM} + C_{IR}H(\frac{S}{n+1}|0)]$. Since the function $f(\tau) = [C_{PM} + C_{IR}H(\tau|0)]/\tau$ is convex, it follows that the optimal $n$ is either $n_{OPT} = [S/\tau^* - 1]$ or $n_{OPT} = [S/\tau^* - 1] + 1$, where $\tau^*$ is the minimizer of $f(\tau)$ and $[a]$ is the integer part of $a$. Putting these considerations together and letting $S \to \infty$, it follows that the optimal PM policy calls for PM actions at every

  $$\tau_{OPT} = \lim_{S\to\infty} \frac{S}{n_{OPT}+1} = B^{-1}(C_{PM}/C_{IR}), \quad (2)$$

  where we have defined $B(t) = th(t|0) - H(t|0) = \int_0^t uh'(u|0)du$.

- **The general case** $(s_0 > 0)$.

  Consider now an optimal policy $M(s_0, S) = (n, \tau_1, \ldots, \tau_n)$. For large $S$ it follows from 2 that $\tau_2 = \ldots = \tau_n = B^{-1}(C_{PM}/C_{IR})$ and $n = (S - s_0 - \tau_1)/B^{-1}(C_{PM}/C_{IR})$. Hence, to obtain the optimal policy we have now to optimize with respect to the remaining variable $\tau_1$. Substituting in 1 and differentiating with respect to $\tau_1$ we get that the optimal policy $M(s_0, S)$ must satisfy

  $$h(\tau_{1,OPT}|s_0) = h[B^{-1}(C_{PM}/C_{IR})|0], \quad (3)$$

  $n_{OPT} = \frac{S-s_0-\tau_{1,OPT}}{B^{-1}(C_{PM}/C_{IR})}$ and $\tau_{2,OPT} = \ldots = \tau_{n,OPT} = B^{-1}(C_{PM}/C_{IR})$.

  Note that in purely dynamical implementation of this solution, the only relevant equation is 3. This is because one monitors the history of the system $(s_0)$ up to a time which solves 3, at which moment a PM action is performed and a renewal occurs. Then, one monitors again the history of the renewed system (again $s_0$) and so on. In other words, we never get to apply the last two equations. For this reason, we call equation 3 the *fundamental law of preventive maintenance*.

Moreover, although 3 may suggest that in order to implement the optimal policy one has to evaluate $h(t|s_0)$ for every possible $s_0$, if PM actions can be scheduled without delay, one only need actually to evaluate $h(t|t)$. More precisely, denote by $\tau_{OPT}(s_0)$ the solution of 3. In implementing the optimal policy, the operator of the system monitors the failure history and at each $s_0$ computes $\tau_{OPT}(s_0)$. In general he or she would have that $\tau_{OPT}(s_0) > s_0$ and will keep going without performing a PM. In other words, the only way that a PM action would be eventually performed is if for some $s_0$ one has that $\tau_{OPT}(s_0) = s_0$. In other words, a PM action would be performed if and only if $\lim_{s_0 \to \tau_1} h(\tau_1|s_0) = h(B^{-1}(C_{PM}/C_{IR})|0)$. This is quite nice, because usually $h(t|t)$ is much easier to compute than $h(t|s_0)$. For instance, for the simple virtual age model, it is easy to show that $h(t|t) == \lambda[t-(1-\theta)t_{N(t)}] = \lambda[V(t)]$, where $\theta$ ($0 \leq \theta \leq 1$) is a parameter which measures the efficacy of the repair, and $V(t)$ denotes the virtual age of the system at time $t$ (see Kijima *et al.*, 1988). Hence, 3 becomes now

$$\lambda[\tau_{1,OPT} - (1-\theta)t_{N(\tau_{1,OPT})}] = h[B^{-1}(C_{PM}/C_{IR})|0], \tag{4}$$

or equivalently,

$$\tau_{1,OPT} - t_{N(\tau_1,OPT)} = \lambda^{-1}\{h[B^{-1}(C_{PM}/C_{IR})|0]\} - \theta t_{N(\tau_1,OPT)}. \tag{5}$$

In other words, a PM action will occur whenever the virtual age attains the value

$\lambda^{-1}\{h[B^{-1}(C_{PM}/C_{IR})|0]\}$.

## 3. Statistical inference for the virtual age model

Consider $K$ independent, identical repairable systems, where $n_i$ failures are observed in the $i$-*th* system ($i = 1, 2, \ldots, K$). We denote by $T_{i,j}$ ($i = 1, 2, \ldots, K$; $j = 1, 2, \ldots, n_i$) the random variable representing the $j$-*th* failure of the $i$-*th* system and by $t_{i,j}$ its observed value. If the $i$-*th* system is time truncated, it is observed until the predetermined time $t_i^*$. If it is failure truncated, it is observed until the predetermined number of failures $n_i$. We consider a *Power Law Process* (PLP) reference intensity $\lambda_R(t) = (\beta/\eta)(t/\eta)^{\beta-1}$ and corresponding $\Lambda_R(t) = \int_0^t \lambda_R(u)\,du = (t/\eta)^{\beta}$.

The vector of model parameters is $\mu = (\beta, \eta, \theta)$, where $\beta$ and $\eta$ are shape and scale parameters, respectively. In order to have SCWO, we further assume that $\beta > 1$. We obtain

$$L(\mu) = \prod_{i=1}^{K} \left\{ \prod_{j=1}^{n_i} \lambda_R[t_{i,j} - (1-\theta)t_{i,j-1}] \times e^{-\Lambda_R[t_{i,j}-(1-\theta)t_{i,j-1}]+\Lambda_R(\theta t_{i,j-1})} \right\} \times \times e^{-\Lambda_R[t_i^* - (1-\theta)t_{i,n_i}]+\Lambda_R(\theta c_i)}, \quad (6)$$

where $c_i = t_i^*$ or $c_i = t_{i,n_i}$ depending on whether the $i$th system is time or failure truncated.

First, we maximize numerically (6) to obtain the maximum likelihood estimates (MLEs) $(\hat{\beta}, \hat{\eta}, \hat{\theta})$. The mean $\Phi(t)$ and ROCOF $\phi(t)$ functions are now (deterministic) functions of the model parameters. Hence, at least in theory, one could substitute the MLEs in their expression in terms of $(\beta, \eta, \theta)$ to obtain the MLE of those functions. However, since there is no closed form for $\Phi(t)$ and $\phi(t)$, we use instead a Monte Carlo step inside the estimation procedure. More precisely, once that $(\hat{\beta}, \hat{\eta}, \hat{\theta})$ have been obtained, we simulate $M$ systems with the estimated parameters $(\hat{\beta}, \hat{\eta}, \hat{\theta})$ and used them to approximate $\hat{\Phi}(t)$ and $\hat{\phi}(t)$ following the procedure explained below. In fact, we take the Monte Carlo size $M$ large enough to make the approximation error at least one order of magnitude less than the MLEs error. Hence, in practice, the relevant uncertainty in the final estimates would depend only on the precision of the MLEs.

To compute the approximations to $\hat{\Phi}(t)$ and $\hat{\phi}(t)$ we use the fact that the SCWO property implies that $\phi(t)$ is increasing and, hence, $\Phi(t)$ is convex. Therefore, once the auxiliary Monte Carlo experiment is run, in order to take into account the convexity restriction, we let $\hat{\Phi}(t)$ be the *Greatest Convex Minorant* (GCM) of the Nelson-Aalen estimator of $\Phi(t)$. Then, since $\phi(t) = \Phi'(t)$, we let $\hat{\phi}(t)$ be the right derivative of $\hat{\Phi}(t)$ (see, for instance, Gilardoni and Colosimo (2011)). This works quite well for the large Monte Carlo size we are using and, unlike other nonparametric procedures, does not require set up of tuning parameters. The estimates $\hat{\Phi}(t)$ and $\hat{\phi}(t)$ can now be used to compute

$$\hat{B}(t) = t\hat{\phi}(t) - \hat{\Phi}(t)\,. \quad (7)$$

Inverting $\hat{B}(t)$ we get an estimate of $\tau_P = B^{-1}(C_M/C_R)$. Likewise, the right hand side of (5) can be estimated now after noting that, for the PLP reference intensity, $\lambda_R^{-1}(x) = \eta[\eta x/\beta]^{1/(\beta-1)}$.

Due to the fact that $\Phi(t)$ and $\phi(t)$ are estimated using an auxiliary Monte Carlo simulation, calculation of standard deviations using for instance the Delta Method is difficult. Instead, we propose to compute confidence intervals based on a parametric Bootstratp. Suppose we want a confidence interval for $\tau_{VA}$. A bootstrap sample of $\tau_{VA}$ can be obtained as follows:

1. Use the MLEs $\hat{\beta}$, $\hat{\eta}$ and $\hat{\theta}$ to generate $K$ systems under the imperfect repair model desired with the same truncation structure as the original data set;
2. Use the generated data set to maximize (6) and compute the MLEs $\hat{\beta}^{(b)}$, $\hat{\eta}^{(b)}$ and $\hat{\theta}^{(b)}$;
3. Use the Monte Carlo simulation described above to compute estimates $\hat{\Phi}^{(b)}(t)$, $\hat{\phi}^{(b)}(t)$ and then $(\hat{B}^{-1})^{(b)}(t)$;
4. Given a cost ratio $C_M/C_R$, $\tau_{VA}^{(b)}$ is obtained.
5. Repeat the procedure above $B$ times to get $(\hat{\tau}_{VA}^{(1)}, \ldots, \hat{\tau}_{VA}^{(B)})$. The sample percentiles can then be used to construct the desired confidence interval for $\tau_{VA}$.

## 4. Comparison with the periodic policy

In this section we present a simulation study conducted in order to give a better understanding of the economic impact of implementing the dynamic policy proposed in the previous sections. Monte Carlo experiments were executed to estimate the costs per unit of time for the dynamic and periodic policies under different scenarios.

The simulations were done using a script written in R, a language and environment for statistical computing (www.R-project.org, v.2.15). The performance of each policy was measured computing the mean cost per unit of time for the periodic and dynamic policy. For each possible scenario, failure times were generated for $N = 100,000$ systems according to an ARA$_1$ model with a PLP reference intensity. The 120 scenarios were defined by combining $\beta = 1.5$, 2.0, 2.5 and 3, $\theta = 0.1$, 0.3, 0.5, 0.7 and $0,9$, $\frac{C_M}{C_R} = \frac{1}{3}$, $\frac{1}{5}$ and $\frac{1}{15}$, and both the periodic and the dynamic policies.

The $\eta$ value remained fixed at $15,000$ units of time since it is essentially a scale parameter and has no important consequence in the comparison of the policies. The procedure to run the simulation for each scenario and policy is described below.

For the periodic maintenance policy, we obtained first the optimal PM period $\tau_P$ for each scenario. Then, for each of the Monte Carlo replications, failures were generated up to $\tau_P$. The observed number of failures was then used to compute the actual cost per unit of time, $[C_M + C_R \times \text{number of failures}]/\tau_P$.

The procedure for the dynamic policy is a little more involved, since the truncation time $\tau_D$ is random. We first compute the right hand side of (5) using $t_{N(t)} = 0$ to obtain a provisional value of $\tau_D$. Then, we generated a failure time for the given scenario and replication. If the failure occurred after the provisional $\tau_D$, we stop and compute the corresponding cost per unit of time $[C_M + C_R \times 0]/\tau_D$. Otherwise, we recompute the right hand side of (5) to get a new provisional $\tau_D$ and generated a new failure. If this new failure occurred after the tentative $\tau_D$, we stop and compute now the cost $[C_M + C_R \times 1]/\tau_D$ and so on, until a PM was effectively realized.

Figure 1 plots the average of the $N = 100,000$ replications of the cost per unit time for each scenario, under each policy, *versus* $\beta$ and $\theta$. The main findings are described below.

- For both policies, the mean costs decrease as the efficiency of repair $\theta$ decreases [see Figures 1(a) to 1(c)]. In other words, the better the repair the lower the mean cost. However, this effect is more evident for the dynamic policy. In addition, for the same degree of repair, the mean costs for the dynamic policy are lower than the respective ones for the periodic policy. We note however that we have assumed that repair actions with different degrees have the same cost. A very important point is that the differences in costs between the two policies are inversely proportional to $\theta$. In general, as we get closer to MR (i.e. $\theta \uparrow 1$), we have lower differences between the costs under the two policies.

- For smaller values of the cost ratio $C_M/C_R$, the mean costs per unit time for a given policy tend to be practically the same, no matter the effect of the repair. This pattern can be observed for both policies, although it is more evident for the dynamic policy. In practice, this result indicates that if one is dealing with much higher $C_R$ costs, there is not much difference if one assumes IR of any degree or MR.

- For both policies, an increase in $\beta$ leads to a decrease in the mean cost. This can be explained by the fact that, under such policies, a stronger convexity for the intensity function determines shorter intervals between PM actions and, consequently, less failures are observed in the systems.

- Finally, the point estimates of the mean costs are consistently lower for the dynamic than for the periodic policy.

In the next section we show an application regarding maintenance of off-road engines used in a mining company.

## 5. Application

Off-road trucks are used to transport loose materials such as minerals and waste in mining operations. Good performance of this equipment is essential to the financial health of these kind of business. Due to the high costs involved, a great concern is the implementation of sound maintenance policies in order to prolong their life and reduce expenses generated by the occurrence of unexpected failures. In the case studied here, the company wanted to adopt a maintenance policy that favored preventive maintenance, as opposed to repair actions taken after failures. More precisely, the company declared that a repair was 23% more expensive than a maintenance (*i.e.* the cost ratio $C_M/C_R = 1/1.23$).

The data set consists of 208 failure times, in hours of operation, recorded for a sample of 193 diesel engines used in off-road trucks. The number of failures per engine ranged from zero up to 4. Out of the 193 engines, 52 were time truncated, since their last inspection time corresponds to a removal for a preventive maintenance.

We fitted the ARA$_1$-PLP model. The MLEs and 95% confidence intervals (computed using the observed information) were $\hat{\beta} = 2.458$ $(2.185; 2.765)$, $\hat{\eta} = 15,586$ $(14,605; 16,633)$ and $\hat{\theta} = 0.471$ $(0.330; 0.673)$. The large value of $\hat{\beta}$ gives strong evidence that the systems are deteriorating, while the fact that the efficiency of repair estimate $\hat{\theta}$ is significantly different from zero and one, suggests that the history has to be considered when determining a maintenance policy.

To compute estimates for $\Phi(t)$, $\phi(t)$ and $B(t)$ we run the Monte Carlo simulation described in Section 3, with $M = 10,000$. An estimate of $\tau_P = B^{-1}(C_M/C_R)$ is obtained. Table 1 shows point estimates and bootstrap 95% confidence intervals using $B = 10,000$ for several cost ratios. Likewise, having computed $\hat{\phi}(t)$ and $\hat{B}(t)$ and using the fact that for the PLP intensity we have that $\hat{\lambda}_R^{-1}(x) = \hat{\eta}[\hat{\eta}x/\hat{\beta}]^{1/(\hat{\beta}-1)}$, one can compute $\hat{\tau}_{VA} = \hat{\lambda}_R^{-1}\ \hat{\phi}[\hat{B}^{-1}(C_M/C_R)]\}$. These values and corresponding 95% confidence intervals are also shown in Table 1.

Consider the cost ratio $C_M/C_R = 1/1.23$. The estimates in Table 1 are interpreted as follows:

- Under a MR PLP model, an estimate of the optimal preventive PM period is $15,815$ hours, with $95\%$ confidence interval from $13,632$ to $18,082$ hours. For a new system, this is how long the company should wait for the first PM action, no matter how many failures occur until there.

- Under an IR $\text{ARA}_1$-PLP model, an estimate of the optimal *virtual time* for the first preventive PM based on the dynamic approach is $\hat{\tau}_{VA} = 11,373$ hours, with $95\%$ confidence interval from $10,978$ to $12,023$ hours. For a new system, the company should wait until $\hat{V}(t) = t - (1-\hat{\theta})\, t_{N(t)} = \hat{\tau}_{VA}$ for the first PM action. In practice, it means that the PM action will occur at $t = \hat{\tau}_{VA}$ if no failure occurs until then. If any failure is observed before $\hat{\tau}_{VA} = 11,373$ hours, the optimal time to the first PM must be recalculated according to (5) and so on.

Table 1: Estimations for the off-road engines data set, under different values of $C_M/C_R$: Optimal preventive PM period ($\hat{B}^{-1}(C_M/C_R)$); Optimal time for preventive PM based on the dynamic approach ($\hat{\tau}_{VA}$), and bootstrap ($B = 10,000$) 95% confidence intervals (values are in hours)

| $C_M/C_R$ | $\hat{B}^{-1}(C_M/C_R)$ | 95% CI | $\hat{\tau}_{VA}$ | 95% CI |
|---|---|---|---|---|
| $1/1.23$ | $15,815$ | $(13,632; 18,082)$ | $11,373$ | $(10,978; 12,023)$ |
| $1/3$ | $9,207$ | $(8,608; 10,173)$ | $8,141$ | $(7,572; 8,647)$ |
| $1/5$ | $7,500$ | $(6,720; 8,125)$ | $6,537$ | $(6,043; 7,097)$ |
| $1/10$ | $5,593$ | $(4,847; 6,227)$ | $4,694$ | $(4,403; 5,404)$ |
| $1/15$ | $4,621$ | $(4,017; 5,386)$ | $4,031$ | $(3,656; 4,596)$ |

## 6. Final remarks

We proposed a dynamic method to estimate the optimal PM policy for repairable systems. This dynamic policy updates the optimal PM time continuously as the information provided by the failure history of the system becomes available. The dynamic policy was derived for a fully general IR model, in the sense of a system whose failures occur according to an arbitrary counting process, with the only restriction that the system is deteriorating continuously over time.

We considered implementation of both the dynamic and periodic policies in the context of the $\text{ARA}_1$ virtual age model (Kijima *et al.* (1988), Doyen and Gaudoin (2004)). We showed how to obtain MLEs for the model parameters and used a Monte Carlo step to compute approximations to the MLEs of the ROCOF and related functions, for which there is no closed form expression. Our proposal also includes a parametric bootstrap to obtain confidence intervals for the model parameters and for the periodic and dynamic maintenance policies.

Finally, to better understand the implications of the dynamic policy, Section 4 reported the main findings of a simulation study conducted in order to compare the performance of the periodic and dynamic policies. It showed that the average operating cost per unit of time may be much lower when using the dynamic policy, specially when either (i) the cost ratio $C_M/C_R$ is large or (ii) the effect of the repair is far away from the MR case.

## References

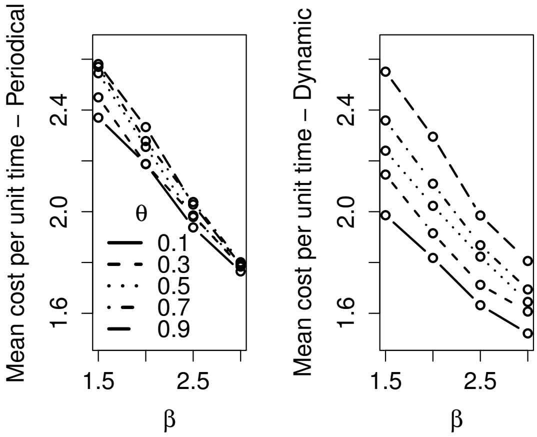
**Barlow, R and Hunter, L.** (1960), Optimum Preventive Maintenance Policies, *Operations Research*, 8, 90-100.

**Doyen, L. and Gaudoin, O.** (2004), Classes of Imperfect Repair Models Based on Reduction of Failure Intensity or Virtual Age, *Reliability Engineering & System Safety*, 84, 45-56.

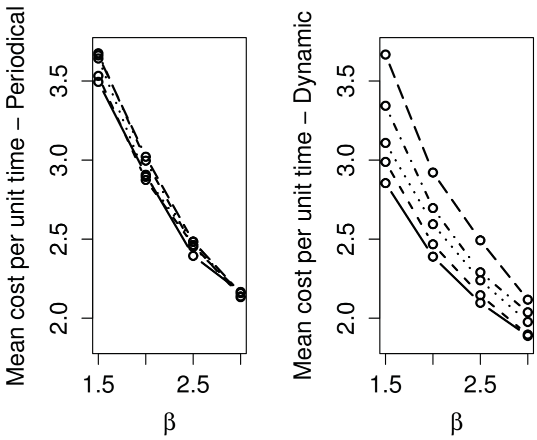
**Gilardoni, G. L. and Colosimo, E. A.** (2011), On the Superposition of Overlapping Poisson Processes and Nonparametric Estimation of their Intensity Function, *Journal of Statistical Planning and Inference*, 171, 3075-3083.

**Kijima, M., Morimura, H and Suzuki, Y.** (1988), Periodical Replacement Problem Without Assuming Minimal Repair, *European Journal of Operational Research*, 37, 194-203.

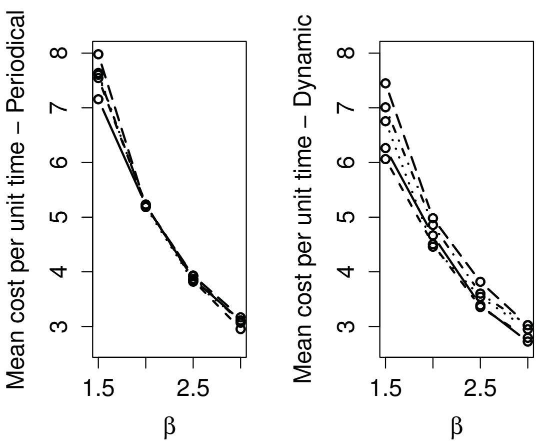
**Pan, R. and Rigdon, S. E.** (2009), Bayes Inference for General Repairable Systems, *Journal of Quality Technology*, 41, 82-94.




(a)




(b)




(c)

Figure 1: Estimates for the mean cost per unit time (periodic and dynamic policies) *versus* $\beta$ values, by efficiency of repair ($\theta$). (a)$C_M/C_R = 1/3$; (b)$C_M/C_R = 1/5$; (c)$C_M/C_R = 1/15$.