CONSENSUS IN MULTIGROUP DECISION PROBLEMS: A CASE STUDY IN A MAYOR ELECTION FOR A BRAZILIAN CITY

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RESUMO

Diante da insatisfação oriunda de diversos resultados eleitorais, uma questão que se coloca é se o sistema eleitoral atual realmente está levando em conta a opinião em grupo dos eleitores. Diante deste contexto, este artigo visa aplicar métodos de apoio à decisão em grupo em eleições, de forma a comparar os resultados obtidos por esta abordagem com o resultado a partir do sistema atual de voto. Baseado em dados reais sobre relações de preferência entre os candidatos, os resultados mostram que as avaliações finais dos mesmos a partir do método multigrupo diferem dos resultados obtidos pelo sistema atual de voto, podendo ocorrer alterações no ordenamento. Além disto, nota-se que a solução que maximiza o consenso entre os eleitores se aproxima do resultado obtido pelo método de apoio à decisão multigrupo.

PALAVRAS CHAVE. Métodos de apoio à decisão multigrupo, Eleições, Consenso.

ADM – Apoio à Decisão Multicritério

ABSTRACT

Whether the dissatisfaction obtained from several electoral results, one question that arises is if the current electoral system is really taking into account the group opinion of voters. In this context, this article aims at applying multigroup decision aiding methods in elections, in order to compare the results obtained by this approach with the one using the current voting system. Based on real data representing preference relations between the candidates, the results indicate that the final evaluations of the alternatives considering the multigroup method differ from the results obtained by the current voting system, which leads to possible changes in the ranking. In addition, it is noted that the solution that maximizes consensus among voters is close to the result obtained by the multigroup decision aiding method.

KEYWORDS. Multigroup decision aiding methods, Elections, Consensus.

ADM - Multicriteria decision aiding

1. Introduction

In the last few years, one may note a worldwide increasing volume of discussions involving politics. These discussions are also present in Brazil and, most of them, are related to the election of a candidate for the post of president, governor or mayor. Although the elected candidate had obtained the majority of votes (voting system considered in Brazil), there is a great discontent for a considerable part of the population. In this context, one question arises: does our voting system, considering maximizing voter's satisfaction, really choose the best candidate?

We might think, initially, of an optimal voting system that would maximize a welfare function among voters. However, as proved by Arrow in the 1950s [Arrow, 1951], if the number of decision makers is greater than 2, there is no social welfare function that rationally expresses the collective decision[1].

Although there is no ideal system, several techniques have been developed in the most varied scenarios involving social decisions [Brandt et al., 2016]. In addition, in a general decision-making context, a number of papers deal with multigroup problems [Herrera-Viedma et al., 2002; Dong et al., 2016; Zhang et al., 2011; Dong et al., 2016a]. Classically, these works aim at reaching a collective decision based on the aggregation of individual opinions. One aspect taken into account in this process is how to aggregate individual opinions in order to achieve a consensus solution among decision makers.

Several aggregation procedures and consensus measures can be exploited, however, the achieved solution depends on the adopted approach. Motivated by this fact, this paper proposes to compare the possible results of an election according to different decision models: the Brazilian electoral system based on direct popular vote (the winner is the one that receives the majority of votes), a group decision aiding technique based on the Analytic Hierarchy Process (AHP) [Saaty, 2008] and an optimization model that maximizes consensus among decision makers. Based on a set of real data collected in the context of the mayor election for a city in the state of São Paulo, we aim at verifying if the order of the candidates remains the same or if it changes according to the adopted approach.

The remainder of this article is divided as follows. In Section 2, we discuss the methodology used in this study. Section 3 contains the experiments performed and the results obtained. Finally, in Section 4, we present the conclusions of this paper and future perspectives.

2. Methodology

In this section, we describe the methodological aspects considered in this paper, which includes an explanation of the real data acquisition, a brief discussion about the Brazilian electoral system and a description of both the group decision aiding technique and the consensus measure considered.

2.1. Data acquisition

As already mentioned, the experiments conducted in this paper are based on real data collected from voters in the context of the election for mayor of a city in the state of São Paulo. Through an opinion survey, participants were invited to answer a set of questions about their preference for one candidate over another. In order to apply the AHP in the obtained data, the preference relations of a candidate i in comparison to the candidate j were based on the fundamental scale considered by such method, described in Table1 [Saaty, 1990].

In the considered election, there were 4 candidates for the mayor post. By considering the possibilities of both blank and null vote, there are $n = 6$ alternatives, which are represented by A_i,

[1] This result is known as Arrow's Impossibility Theorem, which takes into account certain properties that should be satisfied for a rational collective decision.

Table 1: AHP fundamental scale.

Definition	Intensity of preference
Equal importance	1
Moderate importance	3
Strong importance	5
Very strong importance	7
Extreme importance	9

$i = 1, \dots, 6$. This set of alternatives leads to $C_{6,2} = 6!/(2!(6-2)!) = 15$ pairwise comparisons, i.e. 15 preference relation between candidates. Therefore, for each pair of candidates (i, j), $i \neq j$, each participant indicated which was his preference and, then, provided the intensity, according to the scheme shown in Figure 1. For example, if the participant prefers candidate i over candidate j, he also indicates intensity of this preference. However, if it is indifferent between them, the comparison would end for this pair (i, j), not being necessary any intensity.

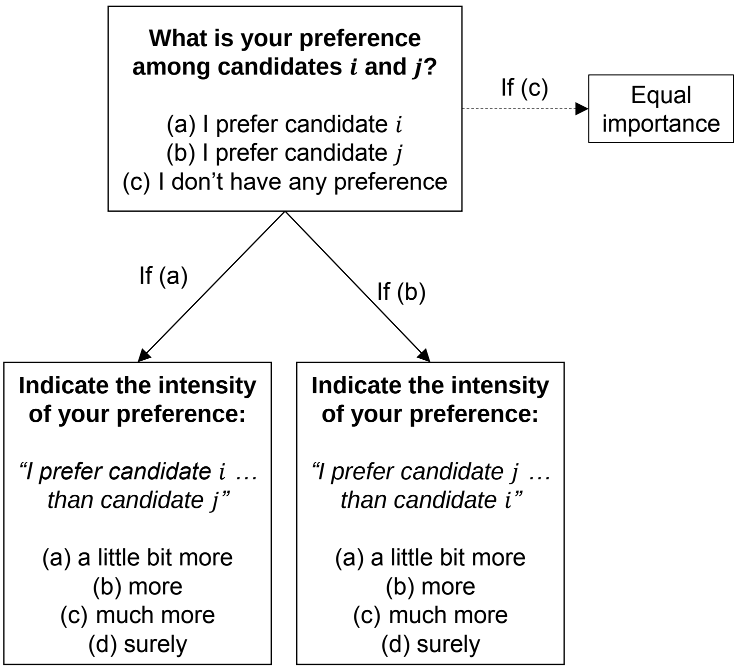

Figure 1: Opinion survey scheme.

In addition to the set of responses based on the opinion survey scheme illustrated in Figure 1, the participants also indicated their intention in the election, i.e. the candidate that he/she were going to vote[2].

2.2. Brazilian electoral system

In the Brazilian electoral system, the elections of a candidate for the post of president, governor or mayor is based on the direct popular vote. Among all the candidates, the one that achieves the majority of the votes is the winner. However, if the majority is not attained, a second

[2]In the next sections, even if an alternative A_i represents the blank vote or the null vote, it will be refered as a "candidate."

round is organized only with the first two candidates in the ranking[3]. If there is no majority even in the second round, the elected candidate is the older one.

The experiments in this paper only consider the general case of a single round. However, there is no restriction to extend the analyzes to the second round case.

2.3. Decision aiding method - AHP multigroup

The AHP, developed by Saaty [Saaty, 1990], is a method that aims at decomposing the problem into hierarchical levels, by taking into account the alternatives, criteria and subcriteria. At each level, pairwise comparisons are performed to determine the priority vector for each option (either alternatives or criteria). Finally, these vectors are aggregated in order to establish the ranking for the problem.

Since the goal in this study comprises the candidates ranking, the analyzes are based only on pairwise comparisons between them, without any criterion. Therefore, from the collected data, we build the set of K decision matrices

$$P^k = \left(p^k_{i,j}\right) = \begin{bmatrix} 1 & p^k_{1,2} & p^k_{1,3} & p^k_{1,4} & p^k_{1,5} & p^k_{1,6} \\ p^k_{2,1} & 1 & p^k_{2,3} & p^k_{2,4} & p^k_{2,5} & p^k_{2,6} \\ p^k_{3,1} & p^k_{3,2} & 1 & p^k_{3,4} & p^k_{3,5} & p^k_{3,6} \\ p^k_{4,1} & p^k_{4,2} & p^k_{4,3} & 1 & p^k_{4,5} & p^k_{4,6} \\ p^k_{5,1} & p^k_{5,2} & p^k_{5,3} & p^k_{5,4} & 1 & p^k_{5,6} \\ p^k_{6,1} & p^k_{6,2} & p^k_{6,3} & p^k_{6,4} & p^k_{6,5} & 1 \end{bmatrix}, \; k = 1, \ldots, K,$$

where K is the number of participants and $p^k_{i,j}$, $i, j = 1, \ldots, 6$, $i \neq j$, represents the preference intensity of candidate i over candidate j for the voter k, according to the scale shown in Table1. Since the AHP method is based on reciprocal matrices[4], each matrix P^k was filled by the 15 preferences indicated by participants and their 15 reciprocal values. The resulting matrix can by represented by

$$P^k = \begin{bmatrix} 1 & p^k_{1,2} & p^k_{1,3} & p^k_{1,4} & p^k_{1,5} & p^k_{1,6} \\ 1/p^k_{1,2} & 1 & p^k_{2,3} & p^k_{2,4} & p^k_{2,5} & p^k_{2,6} \\ 1/p^k_{1,3} & 1/p^k_{2,3} & 1 & p^k_{3,4} & p^k_{3,5} & p^k_{3,6} \\ 1/p^k_{1,4} & 1/p^k_{2,4} & 1/p^k_{3,4} & 1 & p^k_{4,5} & p^k_{4,6} \\ 1/p^k_{1,5} & 1/p^k_{2,5} & 1/p^k_{3,5} & 1/p^k_{4,5} & 1 & p^k_{5,6} \\ 1/p^k_{1,6} & 1/p^k_{2,6} & 1/p^k_{3,6} & 1/p^k_{4,6} & 1/p^k_{5,6} & 1 \end{bmatrix}, \; k = 1, \ldots, K.$$

For each matrix P^k, the AHP establishes the ranking of candidates from its eigenvector $\mathbf{w}^k$ (voter priority vector k containing the weights of each candidate) associated with the largest eigenvalue λ^k_{max}[5]. It is worth mentioning that AHP is based on consistent matrices, i.e. it should be applied in matrices whose consistency ratio $RC = CI/RI \leq 0.1$, where $CI = (\lambda_{max} - n)/(n-1)$ is the consistency index and RI is the index of random reciprocal matrices obtained from Table 2.

In the multigroup analysis, we need firstly to determine the collective matrix that represents the preferences of the group. This matrix, called P^C, is derived, in the multigroup AHP, from

[3]It is worth noting that, in Brazil, the second round in mayor elections is only possible for cities with more than 200.000 voters.

[4]For reciprocal matrices, we have the property $p_{j,i} = 1/p_{i,j}$.

[5]The AHP method considers that the preference of an alternative i over alternative j, in a full consistent scenario, may be represented as w_i/w_j, $i = 1, \ldots, n$, where w_i and w_j are, respectively, the weights of alternative i and alternative j. Therefore, one may represent the decision matrix as $P = (p_{i,j}) = (w_i/w_j)$. Right multiplying P by the vector of weights $\mathbf{w} = [w_1, \ldots, w_n]$, it is simple to verify that we obtain $P\mathbf{w} = n\mathbf{w}$, concluding that $\mathbf{w}$ is an eigenvector of P associated to the eigenvalue n. Moreover, since each row of P is a linear combination of the first row, P is a one-rank matrix. Therefore, only one eigenvalue of P is different to zero, i.e., it is equal to n. Since in real applications P is not a full-rank matrix, one determines the eigenvector $\mathbf{w}$ as the one associated with the largest eigenvalue λ_{max}.

Table 2: Index of random reciprocal matrices.

N	1	2	3	4	5	6	7
RI	0.00	0.00	0.58	0.90	1.12	1.24	1.32

the geometric mean of the individual matrices, i.e.[6]

$$P^C = \left(\prod_{k=1}^{K} p_{i,j}^k \right)^{1/K} . \tag{1}$$

Therefore, the collective ranking of the candidates is obtained from the eigenvector $\mathbf{w}^C$ of P^C associated with the eigenvalue λ_{max}^C.

2.4. Consensus solution

In addition to the AHP multigroup analysis, we also calculate the solution that takes into account the consensus among the voters, i.e., the level of agreement between them regarding all the candidates. Since in real applications it is improbable a full consensus between the decision makers, one may consider a consensus measure in order to evaluate the level of agreement between them [Dong et al., 2016a]. In this context, it is reasonable to define a measure that considers the distance between the collective and the individual rankings. Therefore, based on the hypothesis that the consensus solution reaches its maximum when its distance to the other individual rankings is minimized, the following optimization model was considered to find such a solution:

$$\min_{\mathbf{o}^C} \quad \sum_{k=1}^{K} \sum_{i=1}^{N} \left(o_i^C - o_i^k \right)^2 , \tag{2}$$

where o_i^C and o_i^k represent, respectively, the positions of the alternative i in the collective ranking $\mathbf{o}^C$ and in the ranking $\mathbf{o}^k$ of the voter k. It is noted that we used the Euclidean distance in (2), however, other metrics may be employed.

3. Experiments and results

In this section, we present the experiments and the obtained results. All the analyses were performed using the software MATLAB.

3.1. Real dataset

As described in Section 2.1, we consider a set of real data representing preference relations between candidates. In total, 34 voters participated in the survey. However, 15 provided consistent matrices (according to the definition that is often considered in the context of AHP). Therefore, in order to avoid inconsistent results, only these 15 datasets were considered in the analyzes, which are presented in the Table 3.

[6]The geometric mean must be applied in the multigroup AHP in order to guarantee the reciprocity of the collective matrix.

Table 3: Dataset: vote intentions and preference relations.

Voter 1 – **Vote:** A_3						**Voter** 2 – **Vote:** A_6						**Voter** 3 – **Vote:** A_2					
1	1/5	1/5	3	1	1	1	1/3	1/3	1	3	1/3	1	1/9	1	1	1	1/9
5	1	1/3	9	3	7	3	1	1/3	1	1	1/3	9	1	9	9	9	9
5	3	1	9	5	5	3	3	1	3	9	1/3	1	1/9	1	1	1	1/9
1/3	1/9	1/9	1	1/5	1/7	1	1	1/3	1	1	1/3	1	1/9	1	1	1	1/9
1	1/3	1/5	5	1	1	1/3	1	1/9	1	1	1/5	1	1/9	1	1	1	1/5
1	1/7	1/5	7	1	1	3	3	3	3	5	1	9	1/9	9	9	5	1
Voter 4 – **Vote:** A_2						**Voter** 5 – **Vote:** A_3						**Voter** 6 – **Vote:** A_6					
1	1/5	1	1	1	1	1	1	1/5	1	1/7	1/7	1	1	1	1	1/3	1/3
5	1	9	7	7	5	1	1	1/7	1/7	1/7	1/3	1	1	1	1	1/3	1/3
1	1/9	1	1	1	1	5	7	1	5	3	3	1	1	1	1	1	1/3
1	1/7	1	1	1	1	1	1	1/5	1	1/5	1/7	1	1	1	1	1	1/3
1	1/7	1	1	1	1	7	7	1/3	5	1	1	3	3	1	1	1	1/5
1	1/5	1	1	1	1	7	7	1/3	7	1	1	3	3	3	3	5	1
Voter 7 – **Vote:** A_2						**Voter** 8 – **Vote:** A_3						**Voter** 9 – **Vote:** A_6					
1	1/3	1	1	1	1/5	1	1	1/7	1	1	1	1	1	1	1	1	1/5
3	1	5	5	3	3	1	1	1/7	1	1	1	1	1	1	1	1/5	1/5
1	1/5	1	1	1/5	1/5	7	7	1	7	9	9	1	1	1	1	1	1/9
1	1/5	1	1	1/5	1/5	1	1	1/7	1	1	1	1	1	1	1	1	1/7
1	1/3	5	5	1	1/5	1	1	1/9	1	1	7	1	5	1	1	1	1
5	1/3	5	5	5	1	1	1	1/9	1	1/7	1	5	5	9	7	1	1
Voter 10 – **Vote:** A_3						**Voter** 11 – **Vote:** A_3						**Voter** 12 – **Vote:** A_3					
1	1	1/7	1	1	1	1	1	1/9	1	1	1	1	1	1/5	1	1/5	1
1	1	1/9	1	1	1	1	1	1/9	1	1	1	1	1	1/5	1	1/5	1/5
7	9	1	9	9	9	9	9	1	9	9	9	5	5	1	5	5	5
1	1	1/9	1	1	1	1	1	1/9	1	1	1	1	1	1/5	1	1/5	1/5
1	1	1/9	1	1	1/7	1	1	1/9	1	1	1	5	5	1/5	5	1	1
1	1	1/9	1	7	1	1	1	1/9	1	1	1	1	5	1/5	5	1	1
Voter 13 – **Vote:** A_3						**Voter** 14 – **Vote:** A_4						**Voter** 15 – **Vote:** A_3					
1	1/5	1/9	1	1	1	1	3	1	1/5	5	3	1	1	1/7	5	5	1
5	1	1/5	1	1	1	1/3	1	1/5	1/7	1	3	1	1	1/7	1	1	1
9	5	1	9	9	9	1	5	1	1/5	3	9	7	7	1	7	9	9
1	1	1/9	1	1	1	5	7	5	1	7	9	1/5	1	1/7	1	1	1
1	1	1/9	1	1	1	1/5	1	1/3	1/7	1	1	1/5	1	1/9	1	1	1
1	1	1/9	1	1	1	1/3	1/3	1/9	1/9	1	1	1	5	1/9	1	1	1

3.2. Comparison between the current voting system, AHP multigoup and the consensus solution

Based on the opinions described in Table 3, we may verify that candidate 3 would be elected by the current voting system, since it would receive the highest number of votes (8, in this case). Applying the AHP multigroup method, we obtain the collective matrix preferences

$$P^C = \begin{bmatrix} 1 & 0.582 & 0.340 & 1.076 & 0.978 & 0.569 \\ 1.719 & 1 & 0.472 & 1.492 & 1.005 & 1.040 \\ 2.937 & 2.118 & 1 & 2.790 & 3.104 & 1.859 \\ 0.929 & 0.670 & 0.358 & 1 & 0.741 & 0.472 \\ 1.023 & 0.995 & 0.322 & 1.349 & 1 & 0.651 \\ 1.758 & 0.962 & 0.538 & 2.118 & 1.536 & 1 \end{bmatrix}$$

and, consequently, its eigenvector $\mathbf{w}^C = [0.107, 0.159, 0.328, 0.100, 0.124, 0.182]$ (normalized in order to $\sum_{i=1}^{6} w_i^C = 1$), which provides the following ranking:

$$A_3 \succ A_6 \succ A_2 \succ A_5 \succ A_1 \succ A_4. \tag{3}$$

We may note in (3) that, based on the AHP multigroup method, candidate 3 would also be elected. It is worth mentioning that the ranking obtained by maximizing the consensus between voter, according to (2), is the same that the one provided by the AHP multigroup method, i.e. identical to (3).

In summary, the three approaches considered here provided the same results for the dataset described in Table 3. However, if we analyze the distance between the candidates in the final evaluation, we note that the current voting system and the multigroup AHP method provide different results. Figure 2 illustrates these results.

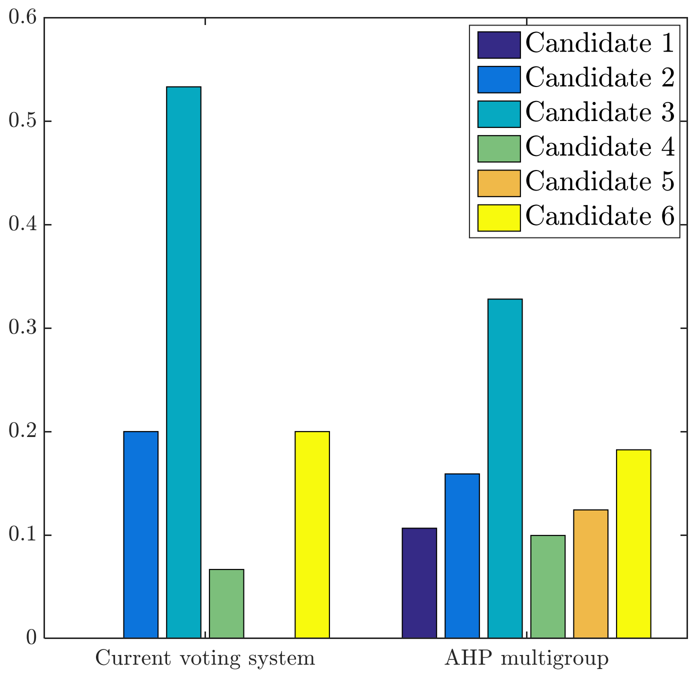

Figure 2: Comparison between the final evaluations of the current voting system and the multigroup AHP method.

Given the difference shown in Figure 2, one question that arises is if there would be any case where the current voting system would elect a candidate and the multigroup AHP method would provide another one as the first position. The experiment presented in the next section illustrates this situation.

3.3. The case of different candidate elected

In order to illustrate the case where the current voting system and the multigroup AHP provide different results when considering the first option in the ranking, we may consider only the opinion collected from voters 2, 3, 5, 6, 7, 10 and 11. Based on the data described in Table 3, we also verify that candidate 3 would have the highest number of votes and would, therefore, be elected by the current voting system. However, when applying the AHP multigroup method, we obtain the collective preferences matrix

$$P^C = \begin{bmatrix} 1 & 0.534 & 0.376 & 1.000 & 0.757 & 0.321 \\ 1.873 & 1 & 0.595 & 1.723 & 1.037 & 0.886 \\ 2.661 & 1.680 & 1 & 2.758 & 2.384 & 0.930 \\ 1.000 & 0.580 & 0.362 & 1 & 0.631 & 0.321 \\ 1.320 & 0.965 & 0.420 & 1.584 & 1 & 0.302 \\ 3.113 & 1.129 & 1.076 & 3.113 & 3.312 & 1 \end{bmatrix}$$

and, consequently, its eigenvector $\mathbf{w}^C = [0.090, 0.166, 0.257, 0.088, 0.120, 0.279]$, which provides the following ranking:

$$A_6 \succ A_3 \succ A_2 \succ A_5 \succ A_1 \succ A_4. \tag{4}$$

Therefore, in this case, candidate 6 would be elected, instead of candidate 3 (the most voted). Figure 3 illustrates the distance between the candidates for the current voting system and the AHP multigroup method.

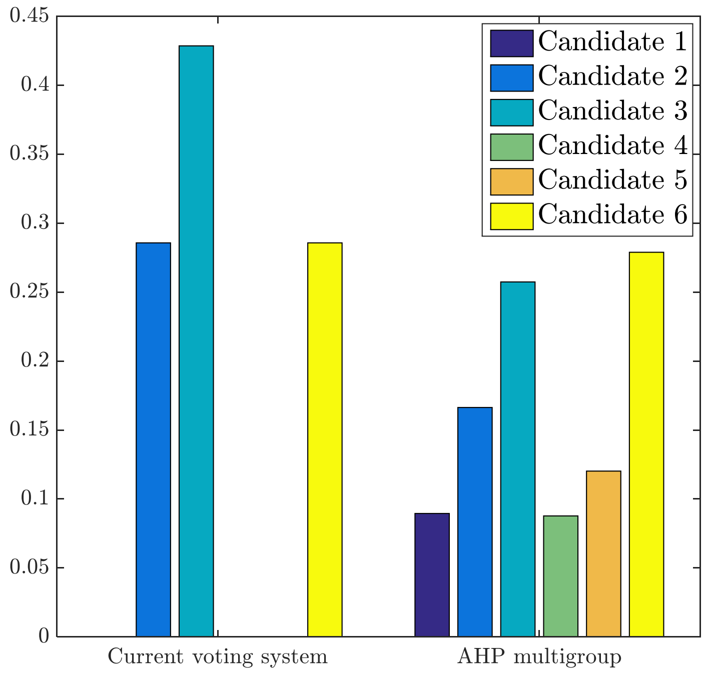

Figure 3: Comparison between the final evaluations of the current voting system and the multigroup AHP method (the case of different winner).

Moreover, if we analyze the solution of maximum consensus, obtained from (2), we find the ranking

$$A_6 \succ A_3 \succ A_5 \succ A_2 \succ A_4 \succ A_1, \quad (5)$$

which indicates that the first position would be the candidate 6.

This difference in the first position can be explained by the fact that candidate 6, although had received only 2 votes (in a total of 7), he/she is well qualified by the voters who did not vote for him as the first option. If we look for candidate 3, he/she has the highest number of votes but he/she is placed next to the last options for those who do not vote in him/her as the first position. Therefore, since the multigroup AHP method and the maximum consensus solution take into account the ranking obtained from the opinion of the voters, candidates who have a good overall positioning are favored. However, those who are the favorites for some voters and the worse ones for others, are penalized.

4. Conclusions

This article investigated the application of multigroup decision aiding techniques in the context of a election for mayor. The results showed that the final evaluations obtained by the Brazilian electoral system, the AHP multigroup method and the maximum consensus solution may differ, which makes possible changes in the ranking of the candidates and, consequently, the winner of the election.

It is worth noting that the experiments performed in this paper were based on a specific dataset. However, the results motivate the question if our current voting system actually represents the voters' opinion in order to maximize their satisfaction (or minimize dissatisfaction) with the elected candidate.

As future perspectives, we aim at applying other multigroup decision aiding methods and other consensus measures, such as the Kendall tau distance, in order to compare the results obtained with the current voting system. Furthermore, other researches may exploit the case in which a second round is needed.

Acknowledgements

The authors would like to thank the São Paulo Research Foundation (FAPESP - process n. 2016/21571-4 and 2015/16325-1).

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