A Column Generation approach for a Compressor Scheduling Problem

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ABSTRACT

The Compressor Scheduling Problem (CSP) can be considered a variation of the classical Capacitated Facility Location Problem, with two types of demands and capacities (gas rate and gas pressure). In the CSP, the objective is to minimize the compressors installation cost and the maintenance/energy loss in the pipeline between all wells and the installed compressors. The problem is characterized by non-linear and non-convex functions. For more efficiency, a piecewise-linear function is used to approximate the objective value. In this paper, we present a Column Generation (CG) approach for the CSP to provide tightened lower bounds for the problem. We run experiments over a set of synthetically generated instances. Experimental results are compared with CPLEX applied to a compact model and its linear relaxation. The same procedures are used for the Single Source Capacitated Facility Location Problem with the objective to compare the degree of difficulty of both problems.

KEYWORDS. Compressor Scheduling Problem, Column Generation, Piecewise-Linear Formulation. Main Area: OC, P&G.

RESUMO

O Problema de Escalonamento de Compressores (CSP) pode ser considerado uma variação do problema clássico de localização de instalações capacitadas, com demandas e capacidades de taxa e volume de gás. O CSP visa minimizar os custos de instalação dos compressores e de manutenção/perda de energia das tubulações. As funções do problema são não-lineares e não-convexas. Para mais eficiência, uma função linear por partes é usada para aproximar o valor objetivo. Nesse artigo é apresentado uma abordagem de Geração de Colunas para o CSP a fim de obter melhores limitantes inferiores para o problema. São executados experimentos sobre um conjunto de instâncias geradas artificialmente. Resultados experimentais são comparados com o CPLEX aplicado ao modelo compacto e sua relaxação linear. Os mesmos procedimentos são usados para o Problema de Localização de Instalações Capacitadas com Fonte Única, com o objetivo de comparar o grau de dificuldade de ambos os problemas.

PALAVRAS CHAVE. Problema de Escalonamento de Compressores, Geração de Colunas, Formulação linear por partes. Área Principal: OC, P&G

1. Introduction

In the process of oil production, after some time of extraction, the internal pressure of the wells may not be sufficient to lift the hydrocarbons (mix of oil, gas and water) to the surface. For this problem, many artificial lifting techniques exist. One of them is called *gas-lift*, that consists of continuous injection of high-pressure gas at the bottom of the production tubing with a certain rate and pressure. Compressors provide the gas, which enters the production tube through valves, mingling with the oil and making it less dense. Consequently, the hydrocarbons can flow through the column fluid to the surface. Figure 1 represents the scheme of the gas-lift technique.

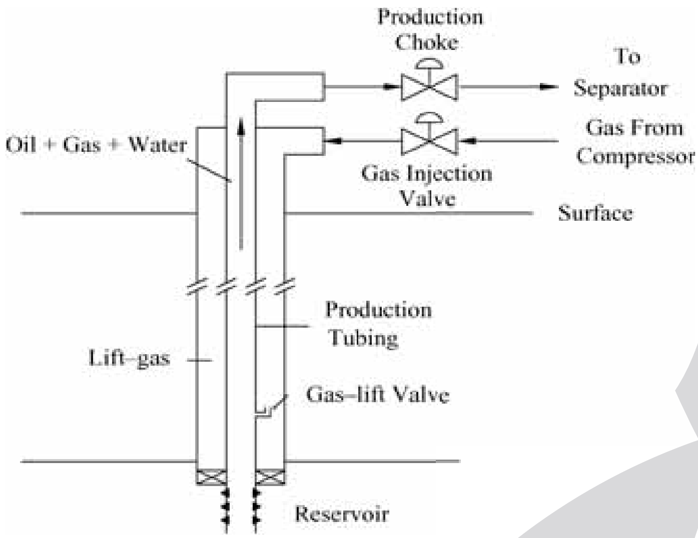

Figure 1: Oil well operating gas-lift (Camponogara & Nakashima, 2006).

The gas-lift is a preferable technique due to its robustness, relatively low installation and maintenance costs, and its wide range of operating conditions (Nakashima & Camponogara, 2006). For instance, if the fluid contains sand, it can damage lift systems as hydraulic pumps, but this is not a problem for the gas-lift.

The oil field needs a study and recovery plan for the gas-lift before its exploration, which establishes the gas-lift injection rates and pressures for the wells along its lifespan to obtain the optimal production. However, these rates and pressures are overestimated due the unpredicted events as compressor failures or dynamic changes in the reservoir conditions. If a compressor needs to reduce its capacity, there is a loss of energy, and consequently the production cost increases. The problem in scheduling the gas-lift to the oil wells, minimizing the installation and maintenance costs, taking into account costs of energy loss, becomes very difficult. The problem was initially proposed by Huppler (1974) and Dutta-Roy et al. (1997). In Kanu et al. (1981) it is presented an economic slope as an approach to define the best oil production function of the gas demand, and a model is proposed for resolving the gas allocation in a field with limited gas. Buitrago et al. (1996) propose a search algorithm to determine the optimal gas injection to maximize the oil production in an environment with a given total amount of gas. In Hamedi & Khamehchi (2012) a particle swarm optimization algorithm and a penalty function are presented for scheduling gas-lift to a group of oil wells considering the gas pressure. The Compressor Scheduling Problem (CSP) is formulated as a generalization of the Capacitated Facility Location Problem (CFLP) with capacity and demand constraints and a non-convex objective function proposed by Camponogara & Nakashima (2006). Based on this model, a column generation approach is proposed in Camponogara & Plucenio (2008). Afterwards the model was improved by Camponogara et al. (2012), where a nonlinear constraint was replaced by a representation of linear inequalities.

The CFLP is a classical integer programming problem with a given set of potential facilities and a set of customers. The problem is to locate a subset of facilities in such a way that the total cost of assigning facilities is minimized when meeting the demands of all customers and respecting the facility capacities. The Single-Source Capacitated Facility Location Problem (SSCFLP) is the variation of the CFLP where the customer must be supplied by only one facility. Applications of the SSCFLP are found in many areas as distribution planning, telecommunications and others. Because of this, many works to resolve the problem at optimality are presented in the literature. Klose & Görtz (2006) embedded a Column Generation (CG) approach in a Branch-and-Bound (B&B) framework for solving the problem exactly. The CG gives improved lower bounds, and the knowledge of a fractional optimal solution of the master problem can be exploited to improve the branch decision in the B&B framework.

This work implements the column generation for the CSP. Unlike the CG proposed in Camponogara & Plucenio (2008), our implementation is based on the model proposed by Camponogara et al. (2012), which has linear constraints and thus tends to be faster than the previously proposed CG approach. Moreover, a simple procedure that takes advantage of the piecewise-linear function is applied to handle the approximation on the pricing subproblem.

This paper is organized as follows: Section 2 presents the formalization of the problem and its piecewise-linear reformulation. Section 3 presents the column generation approach. The CG is also adapted to the classical SSCFLP to compare the difficulty level between the two problems. Section 4 presents results to compare the quality of lower bounds and processing times of the CG considering the compact and relaxed models of CSP and SSCFLP. Finally, in Section 5 we present conclusions and future perspectives of the work.

2. Problem Definition

The Compressor Scheduling Problem becomes a mixed integer nonlinear problem (MINLP). For convenience, we present the notation used in the compact model in the Table 1.

Table 1: Notation used in the compact formulation (1a)-(1h).

Symbol	Definition
Sets	
$j \in N$	: Set of all compressors.
$i \in M$	: Set of all wells.
$j \in N_i$	: Set of compressor that can supply the well i.
$i \in M_j$	: Set of wells that can be supplied by compressor j.
Parameters	
c_j	: Installation cost of compressor j.
d_j	: Energy cost loss of compressor j.
$q_j^{c,min}$	: Minimum gas rate of compressor j.
$q_j^{c,max}$	: Maximum gas rate of compressor j.
$\alpha_{0..4,j}$	: Parameters of discharge pressure of compressor j.
q_i^w	: Gas rate demand of well i.
p_i^w	: Gas pressure demand of well i.
c_{ij}	: Cost of supply or maintenance between compressor j and well i.
l_{ij}	: Pressure loss in the pipeline between compressor j and well i.
$q_{i,j}^{c,max}$	: Max output gas rate of compressor j for well i such that $p_j^c(q_j^c) \geq p_i^w + l_{ij}$.
Variables	
$y_j \in \{0,1\}$	: Indicates whether compressor j is installed.
$x_{ij} \in \{0,1\}$	: Indicates whether the well i is supplied by compressor j.
$q_j^c \in \mathbb{R}^+$	: Gas rate of compressor j.
$p_j^c(q_j^c) \in \mathbb{R}^+$	: Discharge pressure with the gas output rate q_j^c.

The CSP model is formulated as follows:

$$\begin{aligned}
\text{Min} \quad & f = \sum_{j\in N} c_j y_j + \sum_{i\in M}\sum_{j\in N_i} c_{ij}x_{ij} + \sum_{j\in N} d_j q_j^c p_j^c(q_j^c) && \text{(1a)}\\
\text{S.t.} \quad : \quad & x_{ij} \le y_j, \quad i\in M,\ j\in N_i && \text{(1b)}\\
& \sum_{j\in N_i} x_{ij} = 1, \quad i \in M && \text{(1c)}\\
& q_j^c \le q_{ij}^{c,max} x_{ij} + q_j^{c,max}(y_j - x_{ij}), \quad j\in N, i\in M_j && \text{(1d)}\\
& q_j^c \ge q_j^{c,min} y_j, \quad j \in N && \text{(1e)}\\
& \sum_{i\in M_j} q_i^w x_{ij} \le q_j^c, \quad j\in N && \text{(1f)}\\
& y_j \in \{0,1\}, \quad j\in N && \text{(1g)}\\
& x_{ij}\in\{0,1\}, \quad i\in M, j\in N_i. && \text{(1h)}
\end{aligned}$$

The objective function f accounts costs for the compressor installation, supply or maintenance between compressors and wells, and compressors operation. Constraints sets (1b) and (1c) are the same of the SSCFLP. The first determines that a well can be supplied only by a compressor that was installed, and the second imposes that all wells must be supplied. Constraints sets (1d) and (1e) define the upper and lower limits of the compressor gas rate output. Constraint set (1f) imposes that the capacity of the compressor must be respected. In the CSP, the right-hand side (RHS) of this constraint is a variable while in the SSCFLP the RHS is a parameter. Because of the definition of the parameter $q_{i,j}^{c,max}$, the constraint set (1d) is a combination of linear constraints that replaces the non-linear constraint $p_j^c(q_j^c) \ge p_i^w + l_{ij}, j\in N, i\in M_j$ in the formulation of Camponogara et al. (2007). This approach ensures that the model (1a)-(1h) has only linear constraints, and consequently its resolution is expected to be faster than using the previous model.

2.1. Piecewise-Linear Formulation

The CSP is an *NP-Hard* problem (Camponogara et al., 2012). Because the performance curve function pressure $p_j^c(q_j^c) = \alpha_{0,j} + \alpha_{1,j}q_j^c + \alpha_{2,j}(q_j^c)^2 + \alpha_{3,j}(q_j^c)^3 + \alpha_{4,j}ln(1+q_j^c)$ of a compressor j is a non-linear and non-convex function, model (1) can be difficult to be solved. A piecewise-linear function was proposed in (Camponogara et al., 2007; Camponogara & Plucenio, 2008; Camponogara et al., 2012) based on Sherali (2001), to allow the use of techniques and algorithms for linear problems.

We assume that $Q_j = \{(q_j^{c,0}, h_j^{c,0}), ..., (q_j^{c,\kappa(j)}, h_j^{c,\kappa(j)})\}$ are the points of the compressor j with the output gas rate and operating cost $h_j = d_j q_j^c p_j^c(q_j^c)$, where:

i) $\kappa(j)$ is the number of points in the piecewise-linear function;

ii) $q_j^{c,k-1} < q_j^{c,k}$ for all $k\in K(j) = \{1, ..., \kappa(j)\}$;

iii) $q_j^{c,0} = q_j^{c,min}$ and $q_j^{c,\kappa(j)} = q_j^{c,max}$ to maintain the output gas rate in a feasible range;

iv) $h_j^{c,k} = d_j q_j^{c,k} p_j^c(q_j^{c,k})$ to be consistent with the piecewise-linear formulation.

The piecewise-linear formulation of the CSP becomes a mixed-integer linear program

(MILP) as follows:

$$\begin{aligned} \text{Min} \quad & f^1 = \sum_{j\in N} c_j y_j + \sum_{i\in M}\sum_{j\in N_i} c_{ij}x_{ij} + \sum_{j\in N}\sum_{k\in K(j)} (h_j^{c,k-1}\lambda_j^{k,L} + h_j^{c,k}\lambda_j^{k,R}) && \text{(2a)}\\ \text{S.t.} \quad & \text{Equations (1b) to (1d) (1f) to (1j)} && \text{(2b)}\\ & q_j^c = \sum_{k\in K(j)} (q_j^{c,k-1}\lambda_j^{k,L} + q_j^{c,k}\lambda_j^{k,R}), \quad j\in N && \text{(2c)}\\ & \sum_{k\in K(j)} z_j^k = y_j, \quad j \in N && \text{(2d)}\\ & \lambda_j^{k,L} + \lambda_j^{k,R} = z_j^k, \quad j\in N, k\in K(j) && \text{(2e)}\\ & \lambda_j^{k,L}, \lambda_j^{k,R} \geq 0, \quad j\in N, k\in K(j) && \text{(2f)}\\ & z_j^k \in \{0,1\}, \quad j\in N, k\in K(j), && \text{(2g)} \end{aligned}$$

where $\lambda_j^{k,L}$ and $\lambda_j^{k,R}$ are the weights of the left and right points of the segment from $q_j^{c,k-1}$ to $q_j^{c,k}$. Variable z_j^k assumes value 1 if q_j^c is approximated linearly with the points $q_j^{c,k-1}$ and $q_j^{c,k}$, where $q_j^c = q_j^{c,k-1}\lambda_j^{k,L} + q_j^{c,k}\lambda_j^{k,R}$.

The objective value of f^1 is a lower bound of f due to the piecewise-linear formulation. The degree of discrepancy between f^1 and f can be adjusted by increasing the number $\kappa(j)$ of linear segments. This approach needs a numerical analysis to define the optimal number of segments of each compressor. Increasing the number $\kappa(j)$ of segments, the number of variables in the model (2a)-(2g) also increases taking longer to be solved.

3. Column Generation Approach

Solving the compact model (2) directly with a mixed-integer programming solver can take a long time especially when considering large instances of the CSP. Moreover, the compact formulation of the MILP may have a weak linear programming relaxation (Barnhart et al., 1998). An alternative way for producing a lower bound of the CSP efficiently is using a column generation approach.

3.1. Master problem

We consider a column in CSP as an assignment S of the compressors to wells that is represented as a pair (y, x), where $y = (y_j : j \in N)$ and $x = (x_{ij} : i \in M_j, j \in N_i)$ are vectors associated with the decision variables. The *master problem* (MP) can be formulated as:

$$\begin{aligned} & \text{Min} \sum_{j\in N}\sum_{S\in M_j^*} c_{jS}\lambda_{jS} && \text{(3a)}\\ \text{S.t.} \quad : \quad & \sum_{j\in N}\sum_{S\in M_j^*} \delta_{jS}^i \lambda_{jS} = 1, \quad i\in M && \text{(3b)}\\ & \sum_{S\in M_j^*} \lambda_{jS} \leq 1, \quad j\in N && \text{(3c)}\\ & \lambda_{jS} \in \{0,1\}, \quad j\in N, S\in M_j^* && \text{(3d)} \end{aligned}$$

$M_j^* \subseteq 2^{M_j}$ is the set of feasible assignments of the compressor j, where $S \in M_j^*$ if $\sum_{i\in S} q_i^w \leq q_j^{c,max}$ and $max(\sum_{i\in S} q_i^w, q_j^{c,min}) \leq (q_{ij}^{c,max} : i \in S)$. The cost of the assignment/column is: $c_{jS} = c_j + \sum_{i\in S} c_{ij} + d_j \sum_{i\in S} q_i^w p_j^c(max\{\sum_{i\in S} q_i^w, q_j^{c,min}\})$. The parameter δ_{jS}^i is set to 1 if $i \in S, S \in M_j^*$, and 0 otherwise. Variable λ_{jS} is set to 1 if compressor j is assigned to S, and 0 otherwise. Constraint (3b) ensures that all wells are met, each well by a single compressor. Finally, constraint (3c) states that is designated at most one assignment to compressor j.

Solving the master problem (3a)-(3d) with all columns $S \in M_j^*, j \in N$, is not efficient due to the large number of variables, which is potentially exponential, while most of them have zero value in an optimal solution. Then the CG algorithm starts with an initial subset of columns and adds others at each CG iteration.

3.2. Initial columns

The model (3a)-(3d) with a subset of columns is called *restricted master problem* (RMP). For generating the initial columns for the RMP, we consider the vectors $q^c[j][c]$ as the gas rate q_j^c of the compressor j in the column c, and $\delta[j][c][i]$ is set to one if the well i in the column c of compressor j is supplied. The pseudo-code to create the initial subset of columns for the CSP is given in Algorithm 1.

Algorithm 1 Initial RMP of the CSP

1: $\delta[j][c][i] = 0$, for all j, c, and i.
2: **for** each $j \in N$ **do**
3:     int $c = 1$;
4:     $q^c[j][c] = 0$;
5:     **for** each $i \in M_j$ **do**
6:         **if** $(q^c[j][c] + q^w[i] \leq q^{c,max}[i][j])$ **then**
7:             $q^c[j][c] = q^c[j][c] + q^w[i]$;
8:             $\delta[j][c][i] = 1$;
9:         **else**
10:             $c = c + 1$;
11:             $q^c[j][c] = q^w[i]$;
12:             $\delta[j][c][i] = 1$;
13:         **end if**
14:     **end for**
15: **end for**

The algorithm assumes that all $q_i^w \leq q_j^{c,max}$ for all $j \in N, i \in M$. In this procedure, each compressor has a minimum number c of columns such that all clients in M_j are supplied at least once. In Line (6) it is verified if the current column of the compressor j can supply well i. If it cannot, a new column is initialized for this compressor (lines 10 to 12).

The initial RMP must have a feasible relaxation solution to ensure that proper information is passed to the subproblems (Barnhart et al., 1998). Then we insert n slack variables $\lambda_{jS}^{\star}$ with a large c_{jS}, where $\delta_{jS}^i \lambda_{jS}^{\star} = 1$ for all $i \in M, j \in N$. Thus, the initial solution is feasible, but using artificial variables. If we have at least one $\lambda_{jS}^{\star} > 0$ at the end of the CG procedure, the solution of the CSP is infeasible.

3.3. Pricing sub-problem

To add new columns that can improve the solution of RMP, we need to find one with minimum reduced cost. This procedure is known as pricing sub-problem, where we consider two vectors (π, μ) as an optimal dual solution of the linear relaxation of the RMP. The π vector is associated with the constraint set (3b), and the μ vector is associated with the constraint set (3c). The pricing sub-problem of the compressor j is:

$$SP_j : \bar{c}_j = \min_{S \in M_j} c_{jS} - \sum_{i \in S} \pi_i - \mu_j \tag{4a}$$

where $\bar{c}_j$ is the least reduced cost of the compressor j. As we do not consider all columns in the RMP, we need to execute the pricing indirectly, reformulating it as an MILP problem based on the model (2). The pricing sub-problem that approximates SP_j is:

$$\widetilde{SP}_j : \widetilde{c}_j = \text{Min} \quad : \quad c_j - \mu_j + \sum_{i \in M_j} (c_{ij} - \pi_i) x_i + \sum_{k \in K(j)} (h_j^{c,k-1} \lambda^{k,L} + h_j^{c,k} \lambda^{k,R}) \tag{5a}$$

$$\text{S.t.} \quad : \quad q_j^c = \sum_{k \in K(j)} (q_j^{c,k-1} \lambda^{k,L} + q_j^{c,k} \lambda^{k,R}) \tag{5b}$$

$$q_j^c \leq q_{ij}^{c,max} x_i + q_j^{c,max}(1 - x_i), \quad i \in M_j \tag{5c}$$

$$\sum_{i \in M_j} q_i^w x_i \leq q_j^c \tag{5d}$$

$$\sum_{k \in K(j)} z^k = 1 \tag{5e}$$

$$\lambda^{k,L} + \lambda^{k,R} = z^k, \quad k \in K(j) \tag{5f}$$

$$\lambda^{k,L}, \lambda^{k,R} \geq 0, \quad k \in K(j) \tag{5g}$$

$$z^k \in \{0, 1\}, \quad k \in K(j) \tag{5h}$$

$$x_i \in \{0, 1\}, \quad i \in M_j \tag{5i}$$

where $\widetilde{c}_j$ is the reduced cost of the $\widetilde{SP}_j$. The new column is associated with the vector of variables x. After the execution of model (5a)-(5i) , if $\widetilde{c}_j < 0$, then the column found can improve the solution of the RMP and it enters in the basis. Algorithm 2 presents a pseudo-code of an iteration of the CG.

Algorithm 2 Column Generation procedure

```
1:  repeat
2:      Solve the linear relaxation of RMP;
3:      (π, μ) = optimal dual solution of the linear relaxation of RMP;
4:      for each j ∈ N do
5:          Solve SP̃_j with (π, μ);
6:          if (c̃_j < 0) then
7:              RMP ← RMP ∪ column generated by SP̃_j;
8:          end if
9:      end for
10: until (c̃_j : j ∈ N) > 0, for all j.
```

In the Algorithm 2, we first solve the linear relaxation of RMP (line 2). Next, n pricing sub-problems are solved (line 5) considering the dual values. If at least one pricing has negative reduced cost, the RMP is executed again, considering the new inserted column(s). This algorithm runs until all pricing subproblems return a value greater or equal to 0, which means that the solution of the RMP is optimal and thus it cannot be improved.

Because of the discharge pressure curve characteristic, the standard piecewise-linear formulation of the $\widetilde{SP}_j$ may underestimate the cost of the column. Although the generated column is feasible for the original pricing (without piecewise-linear formulation), its reduced cost can be underestimated, and a column entering the RMP, should not enter. This issue is discussed in Camponogara & Plucenio (2008), although it is not explained how to solve it. This phenomenon during the execution of the CG procedure may cause the addition of repeated columns. To handle with this issue, first we calculate the cost of the column taking into consideration the values of q_j^c and the variables x found in the $\widetilde{SP}_j$. Next, we recalculate the reduced cost $\overline{c}_j$ of the column j according to Equation (4). The c_{jS} represents the actual (not approximated) reduced value of the column. This procedure is only possible because the value of q_j^c in the piecewise-linear formulation is exact.

Table 2: Transformation of SSCFLP instances to CSP instances.

Parameter name	SSCFLP	CSP
Facility & Compressor		
Installation cost:	c_j	c_j
Energy cost loss	-	$d_j = rand(1, 7)$
Capacity	b_j	$q_j^{c,min} = b_j/rand([b_j/l^c], [b_j/g^c])$
Capacity	b_j	$q_j^{c,max} = b_j/rand(0.11, 0.44)$
Pressure parameter	-	$\alpha_{0,j} = q_j^{c,max} * rand(0.2887, 0.7983)$
Pressure parameter	-	$\alpha_{1,j} = rand(-1.215, -0.14)$
Pressure parameter	-	$\alpha_{2,j} = \alpha_{1,j} * rand(-0.12, -0.09)$
Pressure parameter	-	$\alpha_{3,j} = \alpha_{1,j} * rand(0.0061, 0.0138)$
Pressure parameter	-	$\alpha_{4,j} = \alpha_{1,j} * rand(-1.6258, -0.976)$
Client & Well		
Demand	d_i	$q_i^w = d_i/rand([d_i/l^w], [d_i/g^w])$
Demand	-	$p_i^w = rand(0.74, 5)$
Client/Well x Facility/Compressor		
Supply cost	c_{ij}	c_{ij}
Pipeline pressure drops	-	$l_{ij} = 0.1$

3.4. Column Generation for the Single Source Capacitated Facility Location

The CG procedure for the SSCFLP is similar to the CG for the CSP. For the SSCFLP, the pricing subproblem consists in the Equation (5) without the last sum of objective value and without the constraints (5b) to (5c), and (5e) to (5h). The RHS of constraint (5d) is replaced from q_j^c to $q_j^{c,max}$, then the maximum output gas rate of compressor j in CSP is equivalent to the capacity of the facility j in the SSCFLP. Furthermore, in Algorithm 2, the reduced cost of the column does not need to be recalculated because no piecewise-linear formulation is used.

4. Computational Results

The Column Generation for the CSP and the SSCFLP were implemented in C++ language using Cplex API version 12.5.0, and it was compiled with the CMake 2.8.10.1. The application run in a computer AMD-FX-8150 (8 cores) running at 3.6GHz, and with 32Gb of RAM.

4.1. Instances

There are three sets of synthetic instances used in the computational tests for the CSP and SSCFLP. Set 1 is composed of 6 instances from Camponogara et al. (2012) and an instance that was made available by the authors. They have different sizes of n and m. Sets 2 and 3 are instances from the SSCFLP, and they were extended to CSP based on data from the first instance of Set 1 (called hereafter as reference instance). Set 2 is composed by instances from Holmberg et al. (1999) and consists in 12 instances named ‘p13’ to ‘p24’, all with $n = 20$ and $m = 50$. Set 3 is composed by 8 instances from Delmaire et al. (1999), named ‘p34’ to ‘p41’, having $n = 30$ and $m = 60$. All experiments were executed with the number of points $\kappa(j) = 10$. For the SSCFLP, we used only the parameters c_j as installation cost and $q_j^{c,max}$ as capacity for the facilities $j = 1..n$, q_i^w as demand for the clients $i = 1..m$, and c_{ij} as assigning cost for $j = 1..n$ and $i = 1..m$.

Instances from SSCFLP were transformed into CSP instances according to the following procedure. Let l^c and g^c be the smaller and higher values of gas rate of all compressors $j \in N$ in the reference instance. Accordingly, let l^w and g^w be the smallest and highest values of gas rate demands of all wells $i \in M$ in the reference instance. The function $rand(x, y)$ represents a random value between x and y, where the seed used corresponds to the index of facility/client. Table 2 demonstrates how the instances were transformed.

4.2. CSP Results

The experimental results for the CSP are presented in Table 3. There are three main columns. The set of columns indicated by CG, LP and MILP represent results for the column generation approach, for the integer relaxation of model (2), and for CPLEX resolution of model (2), respectively. Column Obj represents the objective value of the function at the end of the algorithm, except when it was stopped due the memory overflow, which is signed by '*'. Thus, Obj values signed with '*' are the objective value in hands when the memory overflow happened. The column GAP represents the relative deviation ($100\frac{|MILP-LB|}{LB}$) of the $MILP$ integer solution and the lower bound (LB) found in the columns CG and LP. Column Time(s) corresponds to the total time in seconds to solve the model. The last row of each set represents the average values.

From the results presented in Table 3, we observe that for the small instances of the Set 1, the MILP model has the best time results. However in the two largest instances of this set, the running times grow, and the algorithm does not solve the problem before the memory overflow. In the first instance of this set, the solution found by CG approach is optimal and greater than the solution obtained by MILP because the first does not use the approximated result of the piecewise-linear formulation. In the Set 2, the integer solutions were found in about 30 minutes by CPLEX running the MILP model. This time can be considered derisive for this problem, although the average time for finding the lower bound in CSP is tiny. For Set 3, no instance could be solved by the MILP model while the CG algorithm can find the lower bound in approximated 71 seconds.

Comparing results from CSP and LP model, the execution of the second is quick and faster than the first model in all instances. Nonetheless, the GAP of the lower bound found by the CSP model is always tighter regarding the integer solution.

4.3. SSCFLP results

Table 4 presents results for SSCFLP when applied on the adapted instances.

The results for the SSCFLP are alike the ones presented in Table 3. Differences can be found in the two last instances of Set 1, where the MILP model can find the integer solution. The same happens with the instances 'p34-p35' and 'p37-p41' of Set 3. Only instance 'p36' of this set has memory overflow before the integer optimal solution is found. In Set 2 we can observe that the CG algorithm finds the optimal solution in three instances. In Set 3 the lower bound found by the LP is tighter than the CG lower bound.

For the CSP tests with column generation, the average time spent to solve the relaxation of RMP is on average 0,09% of the total time, while the pricing sub-problem spends 98,55% of the total time. Similarly, in the SSCFLP test, the RMP spends on average 0,1% of the time, while the pricing spends 97,11% of the total time. From these results, one can observe that solving the subproblems is considerable more time demanding than solving the RMP. Strategies to improve the sub-problem can be considered for more efficiency.

Finally, we can observe that it is harder to solve CSP than SSCFLP since more variables and constraints are added due to the piecewise-linear formulation.

5. Conclusion and future works

In this paper, we presented a column generation approach to the compressor scheduling problem. The results show that the CG can be solved quickly, obtaining good lower bounds.

The CSP is a generalization of the single source capacitated facility location problem, and in this work we solve both problems with a common set of instances, which were adapted for CSP. From the results it can be concluded that solving CSP is harder than solving SSCFLP, in the sense that it takes longer. Moreover, the lower bounds obtained by the column generation approaches are in most cases better than the straightforward MILP relaxation. Finally, the MILP can take too long in large instances.

We intend to solve each node of a branch-and-bound algorithm with the CG algorithm, providing a branch-and-price algorithm to obtain exact solutions for the problem. Furthermore, we intend to apply other methods such as Lagrangian decomposition and subgradient method to improve the bounds provided by column generation. This approach is used for the CFLP (Klose & Görtz, 2006) and give good results for the CFLP.

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Table 3: CSP Results.

			CG			LP			MILP	
I	N	M	Time (s)	Obj	GAP(%)	Time (s)	Obj	GAP(%)	Time (s)	Obj
	Set 1									
1	5	6	0.40	275.6	-0.2783	0.11	266.8	3.0075	0.06	274.9
2	7	16	2.93	420.1	5.8315	0.25	369.5	20.3213	1.47	444.6
3	8	18	5.35	452.2	5.8108	0.14	394.5	21.2808	2.59	478.5
4	9	14	1.90	357.9	1.1218	0.20	325.9	11.0676	0.65	361.9
5	14	20	10.56	424.1	0.1929	0.23	380.4	11.6974	5.21	424.9
6	14	32	24.82	621.8	0.4619	0.59	556.5	12.2501	3,805.30	*624.6
7	31	64	100.66	1084.8	0.9620	2.67	910.3	20.3241	577,801.33	*1,095.3
	Avg		20.94		2.0147	0.60		14.2784	83,088.09	
	Set 2									
13	20	50	31.58	18,202.5	0.4170	7.24	14,867.8	22.9395	1,528.90	18,278.4
14	20	50	26.18	15,525.8	0.3588	6.52	12,696.2	22.7257	808.56	15,581.5
15	20	50	29.90	19,424.5	0.4165	7.28	16,069.9	21.3785	1,224.08	19,505.4
16	20	50	34.17	23,216.9	0.3812	7.27	19,269.9	20.9420	3,242.65	23,305.4
17	20	50	30.99	18,202.5	0.4170	7.25	14,867.8	22.9395	1,527.64	18,278.4
18	20	50	26.95	15,525.8	0.3588	6.68	12,696.2	22.7257	809.24	15,581.5
19	20	50	30.15	19,424.5	0.4165	7.23	16,069.9	21.3785	1,224.15	19,505.4
20	20	50	36.27	23,216.9	0.3812	7.24	19,269.9	20.9420	3,247.84	23,305.4
21	20	50	30.15	18,202.5	0.4170	7.19	14,867.8	22.9395	1,528.79	18,278.4
22	20	50	26.99	15,525.8	0.3588	6.58	12,696.2	22.7257	810.21	15,581.5
23	20	50	30.08	19,424.5	0.4165	7.24	16,069.9	21.3785	1,223.91	19,505.4
24	20	50	34.36	23,216.9	0.3812	7.24	19,269.9	20.9420	3,246.85	23,305.4
	Avg		30.65		0.3934	7.08		21.9964	1701.90	
	Set 3									
34	30	60	45.35	15,141.8	6.6802	0.82	14,699.0	9.8938	2,319.47	*16,153.3
35	30	60	70.94	18,021.1	4.0883	3.17	15,779.2	18.8771	4,806.56	*18,757.9
36	30	60	74.91	56,364.3	5.7348	1.03	48,777.7	22.1802	3,214.51	*59,596.7
37	30	60	72.38	50,714.1	6.8391	2.35	43,875.7	23.4909	6,071.16	*54,182.5
38	30	60	76.05	62,894.3	5.7509	0.91	53,864.6	23.4786	3,056.54	*66,511.3
39	30	60	81.07	64,117.2	9.8134	2.64	55,655.3	26.5096	3,136.20	*70,409.3
40	30	60	68.28	90,058.4	8.5377	0.64	78,196.7	25.0018	3,733.23	*97,747.3
41	30	60	79.09	205,173.0	7.2696	1.08	169,814.0	22.8428	7,720.41	*220,088.3
	Avg		71.01		6.8392	1.58		21.5344	4,257.26	

Table 4: SSCFLP Results.

Set 1			CG			LP			MILP	
I	N	M	Time (s)	Obj	GAP(%)	Time (s)	Obj	GAP(%)	Time (s)	Obj
1	5	6	0.07	31.0	0.0000	0.02	29.7	4.3069	0.01	31.0
2	7	16	3.40	23.6	1.6553	0.02	23.9	0.4188	0.19	24.0
3	8	18	5.31	26.1	6.3683	0.11	27.6	0.5225	0.90	27.8
4	9	14	3.10	20.4	6.5751	0.03	21.5	0.8240	0.15	21.7
5	14	20	6.87	25.8	4.3058	0.11	26.4	1.5914	1.35	26.9
6	14	32	16.69	40.5	0.8871	0.11	39.9	2.3703	12.74	40.8
7	31	64	35.63	62.4	3.3778	0.96	63.9	0.9236	4,427.77	64.5
	Avg		10.15		3.3099	0.19		1.5654	634.73	
	Set 2									
13	20	50	16.18	12,343.0	0.0000	0.15	11,502.9	7.3034	2.04	12,343.0
14	20	50	12.11	10,349.9	0.1459	0.34	9,727.7	6.5510	1.35	10,365.0
15	20	50	13.39	13,507.7	0.4242	0.27	12,727.7	6.5786	4.09	13,565.0
16	20	50	14.35	16,624.8	0.6448	0.66	15,727.7	6.3855	6.57	16,732.0
17	20	50	17.55	12,343.0	0.0000	0.16	11,502.9	7.3034	1.98	12,343.0
18	20	50	14.32	10,349.9	0.1459	0.23	9,727.7	6.5510	1.34	10,365.0
19	20	50	13.24	13,507.7	0.4242	0.27	12,727.7	6.5786	4.01	13,565.0
20	20	50	15.76	16,624.8	0.6448	0.47	15,727.7	6.3855	6.56	16,732.0
21	20	50	15.38	12,343.0	0.0000	0.16	11,502.9	7.3034	2.09	12,343.0
22	20	50	12.91	10,349.9	0.1459	0.34	9,727.7	6.5510	1.33	10,365.0
23	20	50	14.49	13,507.7	0.4242	0.27	12,727.7	6.5786	4.02	13,565.0
24	20	50	15.56	16,624.8	0.6448	0.49	15,727.7	6.3855	6.66	16,732.0
	Avg		14.60		0.3037	0.32		6.7046	3.50	
	Set 3									
34	30	60	28.96	12,972.4	0.4209	1.46	12,748.8	2.1822	53,761.60	13,027.0
35	30	60	26.23	12,879.1	3.9203	1.21	12,872.6	3.9728	86,400.80	13,384.0
36	30	60	36.18	45,488.2	1.0174	1.17	45,513.6	0.9610	14,552.74	*45,951.0
37	30	60	33.84	39,786.5	4.2690	1.60	41,269.6	0.5219	311.22	41,485.0
38	30	60	36.50	50,055.0	1.0109	1.14	50,322.0	0.4749	271.85	50,561.0
39	30	60	43.15	51,963.5	2.5912	1.34	53,034.1	0.5202	19,36.00	53,310.0
40	30	60	41.75	74,491.7	1.1173	1.16	74,927.7	0.5289	51,773.20	75,324.0
41	30	60	59.61	163,100.0	2.2164	1.04	166,473.0	0.1454	4,067.01	166,715.0
	Avg		38.28		2.0704	1.26		1.1634	26,634.30	