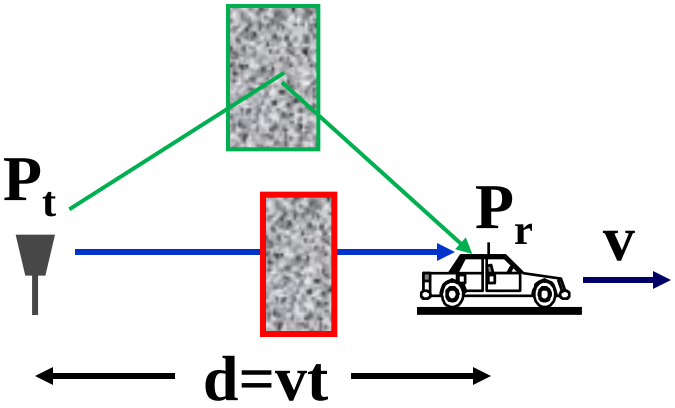
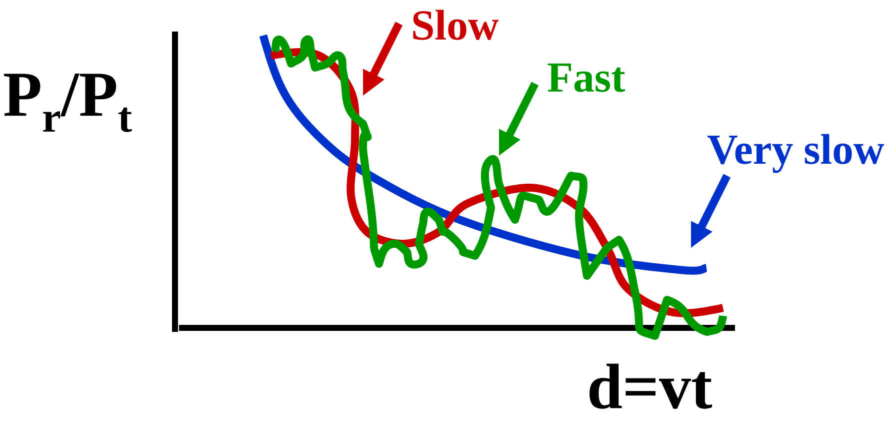
EE359 – Lecture 2 Outline

- TX and RX Signal Models
- Path Loss Models
  - Free-space and 2-Ray Models
  - General Ray Tracing
  - Simplified Path Loss Model
  - Empirical Models
  - Shadowing
  - mmWave Models

Propagation Characteristics

- Path Loss (includes average shadowing)
- Shadowing (due to obstructions)
- Multipath Fading

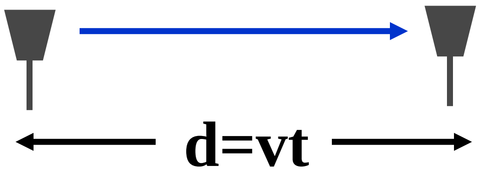




Path Loss Modeling

- Maxwell's equations
  - Complex and impractical
- Free space and 2-path models
  - Too simple
- Ray tracing models
  - Requires site-specific information
- Simplified power falloff models
  - Main characteristics: good for high-level analysis
- Empirical Models
  - Don't always generalize to other environments

Free Space (LOS) Model



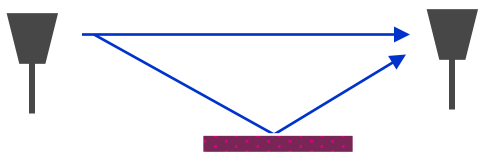
- Path loss for unobstructed LOS path
- Power falls off :

$$\frac{P_r}{P_t} = \left[\frac{\sqrt{G_l}\lambda}{4\pi d}\right]^2$$

  - Proportional to $1/d^2$
  - Proportional to λ^2 (inversely proportional to f^2)
    - This is due to the effective aperature of the antenna
  - Free-space path loss

$$P_L \text{ dB} = 10\log_{10}\frac{P_t}{P_r} = -10\log_{10}\frac{G_l\lambda^2}{(4\pi d)^2}$$

Two Ray Model



- Path loss for one LOS path and 1 ground (or reflected) bounce
- Ground bounce approximately cancels LOS path above critical distance
- Power falls off
  - Proportional to d^2 (small d)
  - Proportional to d^4 ($d>d_c$)
  - Independent of λ (f_c)
    - Two-path cancellation equivalent to 2-element array, i.e. the effective aperature of the receive antenna is changed.

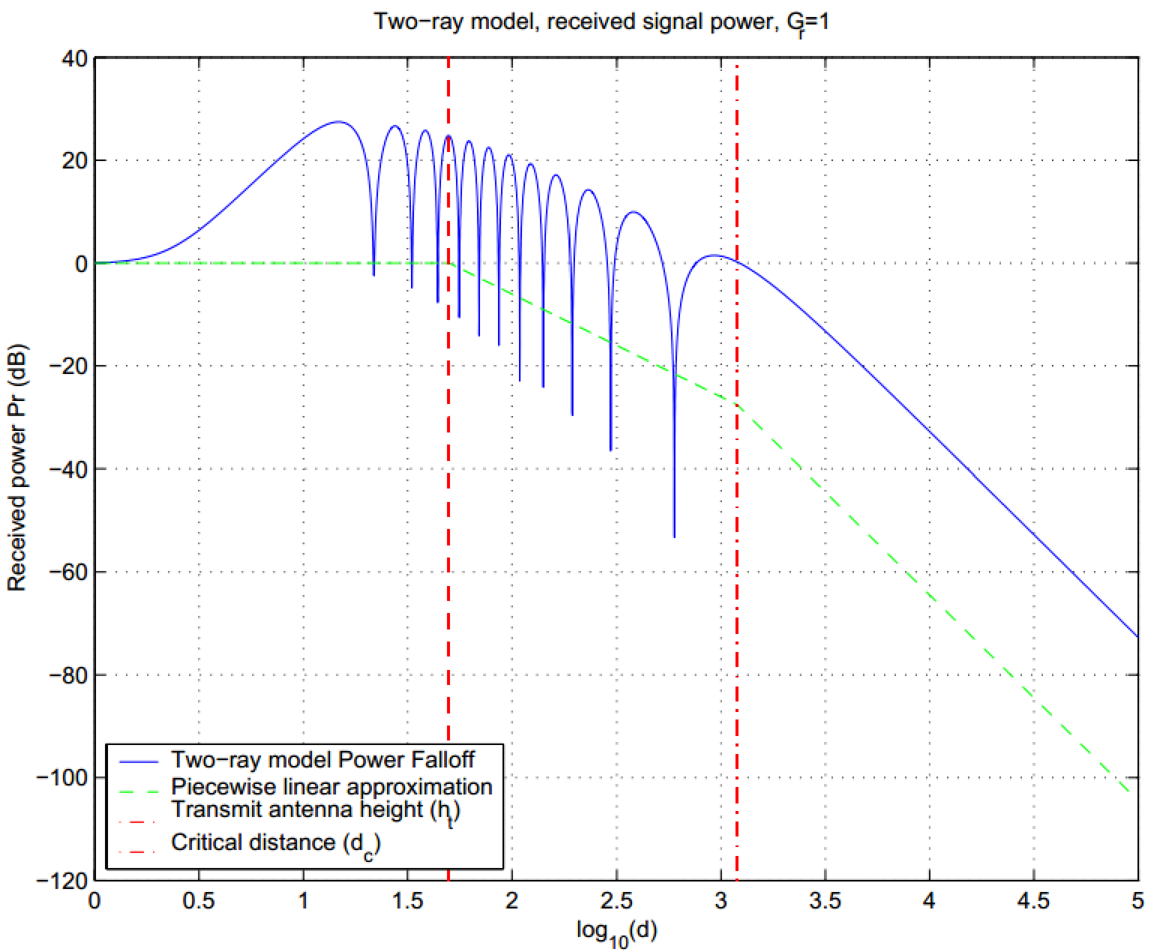
Two Ray Model

- **Received power:**

$$P_r \approx \left[\frac{\sqrt{G_l} h_t h_r}{d^2} \right]^2 P_t$$

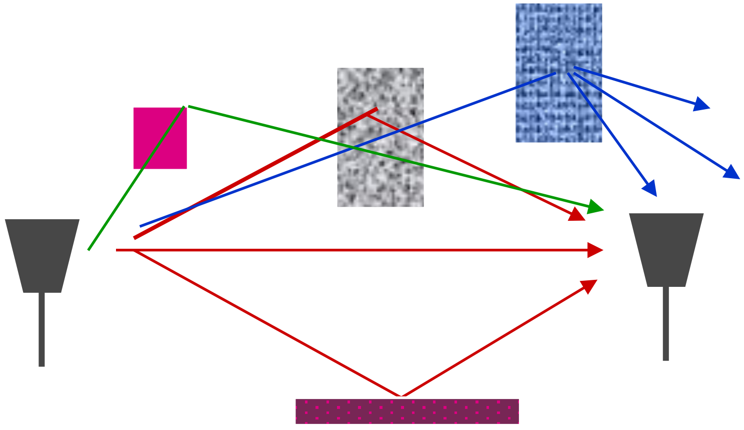
  - **Critical distance:**

$$d_c = 4 h_t h_r / \lambda$$



General Ray Tracing

- Models signal components as particles
  - Reflections
  - Scattering
  - Diffraction



Reflections generally dominate

- Requires site geometry and dielectric properties
  - Easier than Maxwell (geometry vs. differential eqns)
- Computer packages often used

10-ray reflection model explored in HW

Simplified Path Loss Model

- Used when path loss dominated by reflections.
- Most important parameter is the path loss exponent γ, determined empirically.

$$P_r = P_t K \left[\frac{d_0}{d} \right]^{\gamma}, \qquad 2 \le \gamma \le 8$$

Empirical Channel Models

- Cellular Models: Okumura model and extensions:
  - Empirically based (site/freq specific), uses graphs
  - Hata model: Analytical approximation to Okumura
  - Cost 231 Model: extends Hata to higher freq. (2 GHz)
  - Multi-slope model
  - Walfish/Bertoni: extends Cost 231 to include diffraction

- WiFi channel models: TGn
  - Empirical model for 802.11n developed within the IEEE standards committee. Free space loss up to a breakpoint, then slope of 3.5. Breakpoint is empirically-based.

Commonly used in cellular and WiFi system simulations

Empirical Channel Models

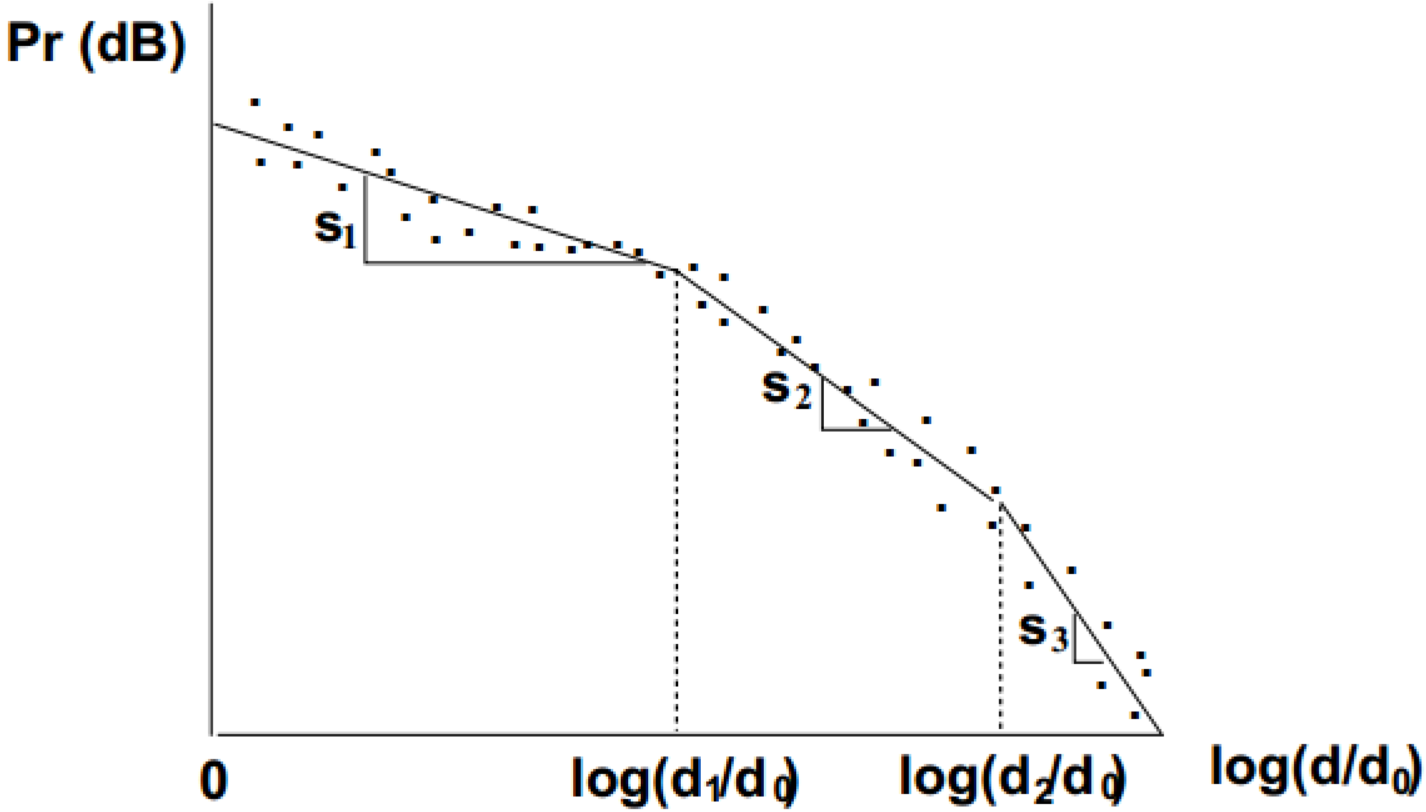
- **Okumura model:**

$$P_L(d)\ \text{dB} = L(f_c, d) + A_{mu}(f_c, d) - G(h_t) - G(h_r) - G_{AREA}$$

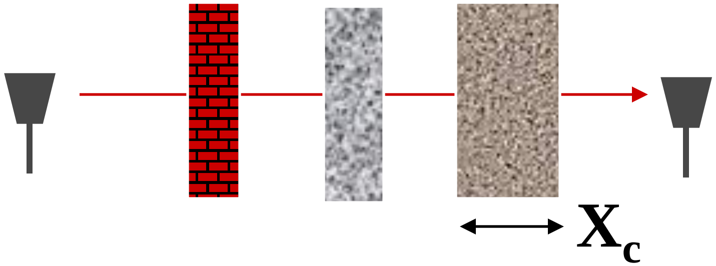
  - **in which d is the distance, f_c is the carrier frequency, $L(f_c,d)$ is free space path loss, $A_{mu}(f_c,d)$ is the median attenuation in addition to free space path loss across all environments, $G(h_t)$ is the base station antenna height gain factor, $G(h_r)$ is the mobile antenna height gain factor, and G_{AREA} is the gain due to the type of environment**

Empirical Channel Models

- **Multi-slope (piecewise linear) model:**



Shadowing



- Models attenuation from obstructions
- Random due to random # and type of obstructions
- Typically follows a log-normal distribution
  - dB value of power is normally distributed
  - $\mu=0$ (mean captured in path loss), $4<\sigma<12$ (empirical)
  - Central Limit Theorem used to explain this model
  - Decorrelates over decorrelation distance X_c

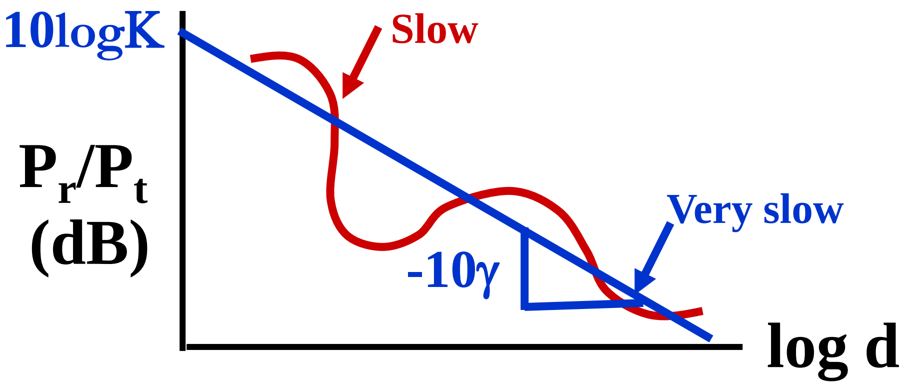
Shadowing

- Log-normal distribution (envelope)
  - PDF:
$$p(\psi_{\mathrm{dB}}) = \frac{1}{\sqrt{2\pi}\sigma_{\psi_{dB}}} \exp\left[-\frac{(\psi_{\mathrm{dB}} - \mu_{\psi_{dB}})^2}{2\sigma^2_{\psi_{dB}}}\right]$$
    - in which ψ_{dB} is the signal envelope, $\mu_{\psi\mathrm{dB}}$ is the mean value, and $\sigma_{\psi\mathrm{dB}}$ is the standard deviation, all given in dB
  - Empirical studies for outdoor channels support a standard deviation $\sigma_{\psi\mathrm{dB}}$ from 4 to 13 dB
  - Mean power $\mu_{\psi\mathrm{dB}}$ depends on the path loss and building properties; it decreases with distance

Combined Path Loss and Shadowing

- **Linear Model: ψ lognormal**

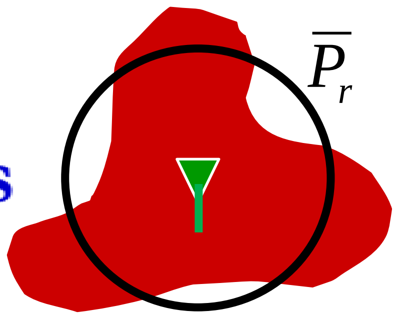
$$\frac{P_r}{P_t} = K\left(\frac{d_0}{d}\right)^{\gamma}\psi$$



- **dB Model**

$$\frac{P_r}{P_t}(dB) = 10\log_{10}K - 10\gamma\log_{10}\left(\frac{d}{d_0}\right) + \psi_{dB}, \quad \psi_{dB} \sim N(0,\sigma_{\psi}^2)$$

Outage Probability



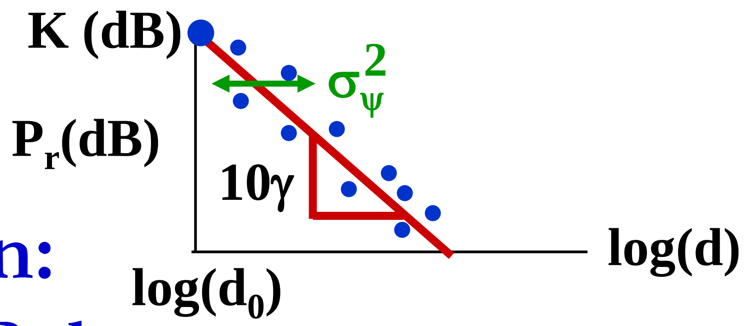
- Path loss only: circular "cells"; Path loss+shadowing: amoeba-shaped cells
- Outage probability: probability received power falls below given minimum:

$$p_{out} = \mathbf{p}(P_r < P_{min})$$

- For log-normal shadowing model

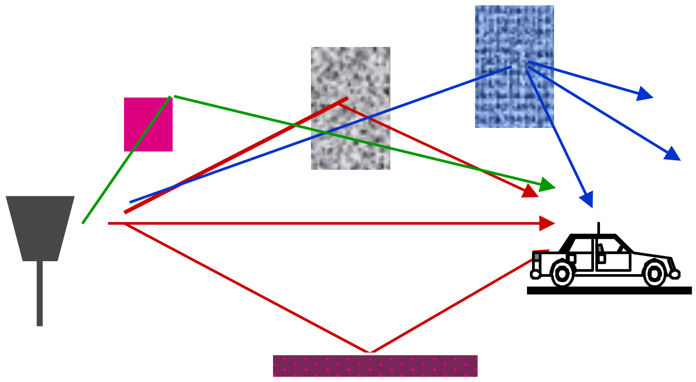
$$p(P_r(d) \leq P_{\min}) = 1 - Q\left(\frac{P_{\min} - (P_t + 10\log_{10} K - 10\gamma\log_{10}(d/d_0))}{\sigma_{\psi_{\mathrm{dB}}}}\right)$$

Model Parameters from Empirical Measurements



- Fit model to data
- Path loss (K,γ), d_0 known:
  - "Best fit" line through dB data
  - K obtained from measurements at d_0.
  - Exponent is Minimal Mean Square Error (MMSE) estimate based on data
  - Captures mean due to shadowing
- Shadowing variance
  - Variance of data relative to path loss model (straight line) with MMSE estimate for γ

Statistical Multipath Model



- Random # of multipath components, each with
  - Random amplitude
  - Random phase
  - Random Doppler shift
  - Random delay
- Random components change with time
- Leads to time-varying channel impulse response

Time Varying Impulse Response

- Response of channel at t to impulse at t-τ:

$$c(\tau,t)=\sum_{n=1}^{N}\alpha_n(t)e^{-j\varphi_n(t)}\delta(\tau-\tau_n(t))$$

  - t is time when impulse response is observed
  - t-τ is time when impulse put into the channel
  - τ is how long ago impulse was put into the channel for the current observation
    - path delay for multipath component currently observed

Received Signal Characteristics

- Received signal consists of many multipath components
- Amplitudes change slowly
- Phases change rapidly
  - Constructive and destructive addition of signal components
  - Amplitude fading of received signal (both wideband and narrowband signals)

Narrowband Model

- Assume delay spread $\max_{m,n}|\tau_n(t)-\tau_m(t)| << 1/B$
- Then $u(t) \approx u(t-\tau)$.
- Received signal given by

$$r(t) = \Re\left\{ u(t) e^{j2\pi f_c t} \left[\sum_{n=0}^{N(t)} \alpha_n(t) e^{-j\phi_n(t)} \right] \right\}$$

- No signal distortion (spreading in time)
- Multipath affects complex scale factor in brackets.
- Assess scale factor by setting $u(t) = e^{j\phi_0}$ (that is, an unmodulated carrier with random phase offset ϕ_0)

In-Phase and Quadrature

under Central Limit Theorem Approximation

- **In phase and quadrature signal components:**

$$r_I(t) = \sum_{n=0}^{N(t)} \alpha_n(t) e^{-j\phi_n(t)} \cos(2\pi f_c t),$$

$$r_Q(t) = \sum_{n=0}^{N(t)} \alpha_n(t) e^{-j\phi_n(t)} \sin(2\pi f_c t)$$

- **For $N(t)$ large, $r_I(t)$ and $r_Q(t)$ jointly Gaussian by CLT (sum of large # of random variables).**
- **Received signal characterized by its mean, autocorrelation, and cross correlation.**
- **If $\varphi_n(t)$ uniform, the in-phase/quad components are mean zero, independent, and stationary.**

Signal Envelope Distribution

- CLT approx. leads to Rayleigh distribution (power is exponential)
- When LOS component present, Ricean distribution is used
- Measurements support Nakagami distribution in some environments
  - Similar to Ricean, but models "worse than Rayleigh"
  - Lends itself better to closed form BER expressions

Signal Envelope Distribution

- **Rayleigh distribution (envelope)**

$$p_Z(z) = \frac{2z}{P_r}\exp[-z^2/P_r] = \frac{z}{\sigma^2}\exp[-z^2/(2\sigma^2)], \quad x \geq 0,$$

  - **in which $P_r = 2\sigma^2$ is the average received signal power of the signal, i.e. the received power based on path loss and shadowing alone**

- **Rayleigh distribution (power)**

$$p_{Z^2}(x) = \frac{1}{P_r}e^{-x/P_r} = \frac{1}{2\sigma^2}e^{-x/(2\sigma^2)}, \quad x \geq 0$$

Signal Envelope Distribution

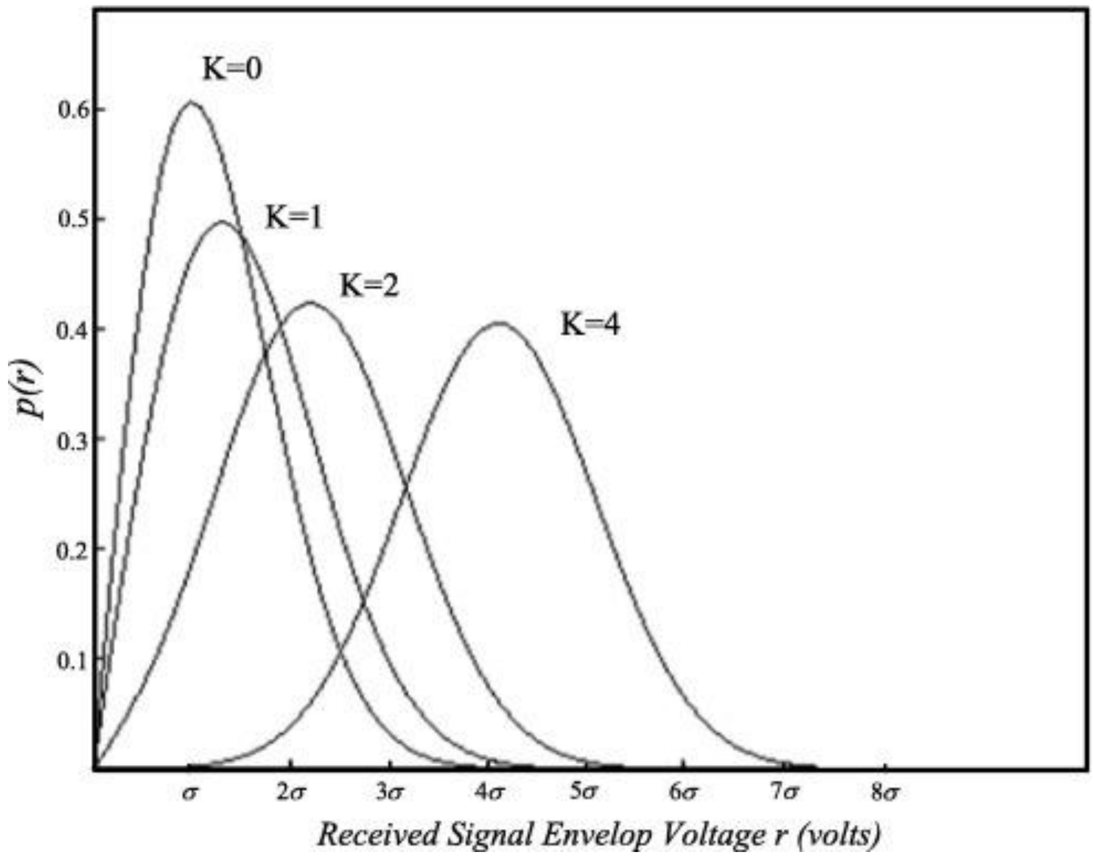
- **Rice distribution (envelope)**

$$f(x) = \frac{2(K+1)x}{\Omega} \exp\left(-K - \frac{(K+1)x^2}{\Omega}\right) I_0\left(2\sqrt{\frac{K(K+1)}{\Omega}}x\right)$$

  - **in which *K* is the ratio between the power in the direct path and the power in the scattered paths, and Ω is the total power from both paths**
  - **If *K* = 0, Rice simplifies to Rayleigh**

Signal Envelope Distribution

- Rice distribution (envelope)



Signal Envelope Distribution

- **Nakagami distribution (envelope)**

$$f(x;\, m, \Omega) = \frac{2m^m}{\Gamma(m)\Omega^m} x^{2m-1} \exp\left(-\frac{m}{\Omega} x^2\right)$$

  - **in which *m* is the fading intensity (m ≥ 0.5), and Ω is a parameter related to the variance**
  - **If *m* = 1, Nakagami simplifies to Rayleigh**

Signal Envelope Distribution

- **Nakagami distribution (envelope)**