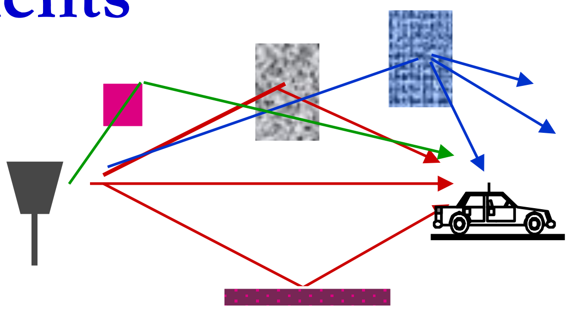
Review of Last Lecture

- Model Parameters from Measurements
- Random Multipath Model
- Channel Impulse Response



$$c(\tau,t)=\sum_{n=1}^{N}\alpha_n(t)e^{-j\varphi_n(t)}\delta(\tau-\tau_n(t))$$

  - Many multipath components, Amplitudes change slowly, Phases change rapidly
- For delay spread $\max|\tau_n(t)-\tau_m(t)| << 1/B$, $u(t) \approx u(t-\tau)$.
  - Received signal given by

$$r(t)=\Re\left\{u(t)e^{j2\pi f_c t}\left[\sum_{n=0}^{N(t)}\alpha_n(t)e^{-j\phi_n(t)}\right]\right\}$$

- No signal distortion in time
- Multipath yields complex scale factor in brackets

Review Continued:
Narrowband Model

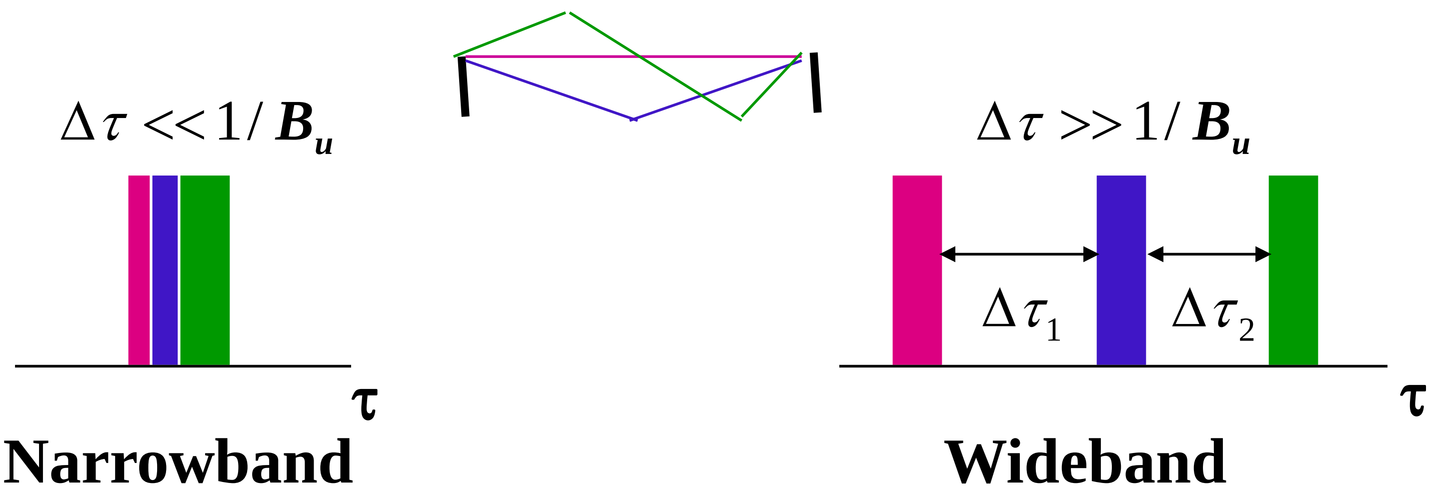
- For $N(t)$ large, in-phase $r_I(t)$ and quadrature $r_Q(t)$ components are jointly Gaussian by CLT (sum of large # of random vars).
- Received signal characterized by its mean, auto-correlation, and cross-correlation.
- If phase distribution is uniform, the in-phase/quad components are zero mean, independent, and stationary

Signal Envelope Distribution

- CLT approx. leads to Rayleigh distribution (power is exponential)
- When LOS component present, Ricean distribution is used
- Measurements support Nakagami distribution in some environments
  - Similar to Ricean, but models "worse than Rayleigh"
  - Lends itself better to closed form BER expressions

Wideband Channels

- Individual multipath components resolvable
- True when time difference between components exceeds signal bandwidth
- Requires statistical characterization of c(τ,t)
  - Assume CLT, stationarity and uncorrelated scattering
  - Leads to simplification of its autocorrelation function

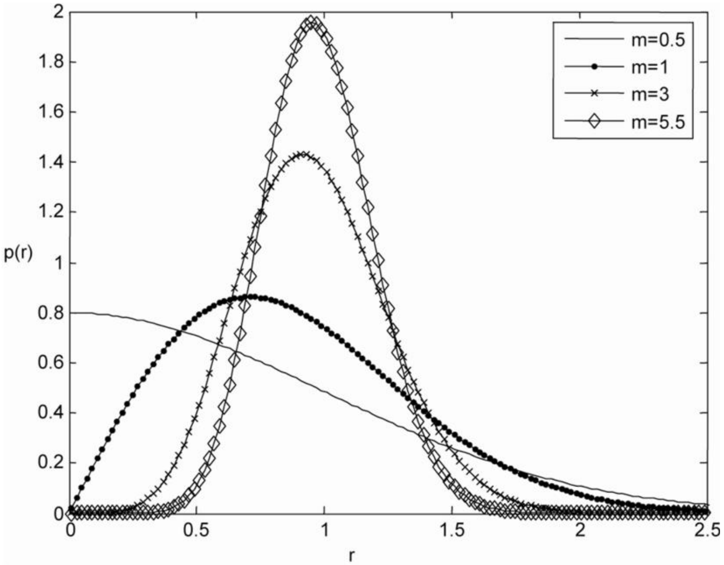


Signal Envelope Distribution

- CLT approx. for no dominant multipath component leads to Rayleigh distribution (power is exponential).
- When LOS component present, Ricean distribution is used
- Some measurements support Nakagami distribution
  - Parameterized by $m > 0.5$, varying from LOS power to multipath
  - Similar to Ricean ($m \approx K$, $K \geq 1$), but models "worse than Rayleigh"
  - Yields closed form BER expressions

$$p_Z(z) = \frac{2m^m z^{2m-1}}{\Gamma(m) P_r^m} \exp\left[\frac{-mz^2}{P_r}\right]$$

$$m \geq .5$$



Shannon Capacity

- Defined as the maximum mutual information of channel
- Maximum error-free data rate a channel can support.
- Theoretical limit (not achievable)
- Channel characteristic
  - Not dependent on design techniques

Capacity of Flat-Fading Channels

- Capacity defines theoretical rate limit
  - Maximum error free rate a channel can support
- Depends on what is known about channel
- Only fading distribution is known
  - Hard to find capacity
- Fading known at receiver only

$$C = \int_0^\infty B \log_2(1+\gamma)p(\gamma)d\gamma \le B\log_2(1+\bar{\gamma})$$

- Fading known at TX and RX
  - Multiplex optimal strategy over each channel state

Capacity with Fading Known at Transmitter and Receiver

- For fixed transmit power, same as with only receiver knowledge of fading
- Transmit power $P(\gamma)$ can also be adapted
- Leads to optimization problem where $\overline{P}$ is the average power constraint

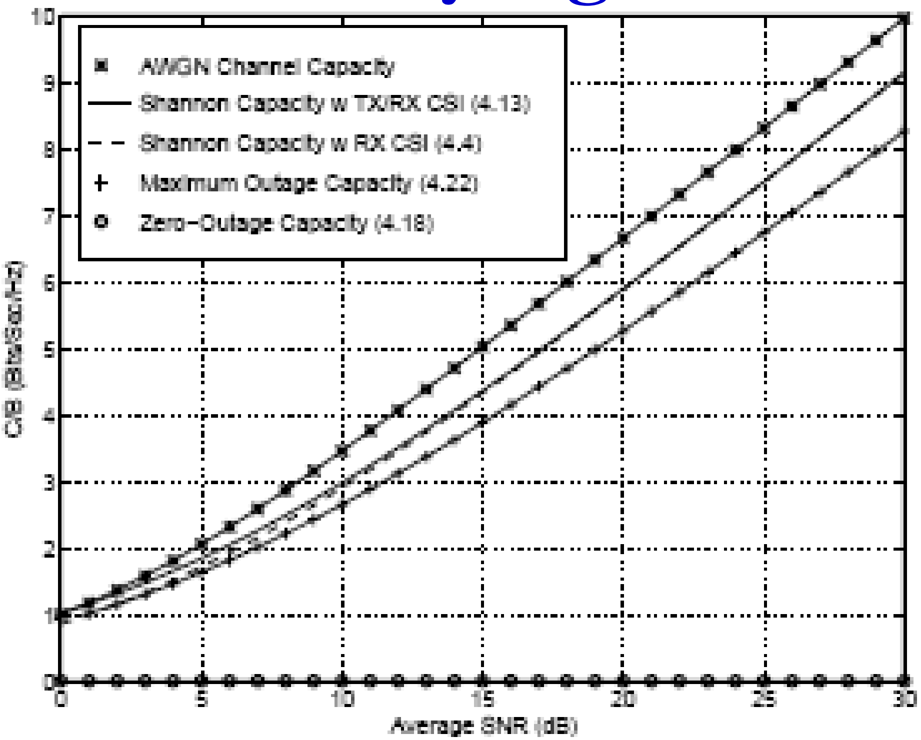
$$C = \max_{P(\gamma):E[P(\gamma)]=\overline{P}} \int_0^\infty B\log_2\left(1+\frac{\gamma P(\gamma)}{\overline{P}}\right)p(\gamma)d\gamma$$

Channel Inversion

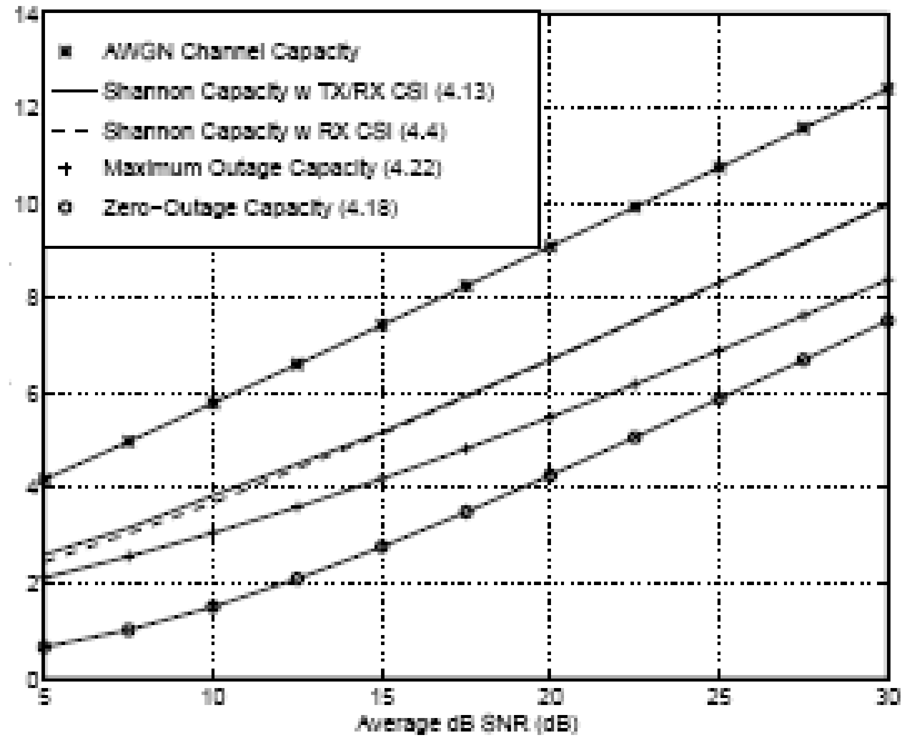
- Fading inverted to maintain constant SNR
- Simplifies design (fixed rate)
- Greatly reduces capacity
  - Capacity is zero in Rayleigh fading
- Truncated inversion
  - Invert channel above cutoff fade depth
  - Constant SNR (fixed rate) above cutoff
  - Cutoff greatly increases capacity
    - Close to optimal

Capacity in Flat-Fading

Rayleigh

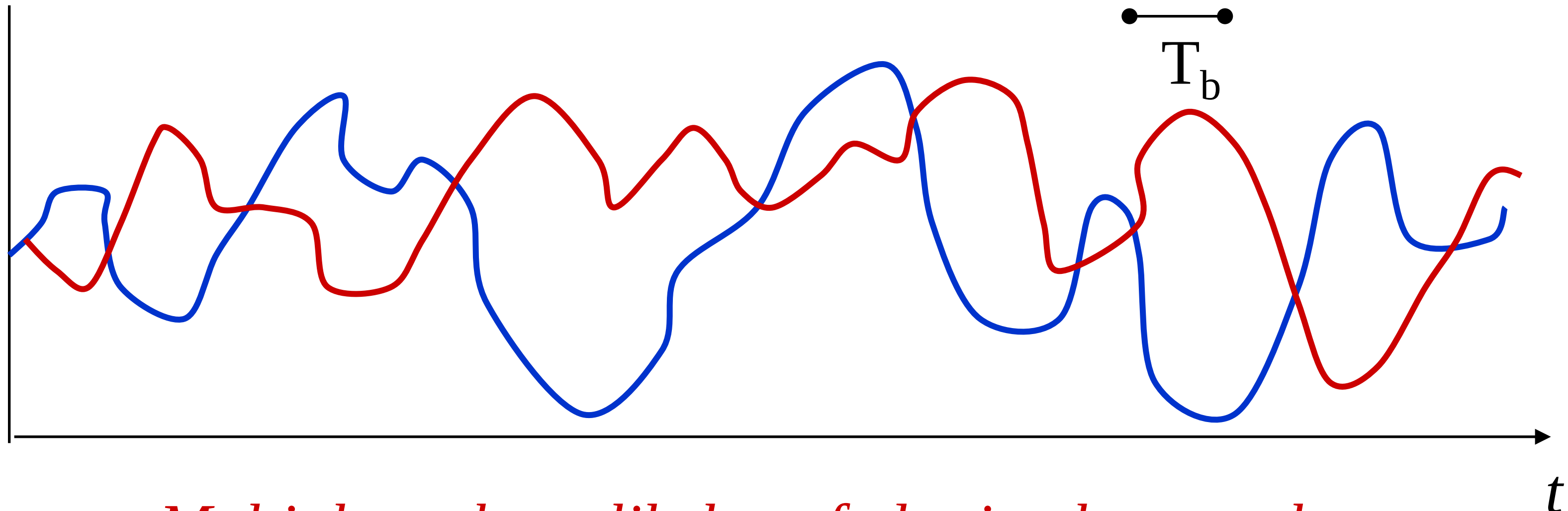


Log-Normal



Introduction to Diversity

- Basic Idea
  - Send same bits over independent fading paths
    - Independent fading paths obtained by time, space, frequency, or polarization diversity
  - Combine paths to mitigate fading effects



Multiple paths unlikely to fade simultaneously

Combining Techniques

- Selection Combining
  - Fading path with highest gain used
- Maximal Ratio Combining
  - All paths cophased and summed with optimal weighting to maximize combiner output SNR
- Equal Gain Combining
  - All paths cophased and summed with equal weighting
- Array/Diversity gain
  - Array gain is from noise averaging (AWGN and fading)
  - Diversity gain is change in BER slope (fading)

Selection Combining Analysis and Performance

- Selection Combining (SC)
  - Combiner SNR is the maximum of the branch SNRs.
  - CDF easy to obtain, pdf found by differentiating.
  - Diminishing returns with number of antennas.
  - Can get up to about 20 dB of gain.

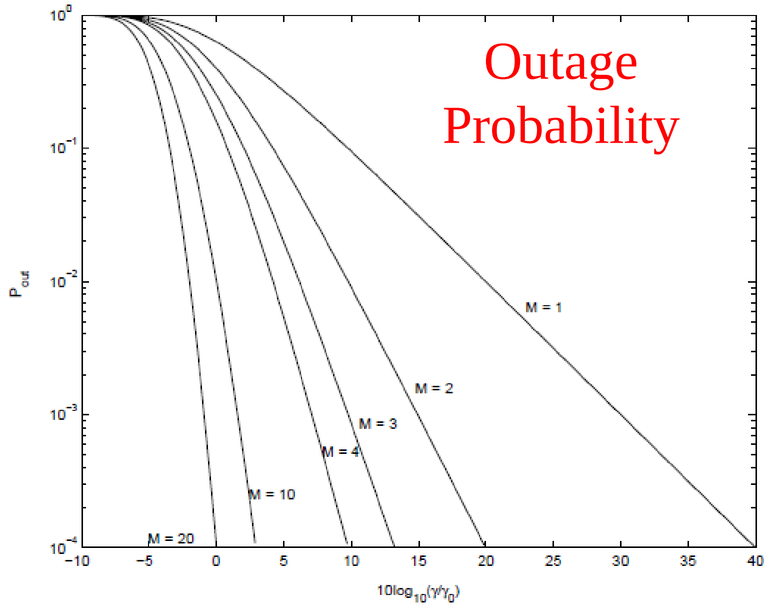


Figure 7.2: Outage Probability of Selection Combining in Rayleigh Fading.

MRC and its Performance

- With MRC, $\gamma_\Sigma = \Sigma \gamma_i$ for branch SNRs γ_i
  - Optimal technique to maximize output SNR
  - Yields 20-40 dB performance gains
  - Distribution of γ_Σ hard to obtain
- Standard average BER calculation

$$\overline{P}_b = \int P_b(\gamma_\Sigma) p(\gamma_\Sigma) d\gamma_\Sigma = \int\int \ldots \int P_b(\gamma_\Sigma) p(\gamma_1) * p(\gamma_2) * \ldots * p(\gamma_M) d\gamma_1 d\gamma_2 \ldots d\gamma_M$$

  - Hard to obtain in closed form
  - Integral often diverges
- Preview: MGF Approach:

$$\overline{P}_b = \frac{1}{\pi} \int_0^{.5\pi} \prod_{i=1}^{M} \mathcal{M}_i \left[\frac{-g}{\sin^2 \varphi} ; \gamma_i \right] d\varphi$$

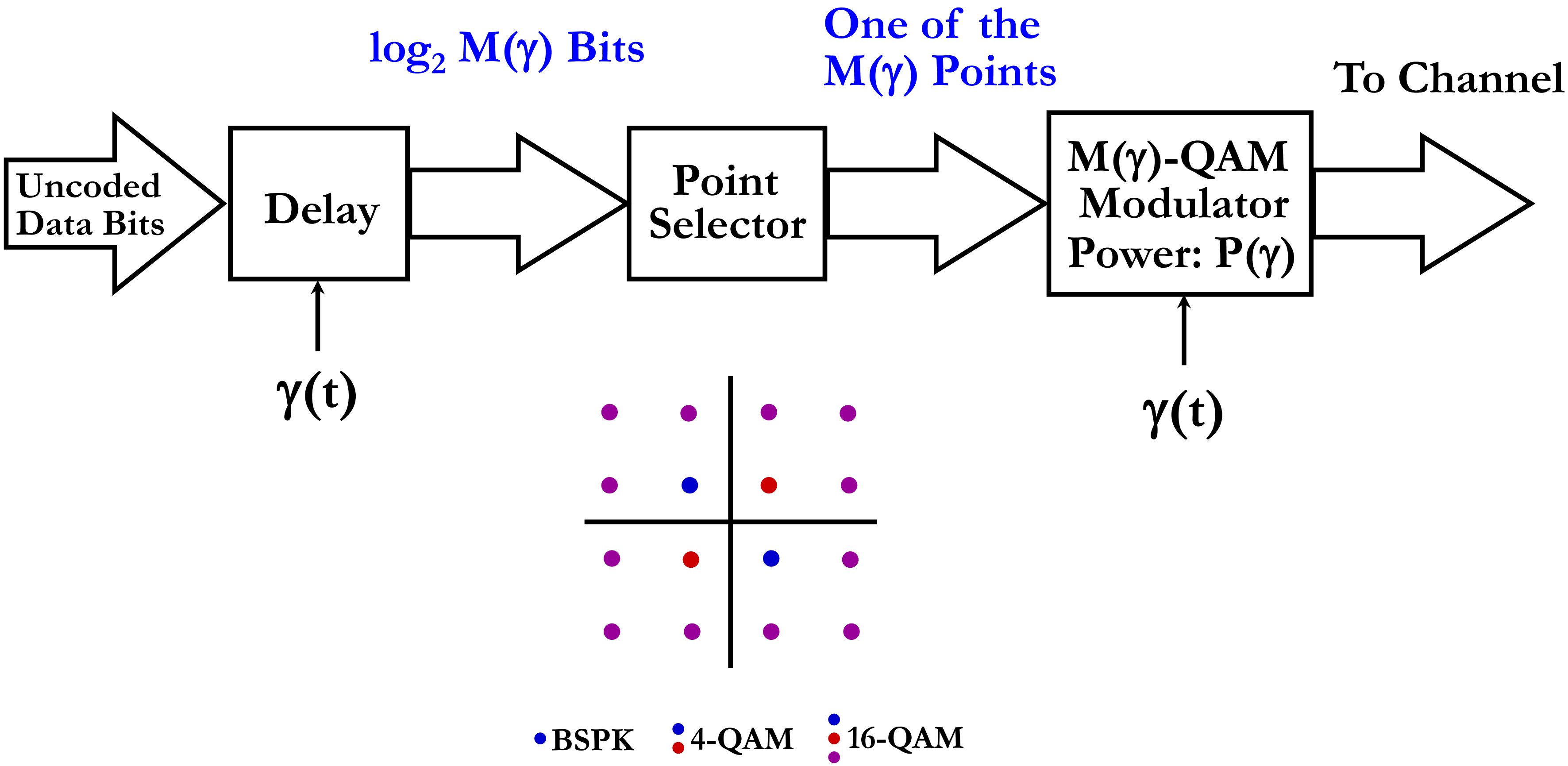
Adaptive Modulation

- Change modulation relative to fading
- Parameters to adapt:
  - Constellation size
  - Transmit power
  - Instantaneous BER
  - Symbol time
  - Coding rate/scheme

Only 1-2 degrees of freedom needed for good performance

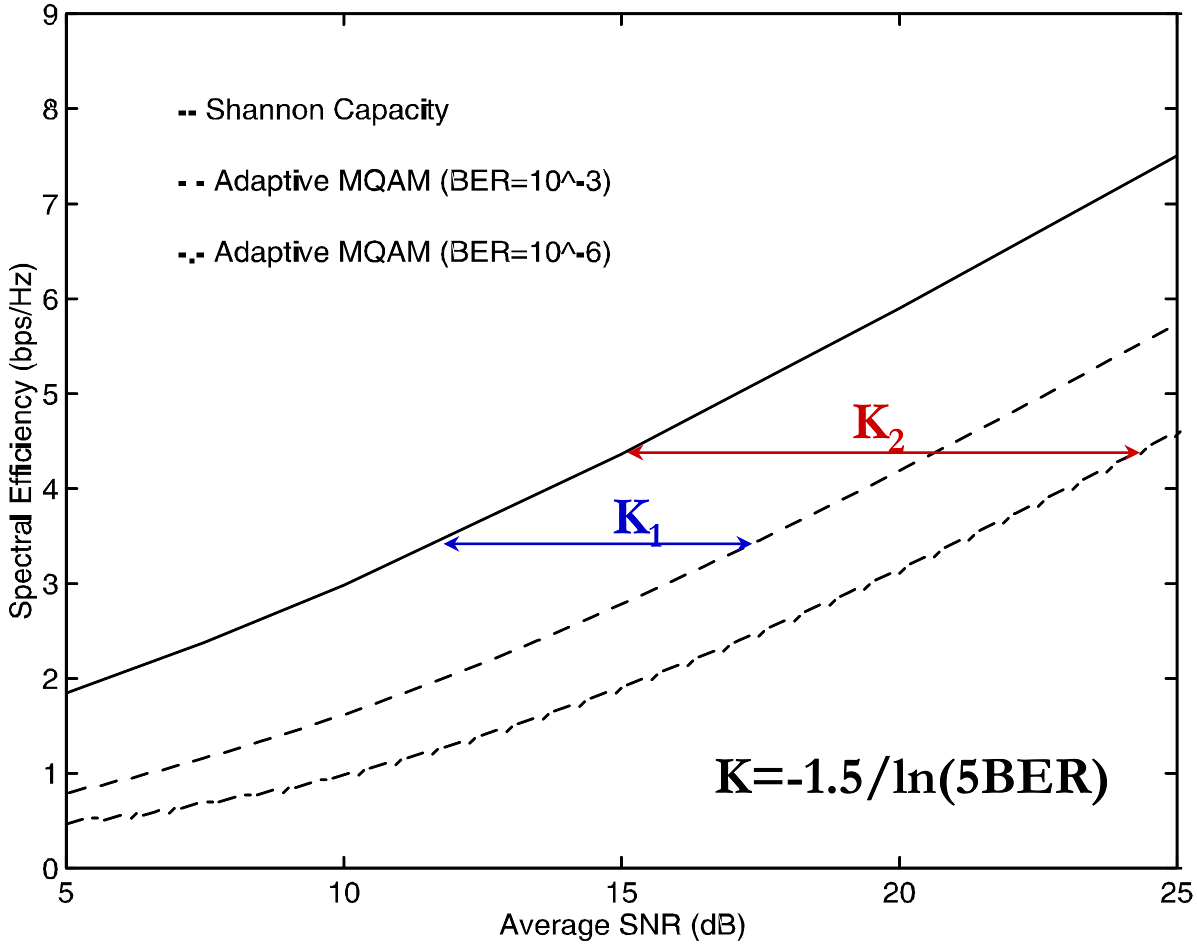
- Optimization criterion:
  - Maximize throughput
  - Minimize average power
  - Minimize average BER

Variable-Rate Variable-Power MQAM



Goal: Optimize P(γ) and M(γ) to maximize R=Elog[M(γ)]

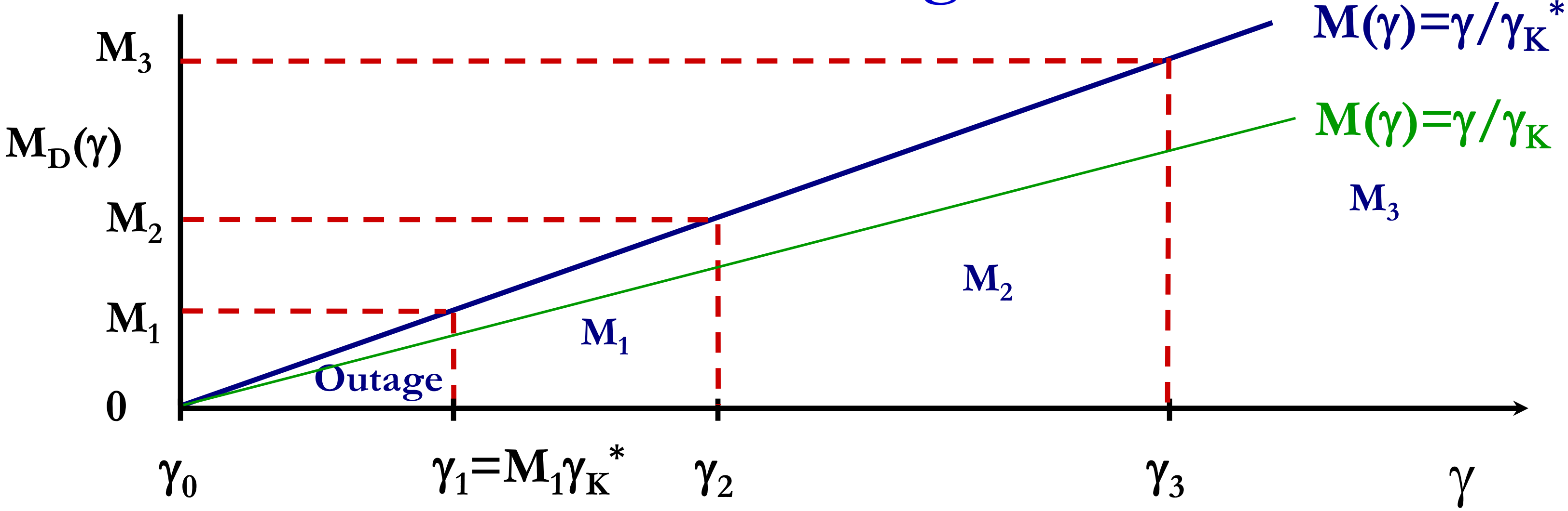
Spectral Efficiency



Can reduce gap by superimposing a trellis code

Constellation Restriction

- Restrict $M_D(\gamma)$ to $\{M_0=0,\ldots,M_N\}$.
- Let $M(\gamma)=\gamma/\gamma_K^*$, where γ_K^* is optimized for max rate
- Set $M_D(\gamma)$ to $\max_j M_j$: $M_j \leq M(\gamma)$ (conservative)
- Region boundaries are $\gamma_j=M_j\gamma_K^*$, j=0,…,N
- Power control maintains target BER



Power Adaptation and Average Rate

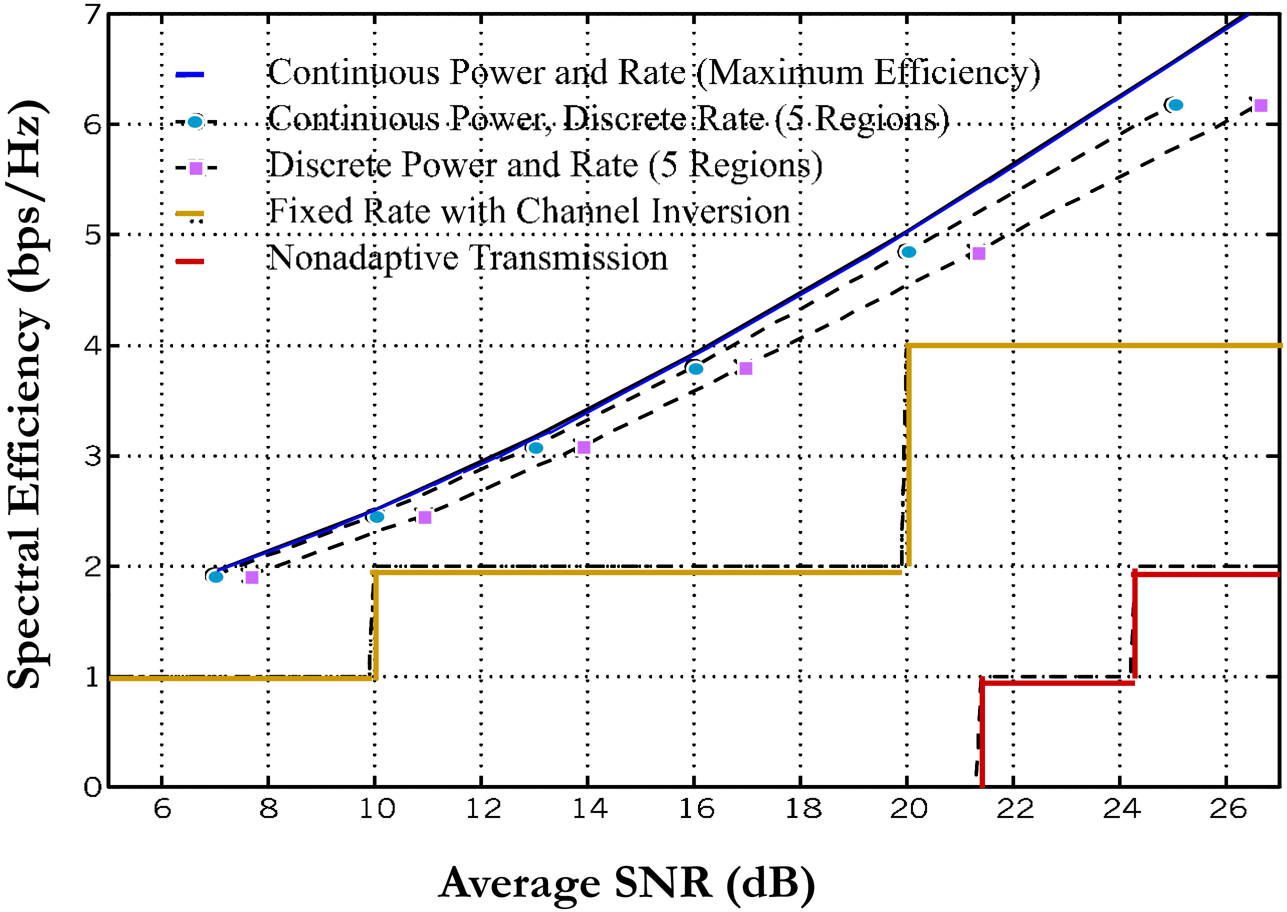
- **Power adaptation:**
  - **Fixed BER within each region**
    - **$E_s/N_0=(M_j\text{-}1)/K$**
    - **Channel inversion within a region**
  - **Requires power increase when increasing M(γ)**

$$\frac{P_j(\gamma)}{P}=\begin{cases}(M_j-1)/(\gamma K) & \gamma_j\le\gamma<\gamma_{j+1}, j>0\\ 0 & \gamma<\gamma_1\end{cases}$$

- **Average Rate**

$$\frac{R}{B}=\sum_{j=1}^{N}\log_2 M_j\, p(\gamma_j\le\gamma<\gamma_{j+1})$$

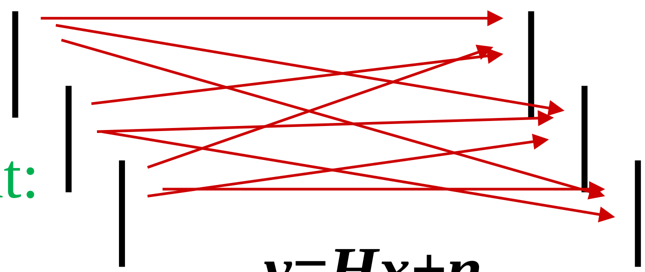
Efficiency in Rayleigh Fading



Multiple Input Multiple Output (MIMO)Systems

- MIMO systems have multiple transmit and receiver antennas (M_t at TX, M_r at RX)

$x=[x_1,\ldots,x_{Mt}]$



$y=[y_1,\ldots,y_{Mr}]$

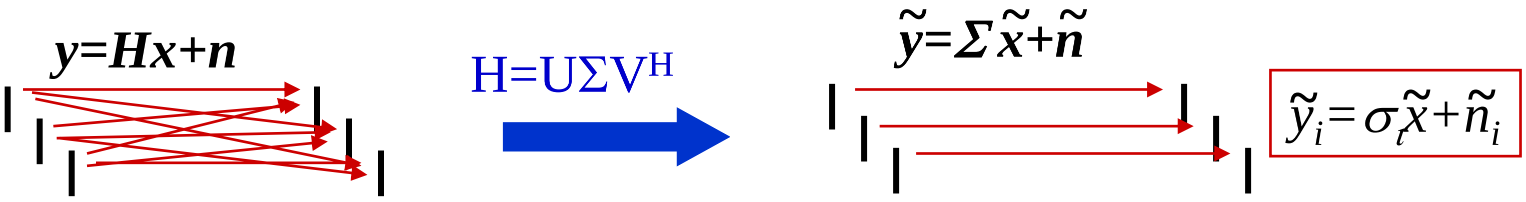
TX power constraint:
$E[\mathbf{xx}^H]=\rho=P/\sigma^2$

$y=Hx+n$

- Input described by vector x of dimension M_t
- Output described by vector y of dimension M_r
- Channel described by $M_r x M_t$ matrix
- Design and capacity analysis depends on what is known about channel H at TX and RX
  - If H unknown at TX, requires vector modulation/demod

MIMO Decomposition

- **Decompose channel through transmit precoding ($x=V\tilde{x}$) and receiver shaping ($\tilde{y}=U^H y$)**



- **Leads to $R_H \leq \min(M_t,M_r)$ independent channels with gain σ_i (i^{th} singular value of H) and AWGN**

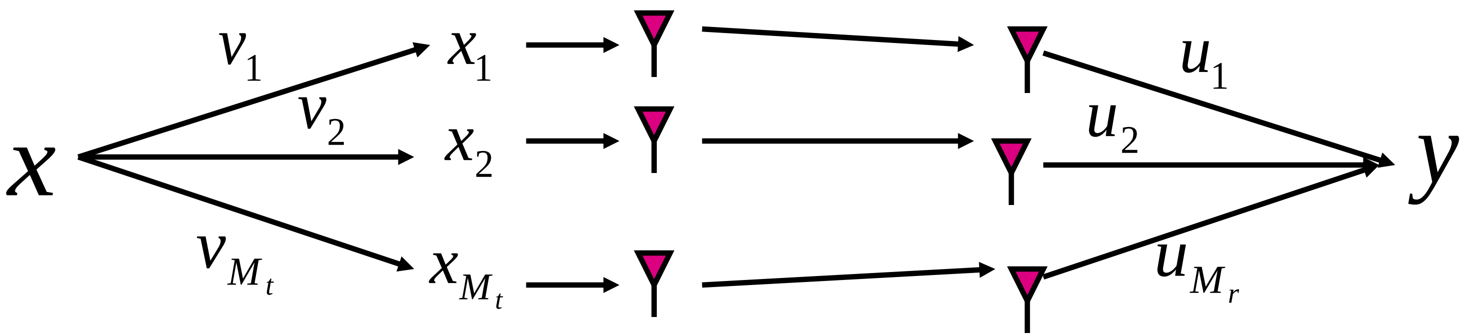
- **Independent channels lead to simple capacity analysis and modulation/demodulation design**

Capacity of MIMO Systems

- Depends on what is known at TX and RX and if channel is static or fading
- For static channel with perfect CSI at TX and RX, power water-filling over space is optimal:
  - In fading waterfill over space (based on short-term power constraint) or space-time (long-term constraint)
- Without transmitter channel knowledge, capacity metric is based on an outage probability
  - P_{out} is the probability that the channel capacity given the channel realization is below the transmission rate.

Beamforming

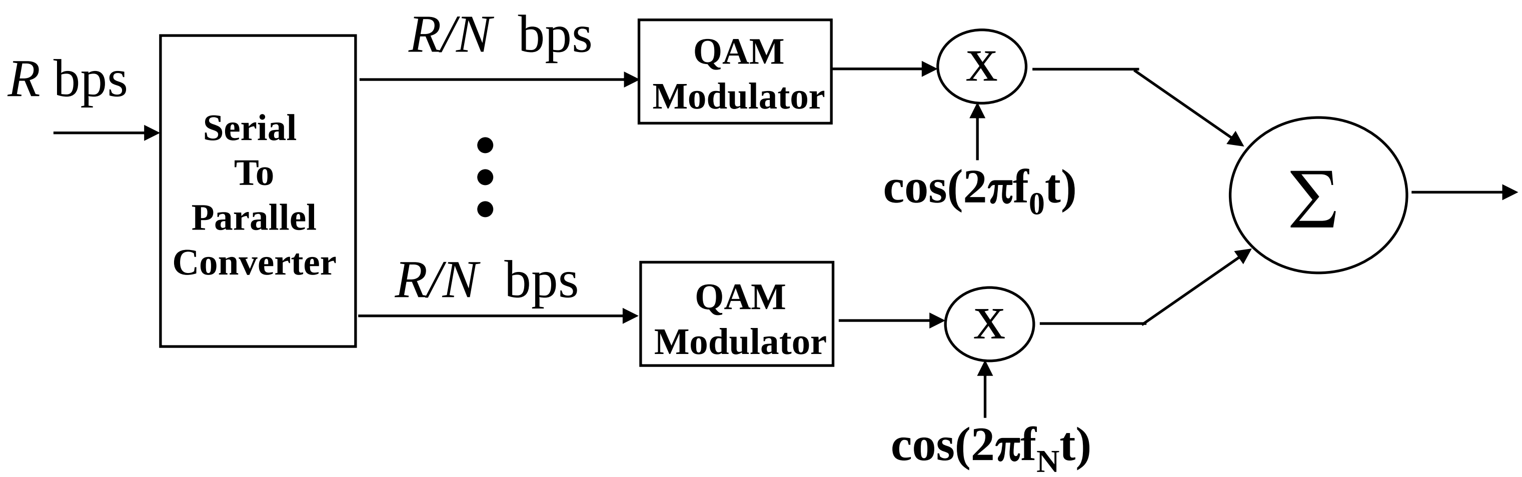
- Scalar codes with transmit precoding



$$y = \mathbf{u}^H \mathbf{H} \mathbf{v} x + \mathbf{u}^H \mathbf{n}$$

- Transforms system into a SISO system with diversity.
  - Array and diversity gain
  - Greatly simplifies encoding and decoding.
  - Channel indicates the best direction to beamform
  - Need "sufficient" knowledge for optimality of beamforming

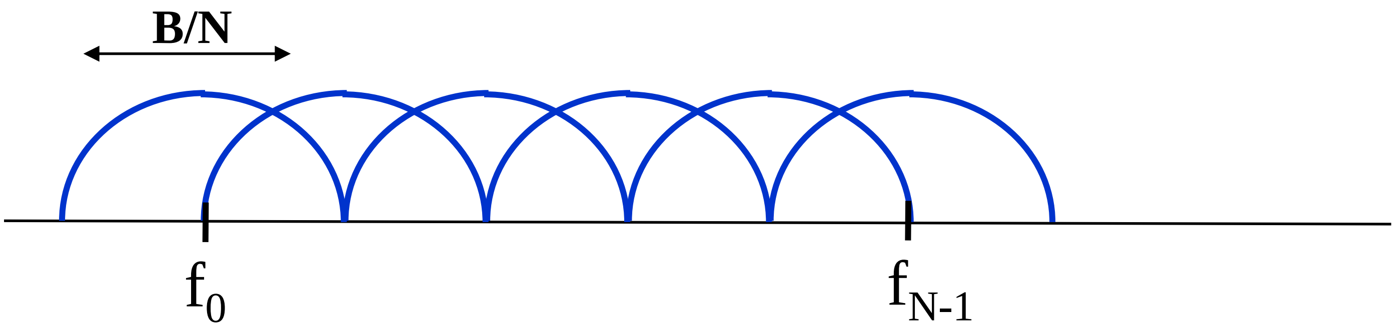
Multicarrier Modulation



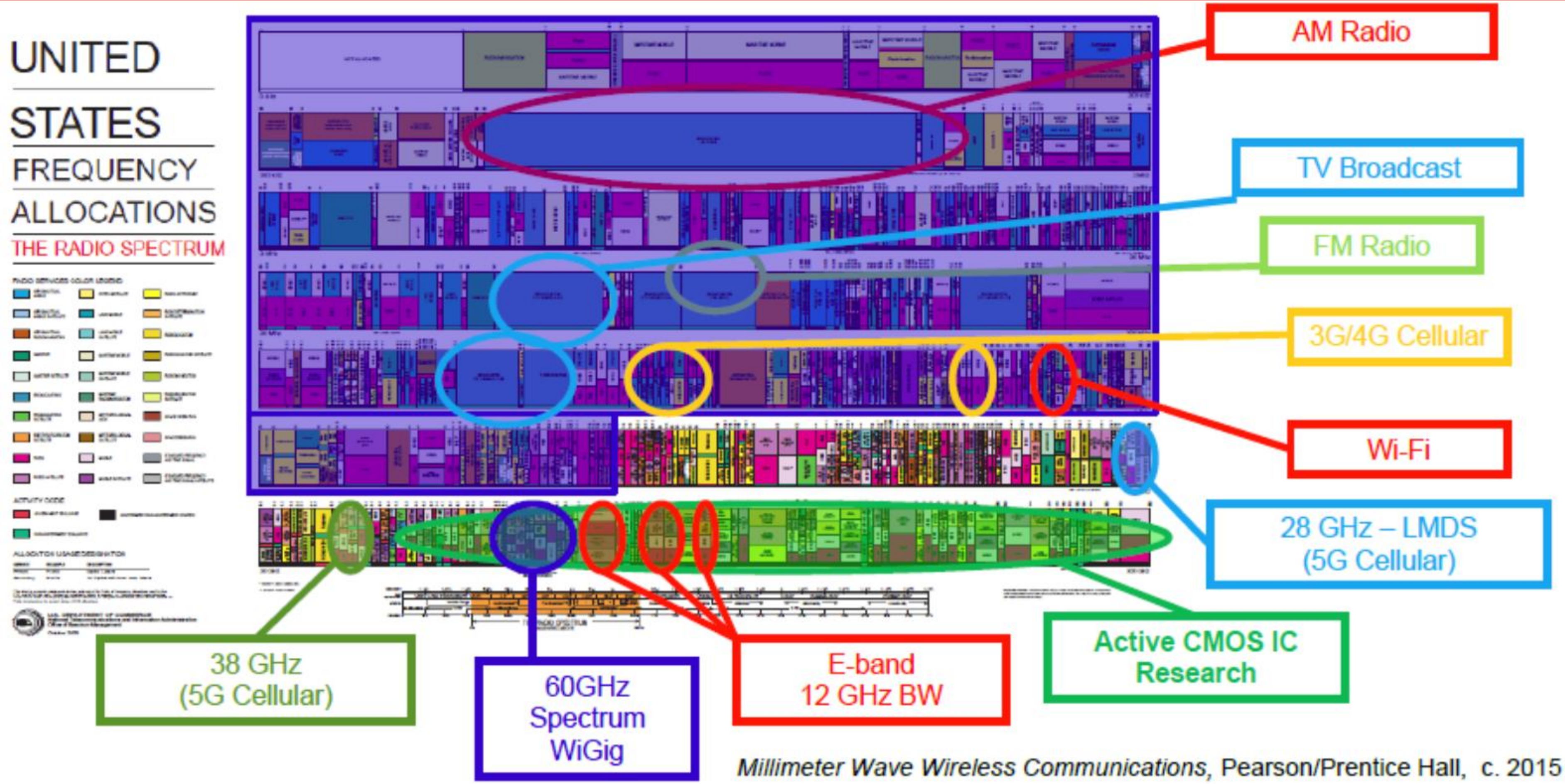
- Breaks data into N substreams
- Substream modulated onto separate carriers
  - Substream passband BW is B/N for B total BW
  - $B/N<B_c$ implies flat fading on each subcarrier (no ISI)

Overlapping Substreams

- Can have completely separate subchannels
  - Required passband bandwidth is B.
- OFDM overlaps substreams
  - Substreams (symbol time T_N) separated in RX
  - Minimum substream separation is $1/T_N$ for rectangular pulses
  - Total required bandwidth is B/2



mmWave: What's the big deal?

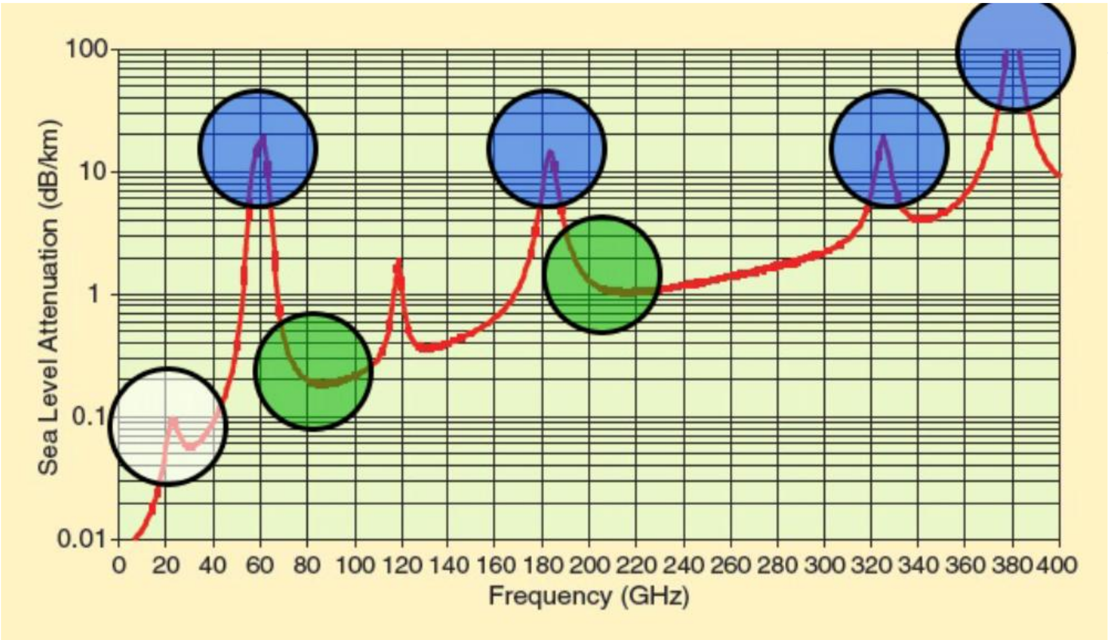


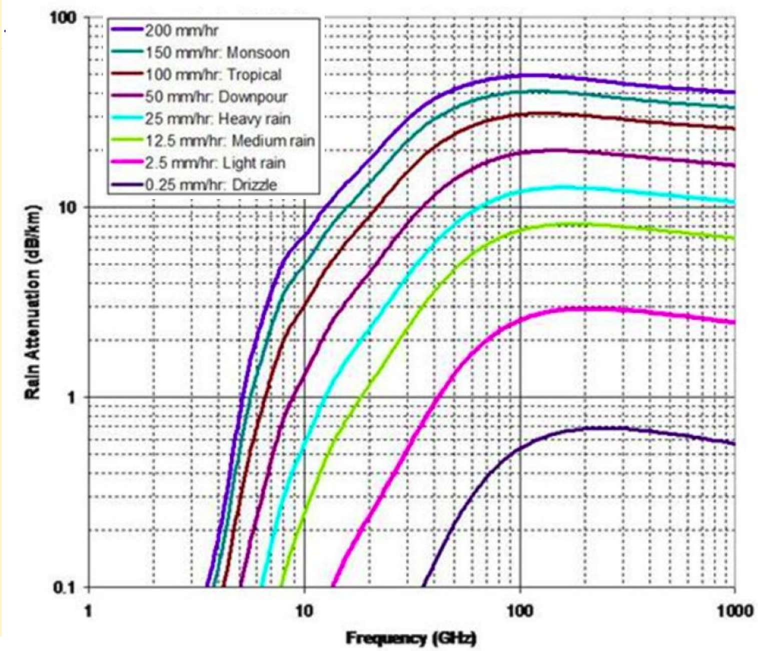
All existing commercial systems fit into a small fraction of the mmWave band

mmWave Propagation (60-100GHz)

mmW Massive MIMO

- Channel models immature
  - Based on measurements, few accurate analytical models
- Path loss proportion to λ^2 (huge)
- Also have oxygen and rain absorbtion





mmWave systems will be short range or require "massive MIMO"